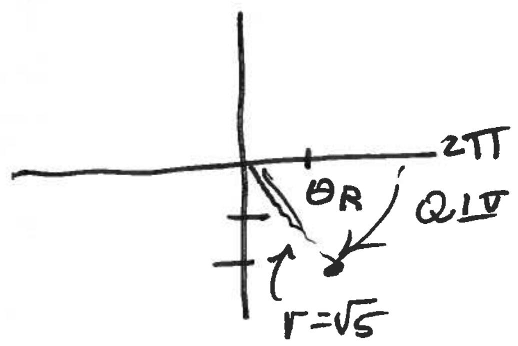
Homework Review

10.3 # 6b) (i) Find polar coordinates (r, θ) of the point where $r > 0$ and $0 \leq \theta < 2\pi$.

(ii) Find polar coordinates (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$.

$(1, -2)$ $\quad x = 1, \; y = -2$

SOLUTION (i) Find r



$$r^2 = x^2 + y^2$$
$$r^2 = (1)^2 + (-2)^2 = 1 + 4 = 5$$
$$r = \sqrt{5}$$

Find θ

$$\tan\theta = \frac{y}{x} = \frac{-2}{1} = -2$$

First find θ_R.

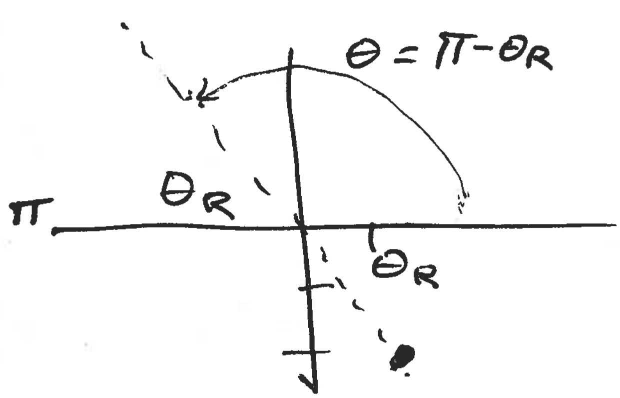
$$\tan\theta_R = 2$$
$$\theta_R = \tan^{-1}(2)$$

notice that negative is dropped

$$\theta = 2\pi - \theta_R \qquad \theta = 2\pi - \tan^{-1}(2)$$

(or $\theta = \tan^{-1}(-2)$)

(i) $(\sqrt{5}, 2\pi - \tan^{-1}(2))$

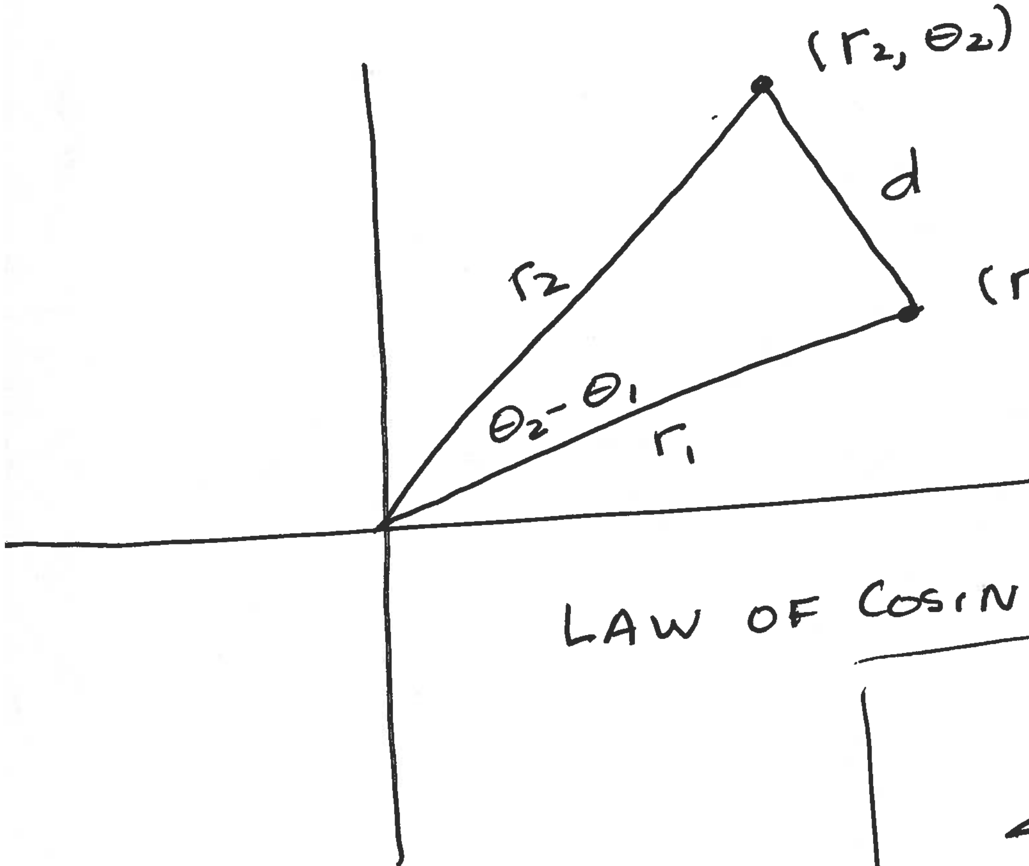


(ii) $r = -\sqrt{5}$

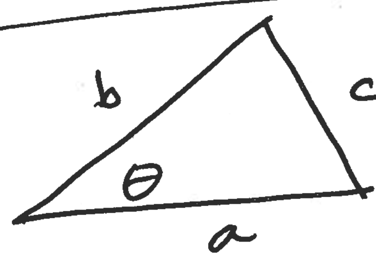
$$\theta = \pi - \theta_R$$
$$= \pi - \tan^{-1}(2)$$

$(-\sqrt{5}, \pi - \tan^{-1}(2))$

10.3 #14 Find a formula for the distance between points with polar coordinates (r_1, θ_1), and (r_2, θ_2).



LAW OF COSINES



$c^2 = a^2 + b^2 - 2ab\cos\theta$

$$d^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)$$

$$d = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos(\theta_2 - \theta_1)}$$

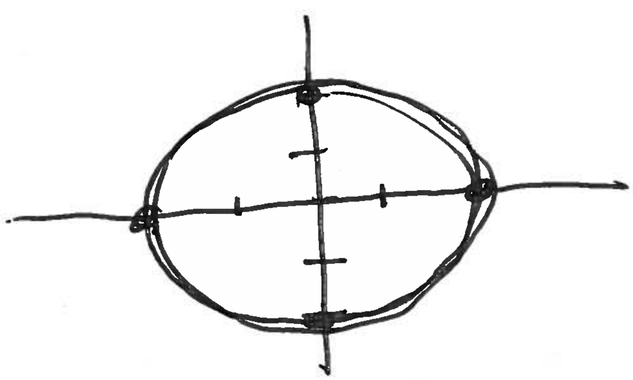
§10.3 Continued.

The Graphs of Curves in Polar Coordinates.

A polar curve can be given by a function $r = f(\theta)$.

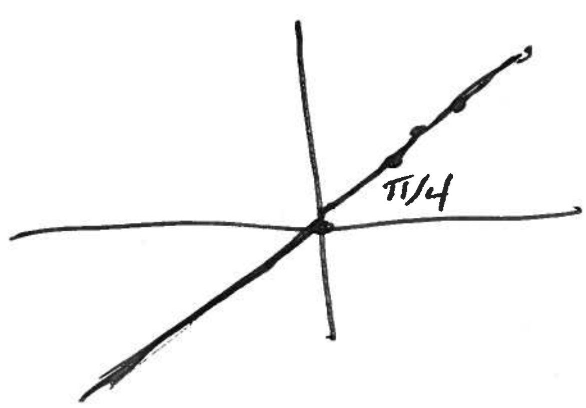
__EXAMPLE__ The equation of a circle of radius $r=2$ centered at the pole ~~origin~~ is

$$\boxed{r = 2.}$$

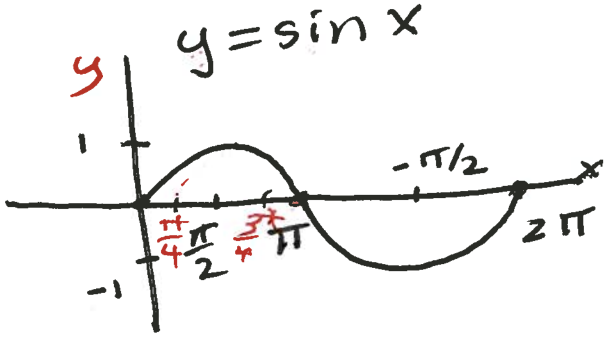


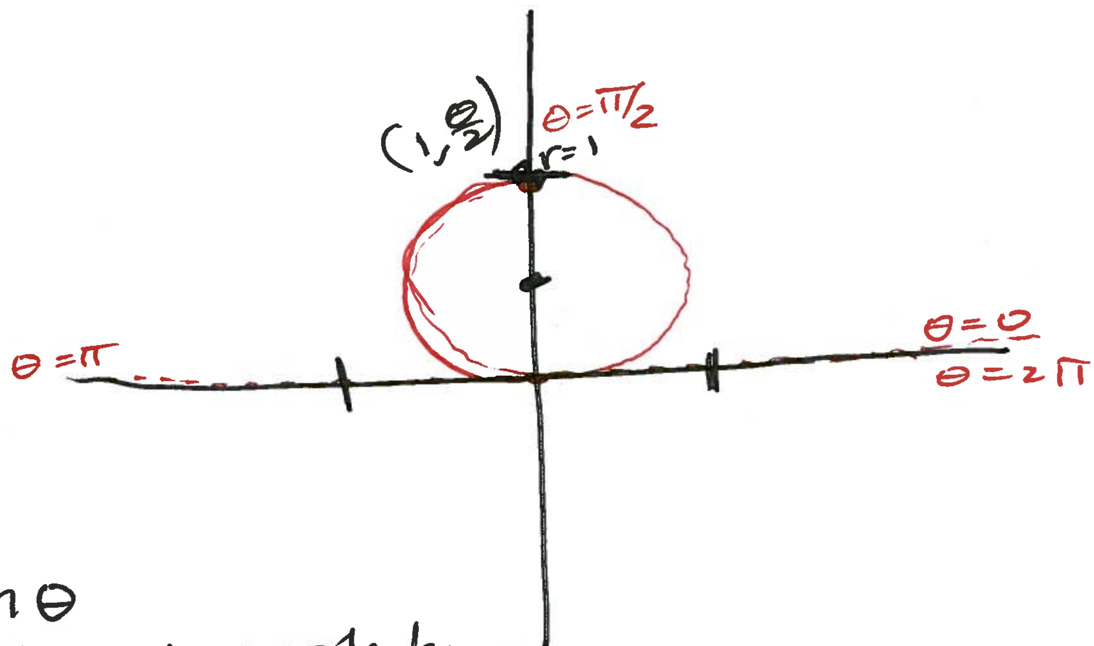
__Example__: A line through the ~~origin~~ pole forming an angle $\theta = \pi/4$ with the x-axis.

$$\boxed{\theta = \frac{\pi}{4}}$$



EXAMPLE: $r = \sin\theta$





OR

$r = \sin\theta$

multiply through by r

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

$$x^2 + (y^2 - \underbrace{y}_{\to\left(\frac{-1}{2}\right)^2} + \tfrac{1}{4}) = 0 + \tfrac{1}{4}$$

$$x^2 + (y - \tfrac{1}{2})^2 = \left(\tfrac{1}{2}\right)^2$$

center $(0, \frac{1}{2})$, $r = \frac{1}{2}$

Standard FORM For circle

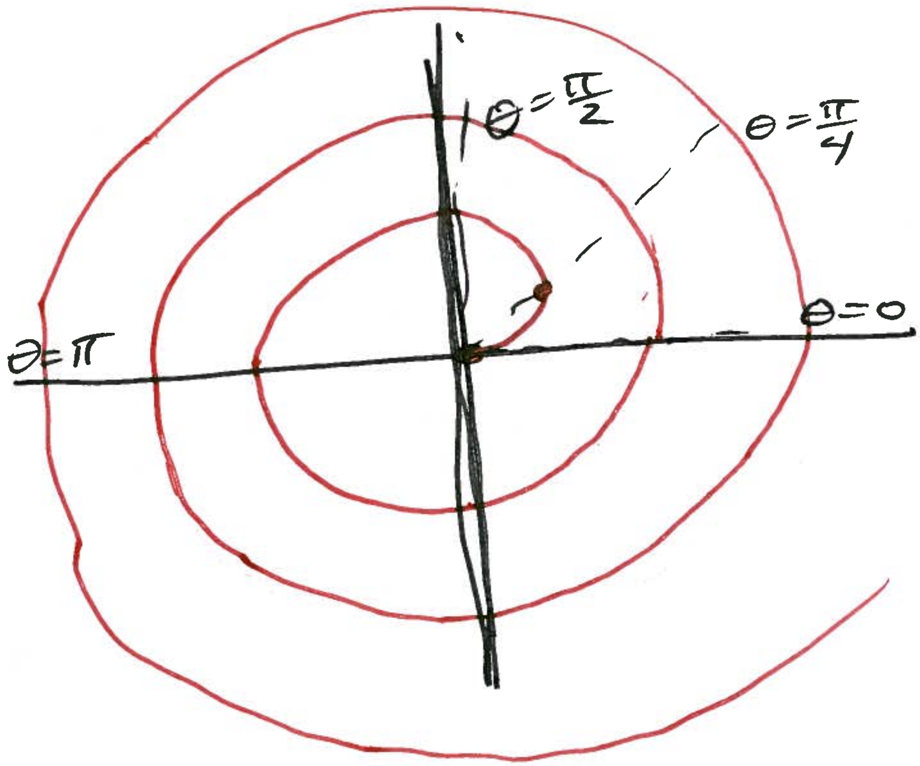
$(x-h)^2 + (y-k)^2 = r^2$

center (h, k)

radius r

EXAMPLE Archimedian Spiral

$r = \theta, \quad \theta \geq 0$

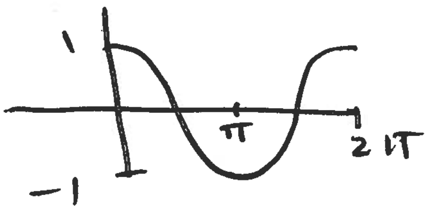


EXAMPLE   Limaçon

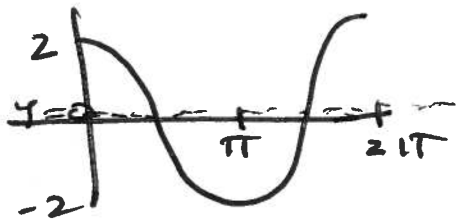
$$r = 3 + 2\cos\theta$$

Scratch work

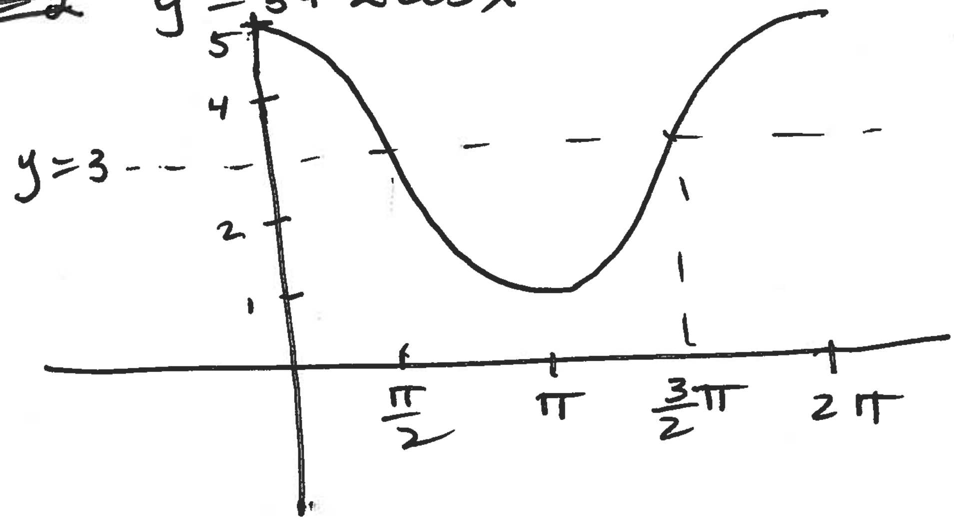
$y = \cos x$



$y = 2\cos x$



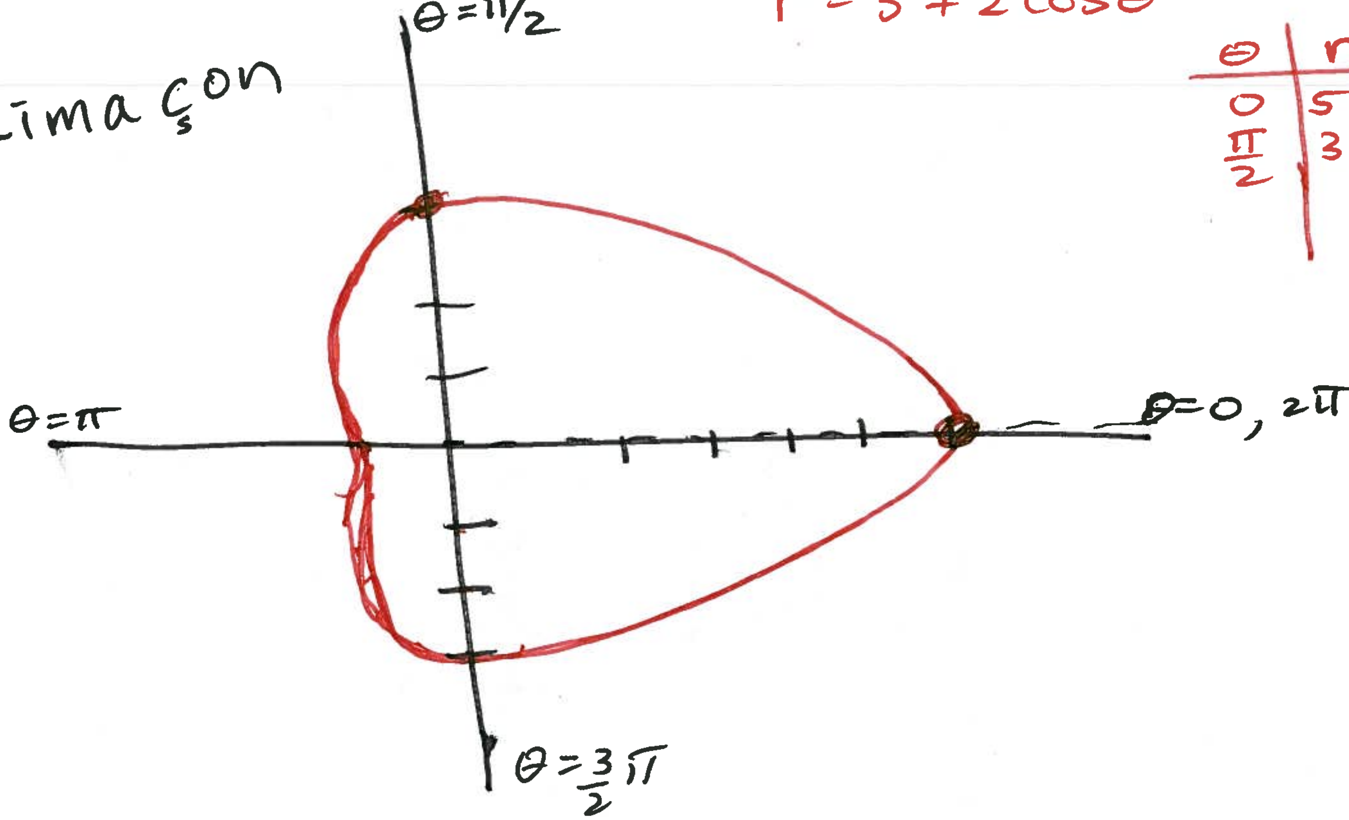
~~$y = 2$~~ $y = 3 + 2\cos x$



Limaçon

$r = 3 + 2\cos\theta$

θ	r
0	5
$\frac{\pi}{2}$	3

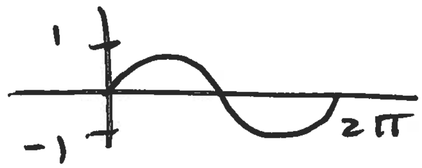


Example: A cardoid

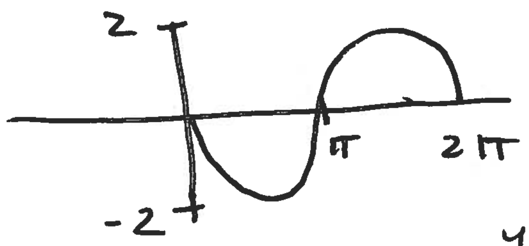
$$r = 2 - 2\sin\theta$$

SOLUTION

$y = \sin x$

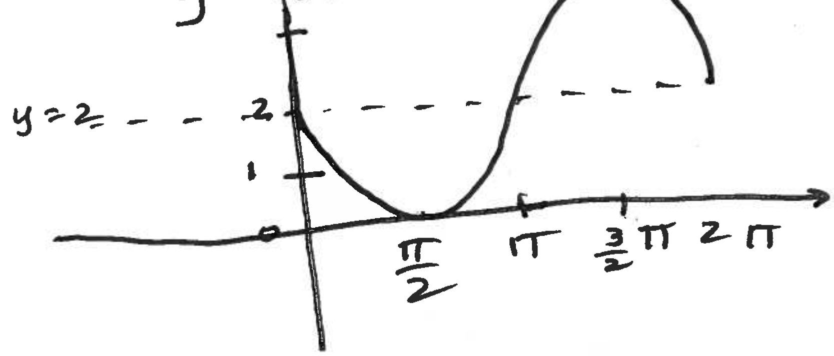


$y = -2\sin x$

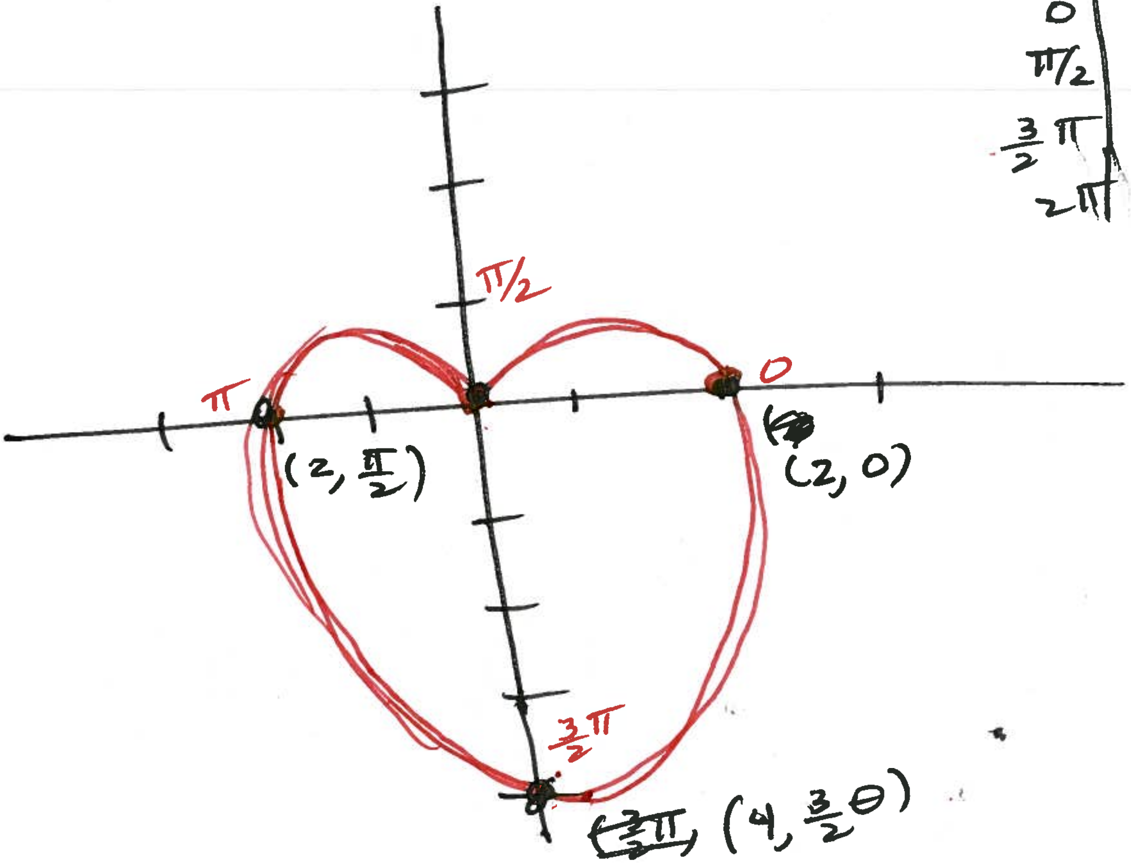


$y = -2\sin(x) + 2$

$y = 2 - 2\sin x$



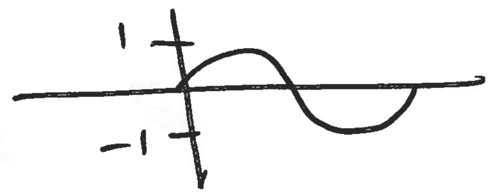
θ	r
0	2
$\pi/2$	0
$\frac{3}{2}\pi$	4
2π	2



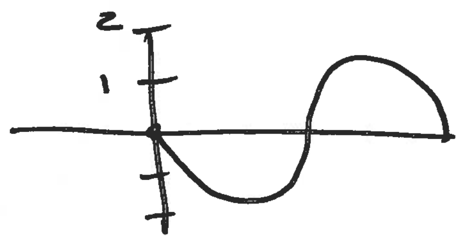
EXAMPLE A Limaçon with a Loop.

$$r = 1 - 2\sin\theta$$

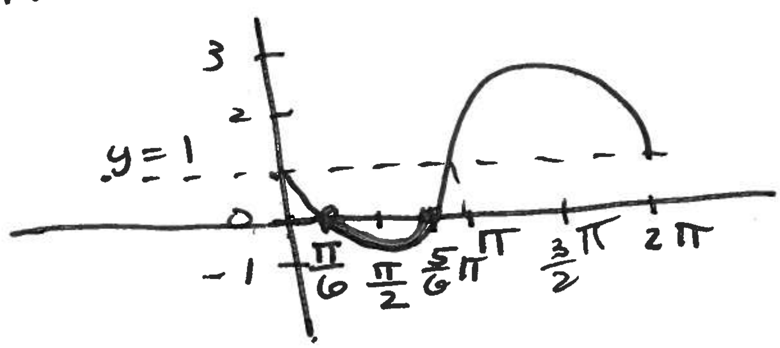
$y = \sin x$



$y = -2\sin x$



$y = 1 - 2\sin x$



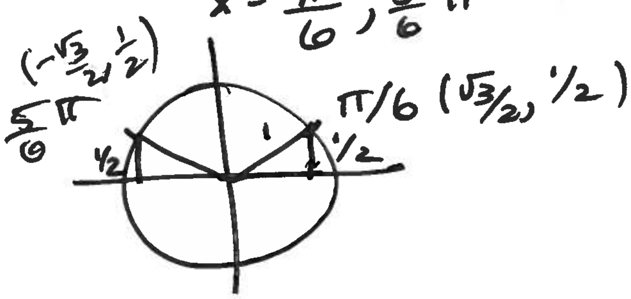
x-int

$1 - 2\sin x = 0$

$1 = 2\sin x$

$\sin x = \frac{1}{2}$

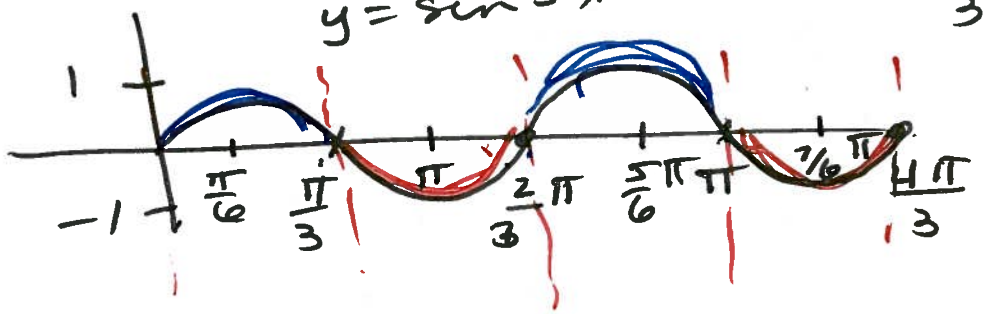
$x = \frac{\pi}{6}, \frac{5}{6}\pi$

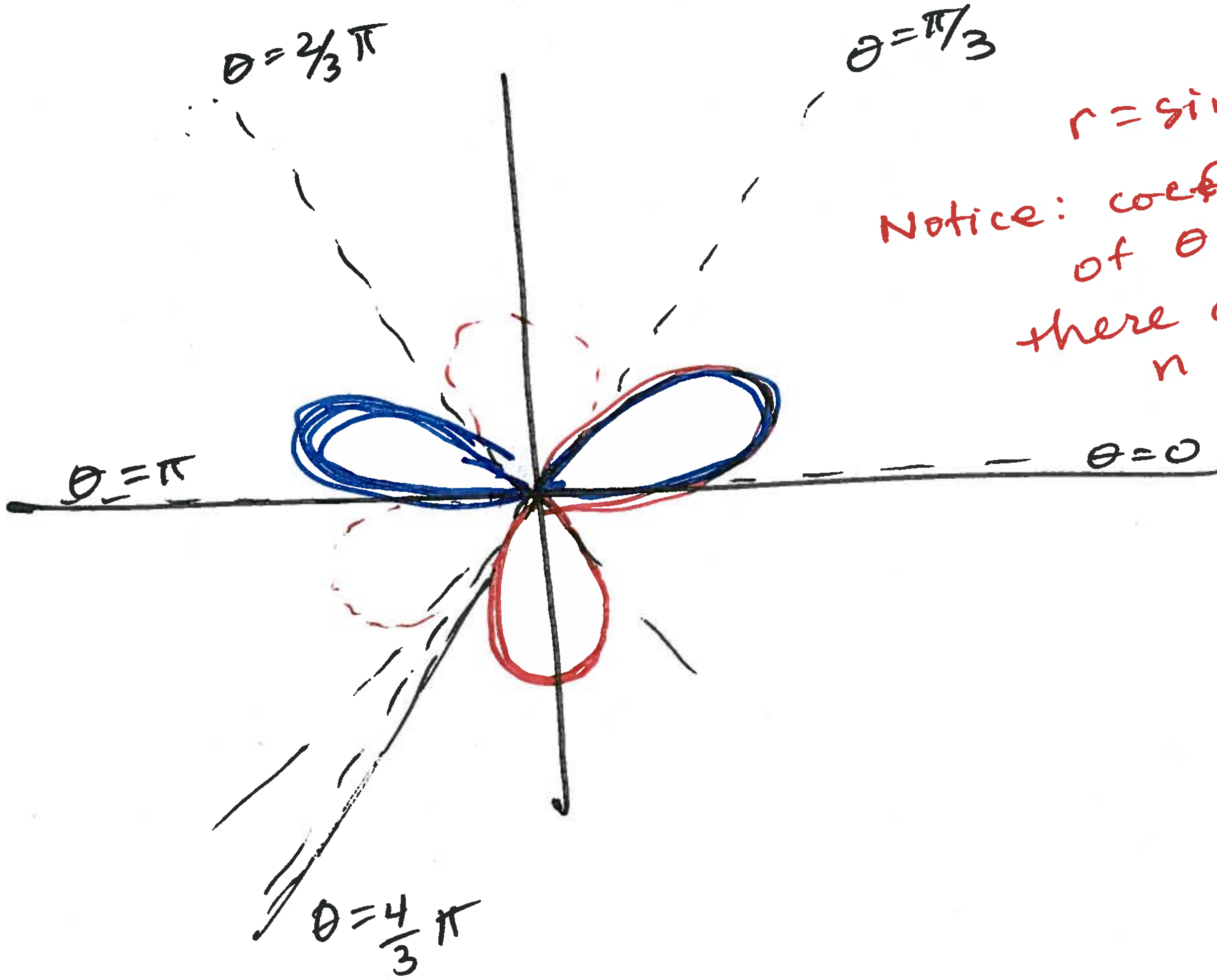


EXAMPLE

$r = \sin 3\theta$

$P = \frac{2\pi}{3}$





$r = \sin n\theta$

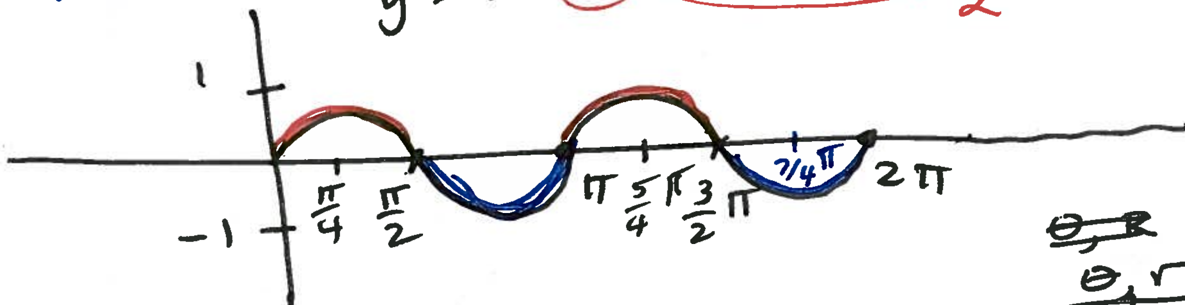
Notice: coefficient of θ is odd, there are n pedals.

EXAMPLE A Four-Leaf Rose

$r = \sin 2\theta$

$r = \sin n\theta$
n is even
2n pedals

$y = \sin 2x \qquad P = \frac{2\pi}{2} = \pi$



~~θ, R~~	
θ	r
0	0
$\frac{\pi}{4}$	1
$\frac{\pi}{2}$	0
$\frac{3}{4}\pi$	-1