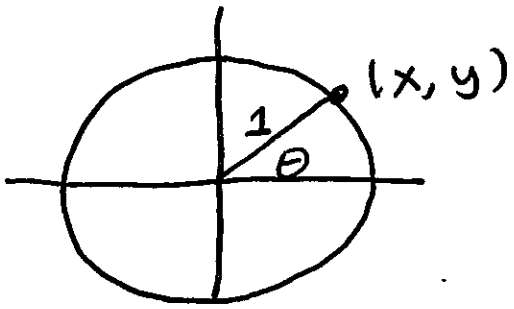
§10.3 Polar Coordinates

HW §10.3 # 1-47 odd

The unit circle

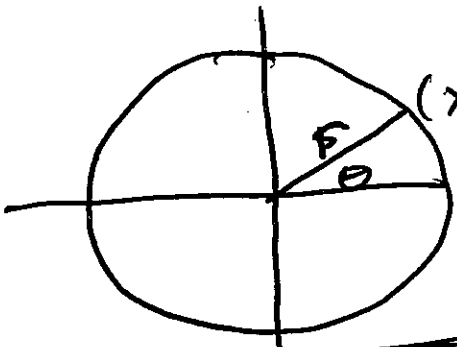


$x = \cos\theta$

$y = \sin\theta$

$r = 1$

The circle of radius r



$$x = r\cos\theta$$
$$y = r\sin\theta$$

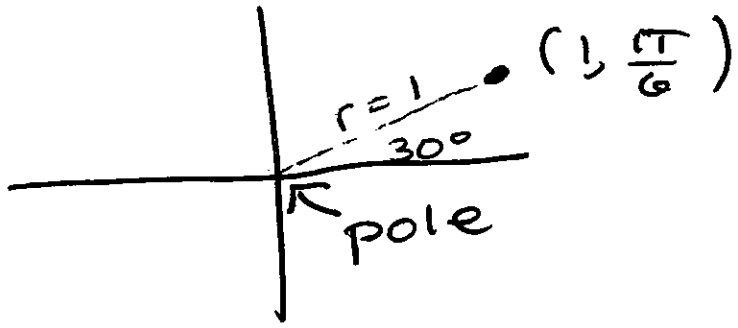
$$x^2 + y^2 = r^2$$

A point $P(x, y)$ can be expressed in polar coordinates (r, θ).

Example: Plot the points in polar coordinates (r, θ).

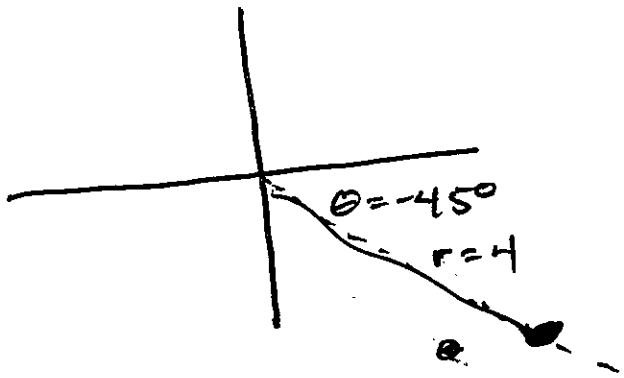
① $(1, \pi/6)$

$r = 1, \quad \theta = \pi/6 = 30°$

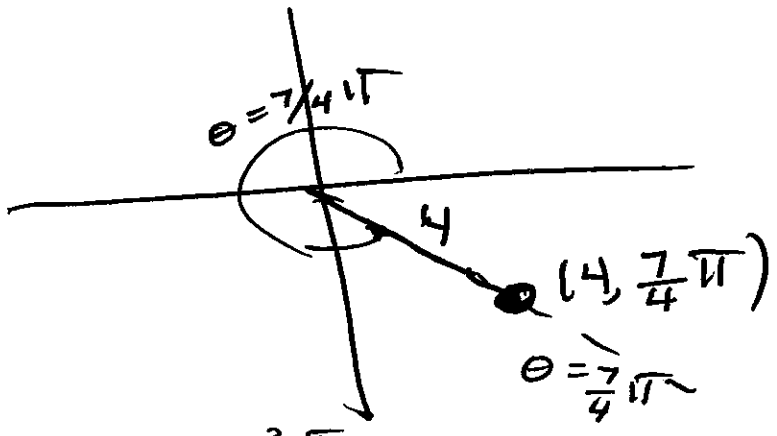


② $\left(4, -\frac{\pi}{4}\right)$

$\theta = -45°$



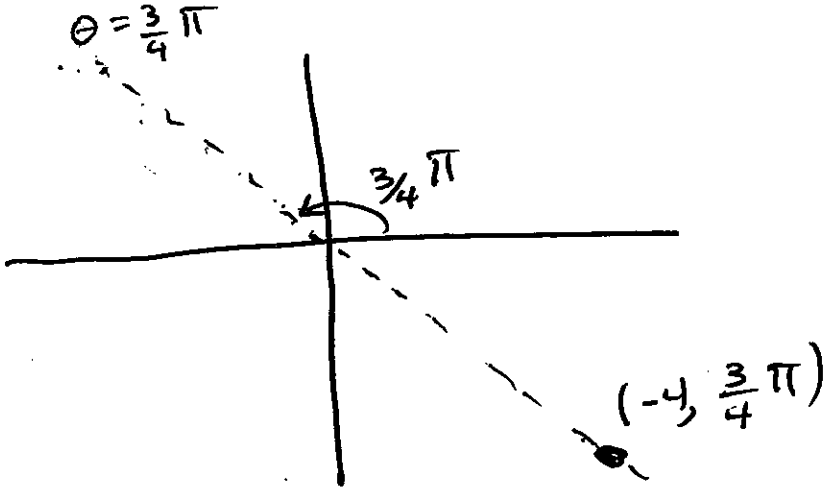
③ $\left(4, \frac{7}{4}\pi\right)$



④ $\left(-4, \frac{3}{4}\pi\right)$

$r = -4, \quad \theta = \frac{3}{4}\pi$

↑ move the opposite direction through the pole



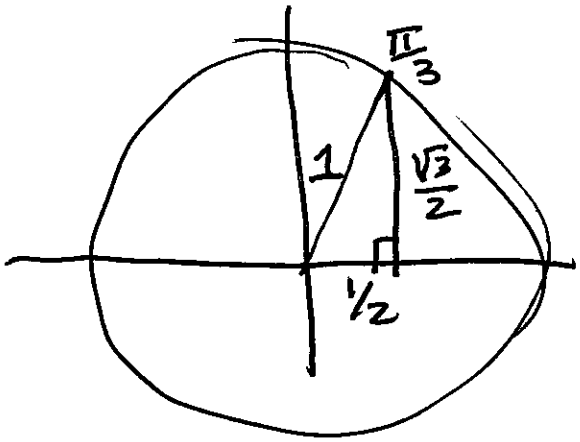
Example: Convert from polar coordinates (r, θ) to rectangular coordinates.

① $\left(1, \frac{\pi}{3}\right)$ $\quad r = 1, \ \theta = \pi/3$

$x = r\cos\theta = 1 \cdot \cos \pi/3 = \frac{1}{2}$

$y = r\sin\theta = 1 \cdot \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$

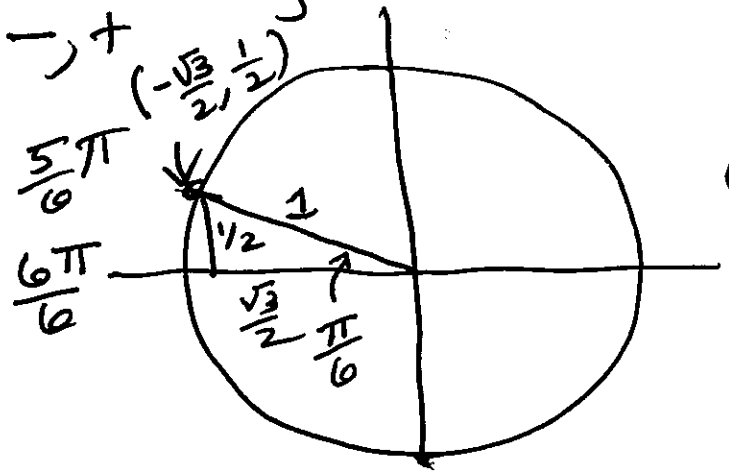
$\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$



② $\left(4, \frac{5\pi}{6}\right)$, $\quad r = 4, \ \theta = \frac{5\pi}{6}$

$x = r\cos\theta = 4\cos\frac{5\pi}{6} = 4\left(-\frac{\sqrt{3}}{2}\right) = -2\sqrt{3}$

$y = r\sin\theta = 4\sin\frac{5\pi}{6} = 4\left(\frac{1}{2}\right) = 2$



$\theta_R = 30°$

$\cos 30° = \frac{\sqrt{3}}{2}$

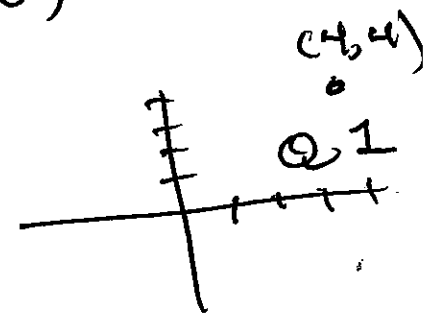
$\sin 30° = \frac{1}{2}$

Example Convert from rectangular coordinates (x, y) to polar coordinates (r, θ).

① $(4, 4)$. $x = 4, y = 4$

$x = r\cos\theta$
$y = r\sin\theta$

Find (r, θ)



$x^2 + y^2 = r^2$ Find r

$(4)^2 + (4)^2 = r^2$

$16 + 16 = r^2$

$r^2 = 32$

$r = \sqrt{32}$

$r = \sqrt{16 \cdot 2}$

$\boxed{r = 4\sqrt{2}}$

Find θ

$x = r\cos\theta$
$y = r\sin\theta$

$4 = 4\sqrt{2}\cos\theta$
$4 = 4\sqrt{2}\sin\theta$

$\cos\theta = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\sin\theta = \frac{4}{4\sqrt{2}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\theta = \cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$

$\theta = 45^\circ$

$\theta = \frac{\pi}{4}$

Answer $\left(4\sqrt{2}, \frac{\pi}{4}\right)$

Note that

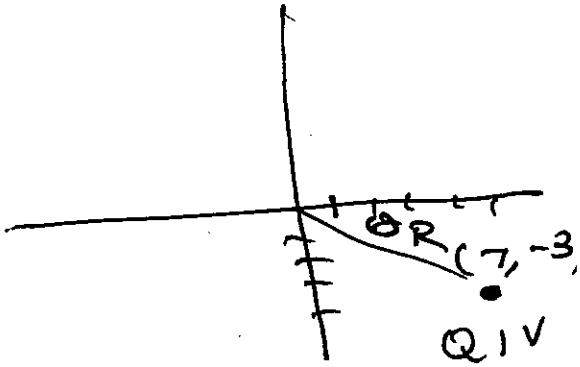
$$\frac{y}{x} = \frac{r\sin\theta}{r\cos\theta} = \tan\theta$$

Formula $\boxed{\tan\theta = \frac{y}{x}}$

(2) $(7, -3)$ $x = 7$, $y = -3$

Find r

$$x^2 + y^2 = r^2$$
$$(7)^2 + (-3)^2 = r^2$$
$$49 + 9 = r^2$$
$$r^2 = 58$$
$$\boxed{r = \sqrt{58}} = \sqrt{2 \cdot 29}$$



Find θ $\tan\theta = \frac{y}{x}$

$$\tan\theta = \frac{-3}{7}$$

First, find the reference angle.

$$\tan\theta_R = \frac{3}{7}$$
$$\theta_R = \tan^{-1}\left(\frac{3}{7}\right)$$

↑ I throw away the negative to get the reference angle.

$$\boxed{\theta = -\tan^{-1}\left(\frac{3}{7}\right)}$$

$$(r, \theta) = \left(\sqrt{58}, -\tan^{-1}\left(\frac{3}{7}\right)\right)$$

(3) Cartesian coord $(-3, -5)$.

Find polar coord.

Find r $\quad r^2 = x^2 + y^2$

$r^2 = (-3)^2 + (-5)^2 = 9 + 25$

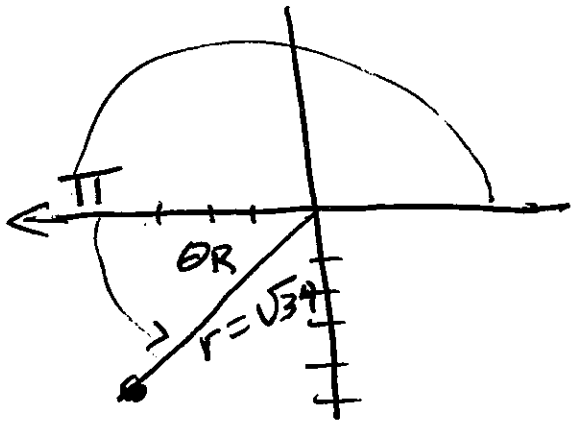
$r^2 = 34$

$\boxed{r = \sqrt{34}}$

Find θ $\quad \tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-5}{-3} = \frac{5}{3}$

$\theta_R = \tan^{-1}\left(\frac{5}{3}\right)$



$\theta = \pi + \tan^{-1}\left(\frac{5}{3}\right)$

$\boxed{\left(\sqrt{34},\ \pi + \tan^{-1}\left(\frac{5}{3}\right)\right)}$