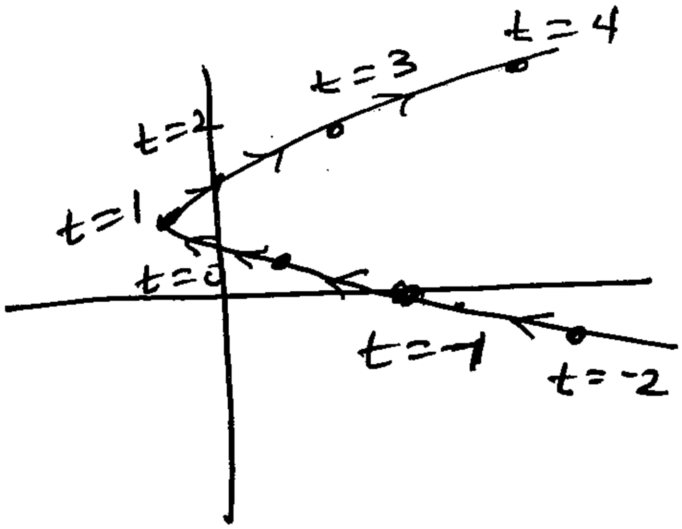
§10.1 Curves Defined by Parametric Equations.

§10.1 #1-22

<u>Example</u>: Sketch and identify the curve defined by the parametric equations

$$x = t^2 - 2t, \quad y = t + 1$$

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	$\cancel{0}$ -1	$\cancel{1}$ 2
2	$\cancel{1}$ 0	$\cancel{2}$ 3
3	3	4
4	8	5



We can <u>eliminate</u> <u>the parameter</u>.

$x = t^2 - 2t,$ $\quad y = t + 1$

- solve for t

  $t = y - 1$

- put $t = y - 1$ into the formula for x.

$$x = (y-1)^2 - 2(y-1)$$

$$x = y^2 - 2y + 1 - 2y + 2$$

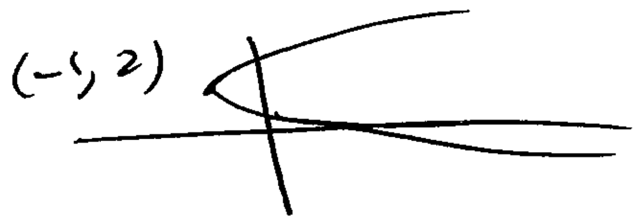
$$x = y^2 - 4y + 3$$

complete the square

$x = (y^2 - 4y + 4) - 4 + 3$

$x = (y-2)^2 - 1$

vertex $(-1, 2)$



<u>Example</u>: • A circle can be given by parametric equations

$x = \cos t, \quad y = \sin t$

$x^2 + y^2 = \cos^2 t + \sin^2 t = 1$

$x^2 + y^2 = 1$ is a circle with center origin, $r = 1$.

• Circle with radius r.

$x = r\cos t, \qquad y = r\sin t$

$$\begin{aligned} x^2 + y^2 &= (r\cos t)^2 + (r\sin t)^2 \\ &= r^2\cos^2 t + r^2 \sin^2 t \\ &= r^2(\cos^2 t + \sin^2 t) \\ &= r^2(1) = r^2. \end{aligned}$$

Example An ellipse:

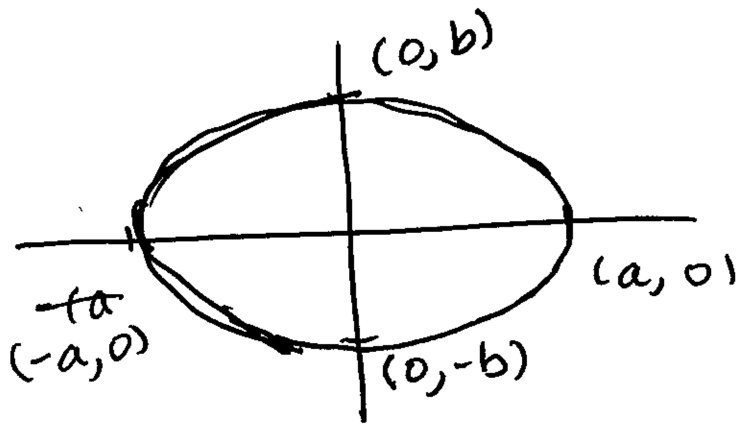
$$x = a\cos t, \quad y = b\sin t.$$

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{(a\cos t)^2}{a^2} + \frac{(b\sin t)^2}{b^2}$$

$$= \frac{a^2\cos^2 t}{a^2} + \frac{b^2\sin^2 t}{b^2}$$

$$= \cos^2 t + \sin^2 t = 1$$

So $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$



x-intercepts

$y=0$

$\frac{x^2}{a^2} + 0 = 1$

$x^2 = a^2$

$x = \pm a$

y-int

$x=0$

$0 + \frac{y^2}{b^2} = 1$

$y^2 = b^2$

$y = \pm b$

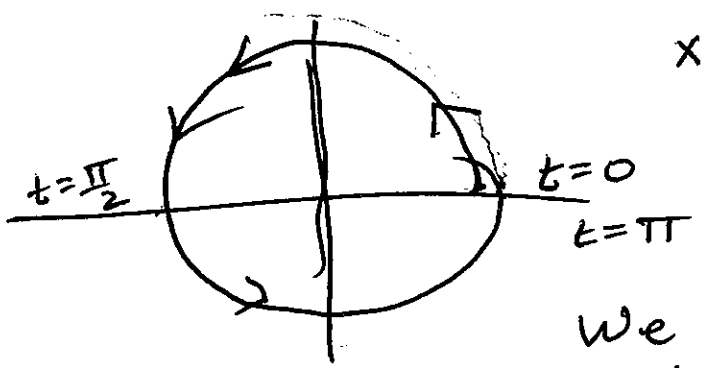
__Example__: What curve do equations

$x = \cos 2t, \quad y = \sin 2t$

give.

__SOLUTION__

$$x^2 + y^2 = (\cos 2t)^2 + (\sin 2t)^2$$
$$= \cos^2 2t + \sin^2 2t$$
$$= 1$$



$x^2 + y^2 = 1$

We go around the circle twice as fast.

Example Find the parametric equations for a circle centered at (h,k) with radius r.

SOLUTION: The circle with radius r centered at the origin

$$x = r\cos t, \quad y = r\sin t.$$

To ~~give~~ make the center (h,k) we shift x by h and y by k.

$$x = h + r\cos t, \quad y = k + r\sin t$$

Check it out:

$$\begin{aligned}
&(x-h)^2 + (y-k)^2 \\
&= ((h + r\cos t) - h)^2 + ((k + r\sin t) - k)^2 \\
&= (r\cos t)^2 + (r\sin t)^2 \\
&= r^2(\cos^2 t + \sin^2 t) = r^2(1) = r^2
\end{aligned}$$

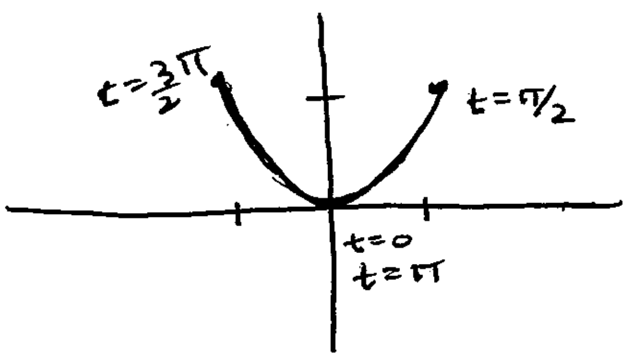
$(x-h)^2 + (y-k)^2 = r^2$ is a circle with center (h,k), radius r.

EXAMPLE Sketch the curve with parametric equations $x = \sin t$, $y = \sin^2 t$.

SOLUTION:

$$y = \sin^2 t = (\sin t)^2 = (x)^2$$

$$y = x^2$$



Note

$-1 \le \sin t \le 1$

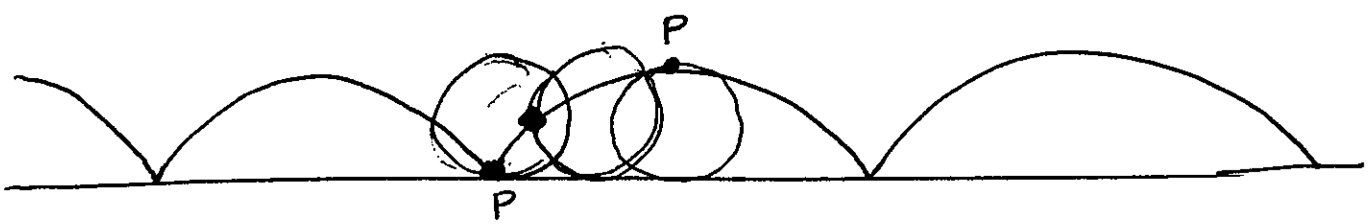
$-1 \le x \le 1$

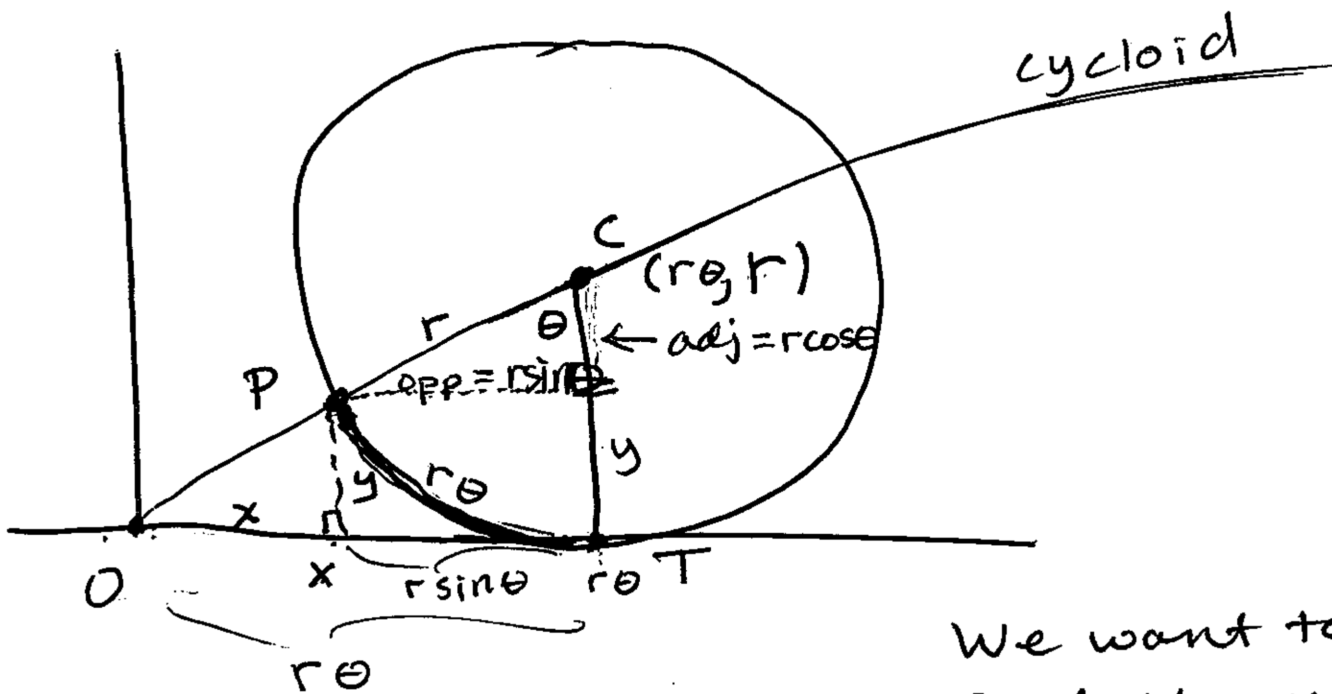
t	x	y
0	0	0
$\frac{\pi}{2}$	$\sin \frac{\pi}{2} = 1$	1
π	$\sin \pi = 0$	0
$\frac{3}{2}\pi$	$\sin \frac{3}{2}\pi = -1$	1
2π	$\sin 2\pi = 0$	0

The Cycloid

The curve traced by a point P on the circumference of a circle as the circle rolls along a straight line is called a <u>cycloid</u>. If the radius is r, and one point of P is the origin, we get parametric equations

$$x = r(\theta - \sin\theta) \quad y = r(1 - \cos\theta)$$
$$\theta \text{ in } \mathbb{R}$$





We want to find the x- and y-coord. of P.

Assume radius r

$\sin\theta = \dfrac{opp}{hyp}$, $opp = (\sin\theta)(hyp)$
$= r\sin\theta$

$\cos\theta = \dfrac{adj}{hyp}$

$adj = (hyp)\cos\theta$
$= r\cos\theta$

$$x = r\theta - r\sin\theta$$
$$x = r(\theta - \sin\theta)$$

$$y = r - r\cos\theta$$
$$y = r(1 - \cos\theta)$$