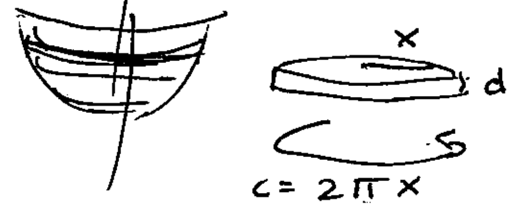
§8.2 #16 The given curve is rotated about the y-axis. Find the area of the resulting surface.

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \le x \le 2$$

Aside



• $A = \int 2\pi x \, ds$

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

SOLUTION

$$A = \int 2\pi x \, ds$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A = \int 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{1}{4}(2x) - \frac{1}{2}\frac{1}{x}$$

$$= \frac{1}{2}x - \frac{1}{2x}$$

$$A = \int_1^2 2\pi x \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{1 + \left(\frac{x^2}{4} - 2\left(\frac{x}{2}\right)\left(\frac{1}{2x}\right) + \frac{1}{4x^2}\right)}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}}\, dx = 2\pi \int x \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2}\, dx$$

$$= 2\pi \int_1^2 x \left(\frac{x}{2} + \frac{1}{2x}\right) dx = 2\pi \int_1^2 \left(\frac{x^2}{2} + \frac{1}{2}\right) dx$$

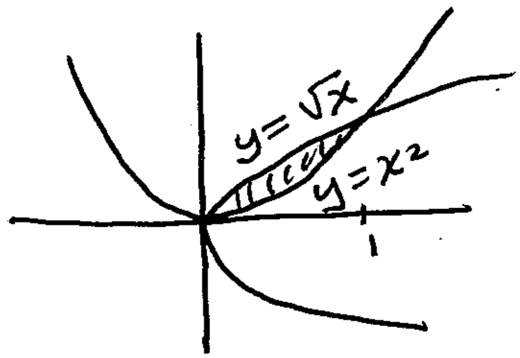
$$= 2\pi \left[\frac{x^3}{2 \cdot 3} + \frac{x}{2}\right]_1^2 = 2\pi \left[\frac{2^3}{6} + \frac{2}{2}\right] - 2\pi \left[\frac{1^3}{6} + \frac{1}{2}\right]$$

$$= 2\pi \left[\frac{8}{6} + 1 - \frac{1}{6} - \frac{1}{2}\right] = 2\pi \left[\frac{8}{6} + \frac{6}{6} - \frac{1}{6} - \frac{3}{6}\right] = 2\pi\left(\frac{10}{6}\right) = \frac{10}{3}\pi$$

$$A = \frac{10}{3}\pi$$

§8.3 #29 Find the centroid of the region bounded by the given curves.

$$y = x^2, \quad x = y^2$$



$$\text{Area} = \int_a^b (f(x) - g(x))\,dx$$

Centroid:

$$\bar{x} = \frac{1}{A}\int_a^b x\,[f(x) - g(x)]\,dx$$

$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}\left[(f(x))^2 - (g(x))^2\right]dx$$

Area

$$A = \int_0^1 (\underset{\text{top}}{x^{1/2}} - \underset{\text{bottom}}{x^2})\,dx$$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^3}{3}\right]_0^1 = \left[\frac{2}{3}(1)^{3/2} - \frac{(1)^3}{3}\right] - \left[\frac{2}{3}(0)^{3/2} - \frac{0^3}{3}\right]$$

$$= \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$$

$$A = \frac{1}{3}$$

$$\bar{x} = \frac{1}{A}\int_a^b x\left[f(x)-g(x)\right]dx$$

$$= \frac{1}{(1/3)}\int_0^1 x(x^{1/2}-x^2)\,dx$$

$$= 3\int_0^1 (x^{3/2}-x^3)\,dx$$

$$= 3\left[\frac{x^{5/2}}{(5/2)} - \frac{x^4}{4}\right]_0^1$$

$$= 3\left[\frac{2}{5}(1)^{5/2} - \frac{(1)^4}{4}\right] - 3\left[\frac{2}{5}(0)^{5/2} - \frac{(0)^4}{4}\right]$$

$$= 3\left[\frac{2}{5}-\frac{1}{4}\right] = 3\left[\frac{8}{20}-\frac{5}{20}\right] = 3\left(\frac{3}{20}\right) = \frac{9}{20}$$

$$\bar{x} = \frac{9}{20}$$

$$\bar{y} = \frac{1}{A}\int_a^b \frac{1}{2}\left[(f(x))^2 - (g(x))^2\right]dx$$

$$= \frac{1}{\left(\frac{1}{3}\right)}\int_0^1 \frac{1}{2}\left[(x^{1/2})^2 - (x^2)^2\right]dx$$

$$= \frac{3}{2}\int_0^1\left[x - x^4\right]dx = \frac{3}{2}\left[\frac{x^2}{2} - \frac{x^5}{5}\right]_0^1$$

$$= \frac{3}{2}\left[\frac{(1)^2}{2} - \frac{(1)^5}{5}\right] - \frac{3}{2}\left[\frac{0^2}{2} - \frac{0^5}{5}\right]$$

$$= \frac{3}{2}\left[\frac{1}{2}-\frac{1}{5}\right] = \frac{3}{2}\left[\frac{5}{10}-\frac{2}{10}\right] = \frac{3}{2}\left(\frac{3}{10}\right) = \frac{9}{20}$$

$$\bar{y} = \frac{9}{20}$$

Centroid $\left(\frac{9}{20}, \frac{9}{20}\right)$

Separable DE's.

§ 9.3 #15 Find the solution of the differential equation that satisfies the given initial conditions

$$\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}, \qquad u(0) = -5$$

SOLUTION

Use multiplication and division to move the u's to the left and the t's to the right.

$$2u\,du = (2t + \sec^2 t)\,dt$$

Integrate

$$\int 2u\,du = \int (2t + \sec^2 t)\,dt$$

$$\cancel{2}\frac{u^2}{\cancel{2}} = \cancel{2}\frac{t^2}{\cancel{2}} + \tan t + C$$

$$u^2 = t^2 + \tan t + C$$

Solve for C when $t = 0$, $u = -5$

$$(-5)^2 = (0)^2 + \tan(0) + C$$

$$25 = C$$

$$C = 25$$

$$u^2 = t^2 + \tan t + 25$$

$$\boxed{u = -\sqrt{t^2 + \tan t + 25}}$$

We put negative in front of square root because u is neg when $t = 0$

Linear First Order DE's

§9.5 #5 $y' + 2y = 2e^x$

SOLUTION $y' + P(x)y = Q(x)$

$$I = e^{\int P(x)\,dx}$$

- Find the integrating factor, I.

$P(x) = 2$

$$I = e^{\int 2\,dx} = e^{2x}$$

- Multiply through by I

$$e^{2x}y' + 2e^{2x}y = 2e^x \cdot e^{2x}$$

- Write the left hand side as $(Iy)'$

$$(e^{2x}y)' = 2e^{3x}$$

- Integrate both sides.

$$\int (e^{2x}y)'\,dx = \int 2e^{3x}\,dx + C$$

$$e^{2x}y = \frac{2}{3}e^{3x} + C$$

$$y = \frac{2e^{3x} + 3C}{3e^{2x}}$$

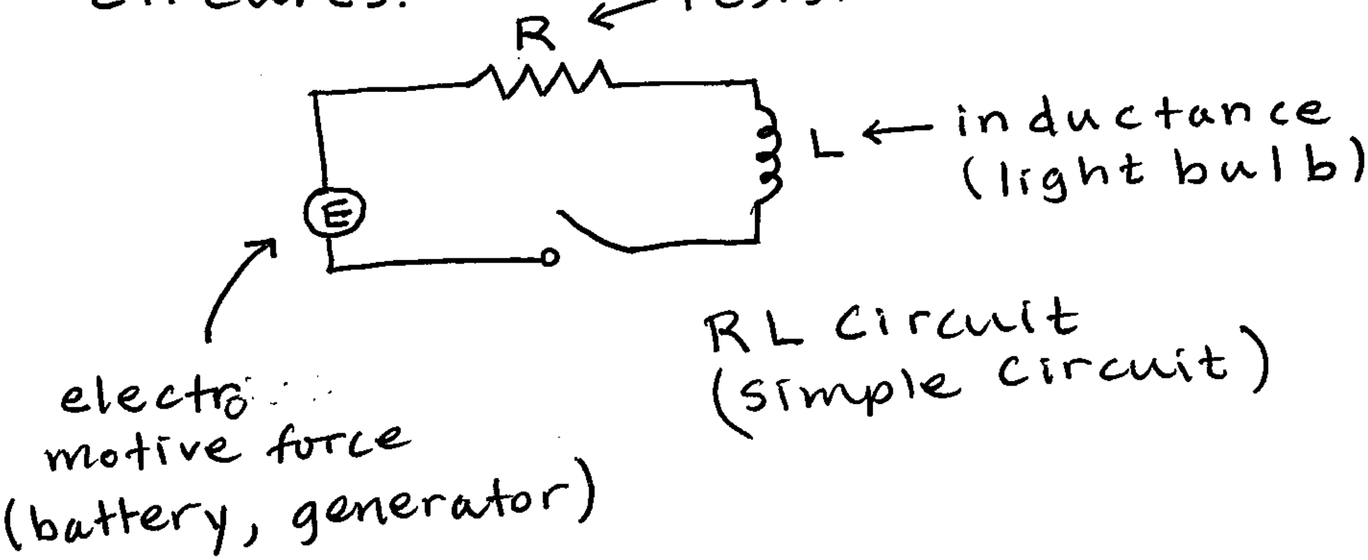
$$y = \frac{2e^{3x}}{3e^{2x}} + \frac{3C}{3e^{2x}}$$

$$y = \frac{2}{3}e^{3x} \cdot e^{-2x} + Ce^{-2x}$$

$$y = \frac{2}{3}e^{x} + Ce^{-2x}$$

§9.5 Notes (continued)

Application to Electric Circuits.



An electromotive force (usually a battery or generator) produces a voltage of $E(t)$ volts (V) and a current of $I(t)$ amperes (A) at time t. The circuit also contains a resistor with a resistance of R ohms (Ω) and an inductor with an inductance of L henries (H).

The system is governed by the equation

$$L \frac{dI}{dt} + RI = E(t)$$

This is a first order linear DE in $I(t)$.

p. 605 Stewart.

EXAMPLE Suppose that in the simple circuit the resistance is 12 Ω and the inductance is 4 H. If a battery gives a constant voltage of 60 V and the switch is closed when $t=0$, so that $\underset{\text{(current)}}{I(0)=0}$, find

(a) $I(t)$

(b) The current after 1 s

(c) The limiting of the current.

(a) $L=4$, $R=12$, $E(t)=60$

$$L\frac{dI}{dt}+RI=E(t)$$

$$4\frac{dI}{dt}+12I=60, \quad I(0)=0$$

divide through by 4.

$$\frac{dI}{dt}+3I=15$$

• integrating factor $e^{\int 3\,dt}=e^{3x}$

Multiply through by int. factor e^{3x}

$$e^{3t}I' + 3e^{3t}I = 15e^{3t}$$

$$\int(e^{3t}I)'=\int 15e^{3t}\,d$$

$$e^{3t}I = 15\frac{e^{3t}}{3}+C$$

$$e^{3t}I = 5e^{3t}+C$$

$$I = \frac{5e^{3t} + C}{e^{3t}}$$

$$I = 5 + Ce^{-3t}$$

Solve for C.

when $t = 0$, $I = 0$

$$0 = 5 + Ce^{0}$$

$$0 = 5 + C$$

$$C = -5$$

$$I = 5 - 5e^{-3t}$$