§11.8 #19 $\sum_{n=1}^{\infty} \frac{(x-2)^n}{n^n}$

$a_n = \frac{(x-2)^n}{n^n}$

Root Test?

$$\lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\left|\frac{(x-2)^n}{n^n}\right|}$$

$$= \lim_{n\to\infty} \frac{|x-2|}{n} = 0 < 1$$

Conv. for all x.

$R = \infty$

$(-\infty, \infty)$

§11.6 #23 $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^{n^2}$

$$a_n = \left(1+\frac{1}{n}\right)^{n^2}$$

<u>Root Test</u>

$$\lim_{n\to\infty} \sqrt[n]{a_n} = \lim_{n\to\infty} (a_n)^{1/n}$$

$$= \lim_{n\to\infty} \left(\left(1+\frac{1}{n}\right)^{n^2}\right)^{1/n}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^{n} = e > 1$$

Divergent.

§11.6 # 27 $\displaystyle\sum_{n=1}^{\infty} \frac{2\cdot4\cdot6\cdots(2n)}{n!}$

Use Ratio or Root Test to check for Abs convergence, ~~Divergence~~, Conditional convergence, Divergent.

$$a_n = \frac{2\cdot4\cdot6\cdots 2n}{n!}$$

$$a_{n+1} = \frac{2\cdot4\cdot6\cdots 2n\cdot\underbrace{2(n+1)}_{2n+2}}{(n+1)!}$$

$$(n+1)! = (n+1)n!$$

Ratio Test $\displaystyle\lim_{n\to\infty}\left|\frac{a_{n+1}}{a_n}\right| = \lim \frac{(2\cdot4\cdot6\cdots2n)(2n+2)}{(n+1)n!}\cdot\frac{n!}{(2\cdot4\cdot6\cdots2n)}$

$$\lim_{n\to\infty} \frac{2n+2}{n+1} = 2$$

Divergent

Aside:

$$2\cdot4\cdot6\cdots 2n = (2\cdot1)\cdot(2\cdot2)(2\cdot3)\cdots(2n)$$

$$\cancel{= 2^n n} = 2^n\cdot(1\cdot2\cdot3\cdots n)$$

$$= 2^n\, n!$$

$$\sum_{n=1}^{\infty}\frac{2\cdot4\cdot6\cdots2n}{n!} = \sum_{n=1}^{\infty}\frac{2^n\,\cancel{n!}}{\cancel{n!}} = \sum 2^n$$

Geom. $r = 2 > 1$

DIV.

§11.8 #13 $\quad \sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \ln n}$

Ratio Test $\quad a_n = \frac{(-1)^n x^n}{4^n \ln n}$

$$\lim_{n\to\infty} \left| \frac{a_{n+1}}{a_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{(-1)^{n+1} x^{n+1}}{4^{n+1} \ln(n+1)} \cdot \frac{4^n \ln n}{(-1)^n x^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{x^{n+1}}{x^n} \right| \cdot \frac{4^n}{4^{n+1}} \cdot \frac{\ln n}{\ln(n+1)}$$

$$= \lim_{n\to\infty} |x| \cdot \frac{1}{4} \cdot 1 = \frac{|x|}{4}$$

<u>Aside</u>: $\lim_{n\to\infty} \frac{\ln n}{\ln(n+1)} = \lim_{n\to\infty} \frac{\ln x}{\ln(x+1)} \stackrel{L'H}{=} \lim_{x\to\infty} \frac{1/x}{1/(x+1)}$

$$= \lim_{x\to\infty} \frac{x+1}{x} = 1$$

<u>Aside</u>

$\ln\left(\frac{A}{B}\right) = \ln A - \ln B$

$\ln(AB) = \ln A + \ln B$

$\ln A^x = x \ln A$

True or False?

no false.

$\ln 2 = \cancel{\ln(1+)}\ \ln(1+1) \neq \ln 1 + \ln 1 = 0 + 0.$

$f(e) = e > 1$, f incr.

so $f(x) > 1$

$$\frac{x}{\ln x} > 1$$

$$x > \ln x$$

Enpt $x = 4$

$$\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{4^n \ln n} = \sum_{n=1}^{\infty} \frac{(-1)^n \cancel{4^n}}{\cancel{4^n} \ln n}$$

$$= \sum_{n=1}^{\infty} \frac{(-1)^n}{\ln n}$$

Alt. Series Test.

$$b_n = \frac{1}{\ln n}$$

- $\lim_{n \to \infty} b_n = \lim_{n \to \infty} \frac{1}{\ln n} = 0$
- $b_{n+1} \leq b_n$

$$\frac{1}{\ln n+1} \leq \frac{1}{\ln n}$$

$$\ln(n) \leq \ln(n+1)$$

$$n \leq n+1$$

$$0 \leq 1 \quad \checkmark$$

Conv. by A.S.T.

Interval of Convergence $(-4, 4]$

$$-4 < x \leq 4$$

$$R = 4$$

conv. for $\frac{|x|}{4} < 1$

$|x| < 4$ $\boxed{R=4}$

$-4 < x < 4$

<u>Endpts</u>

$x=-4$

$$\sum_{n=2}^{\infty} \frac{(-1)^n x^n}{4^n \ln n} = \sum_{n=2}^{\infty} \frac{(-1)^n(-4)^n}{4^n \ln n}$$

$$= \sum_{n=2}^{\infty} \frac{(-1\cdot -4)^n}{4^n \ln n} = \sum_{n=2}^{\infty} \frac{\cancel{4^n}}{\cancel{4^n} \ln n}$$

$$= \sum_{n=2}^{\infty} \frac{1}{\ln n}$$

$a_n = \frac{1}{\ln n}$, $b_n = \frac{1}{n}$

$\boxed{\cancel{n < \ln n} \quad \cancel{\frac{1}{\ln n} < \frac{1}{n}}}$

~~dead end~~

$\cancel{\lim_{n\to\infty} \frac{a_n}{b_n} = \lim}$

- $\ln n < n$

  $\frac{1}{n} < \frac{1}{\ln n}$

- $\sum \frac{1}{n}$ harmonic series div.

Com Comp Test Div.

<u>Claim</u> (Extra, Optional)

$\ln n < n$, $n \geq 2$

$1 < \frac{n}{\ln n}$

$f(x) = \frac{x}{\ln x}$

$$f'(x) = \frac{(1)\ln x - x\cdot\frac{1}{x}}{(\ln x)^2}$$

$$= \frac{\ln(x) - 1}{(\ln(x))^2} > 0$$

So f is increasing. $f(e) = \frac{e}{\ln e} = \frac{e}{1} = e$

§11.7 #25 $\displaystyle\sum_{n=1}^{\infty} \frac{n!}{e^{n^2}}$

Ratio Test

$$\lim_{n\to\infty} \left| \frac{(n+1)!}{e^{(n+1)^2}} \cdot \frac{e^{n^2}}{n!} \right|$$

$$= \lim_{n\to\infty} \frac{(n+1)!}{n!} \cdot \frac{e^{n^2}}{e^{(n+1)^2}}$$

$$= \lim_{n\to\infty} \frac{(n+1)n!}{n!} \frac{e^{n^2}}{e^{n^2+2n+1}}$$

$$= \lim_{n\to\infty} (n+1) \cdot \frac{\cancel{e^{n^2}}}{\cancel{e^{n^2}} e^{2n+1}}$$

$$= \lim_{n\to\infty} \frac{n+1}{e^{2n+1}} = \lim_{x\to\infty} \frac{x+1}{e^{2x+1}}$$

$$\overset{L'H}{=} \lim_{x\to\infty} \frac{1}{e^{2x+1}\cdot 2} = 0 < 1$$

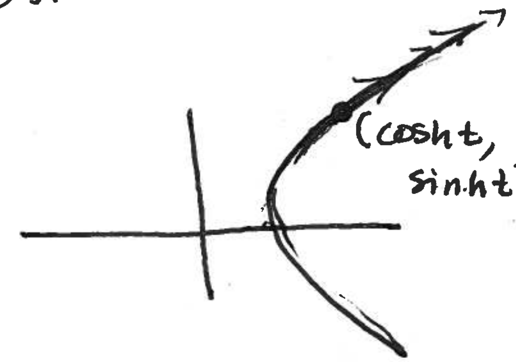
Abs Conv.

§11.7 #29 $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^n}{\cosh n}$

$n \to \infty$

$$\cosh x = \frac{\overset{\nearrow\infty}{e^x} + \overset{\nearrow 0}{e^{-x}}}{2}$$

$\cosh n$



Alt. Series Test.

$$b_n = \frac{1}{\cosh n}$$

- $\displaystyle\lim_{n\to\infty} b_n = \lim_{n\to\infty} \frac{1}{\cosh n}$

$$= \lim_{n\to\infty} \frac{1}{\left(\frac{\overset{\nearrow\infty}{e^n} + \overset{\nearrow 0}{e^{-n}}}{2}\right)} = 0$$

- $\cancel{\lim_{n\to\infty}}$ $b_{n+1} \le b_n$

$$\frac{1}{\cosh(n+1)} \le \frac{1}{\cosh(n)}$$

$$\cosh n \le \cosh(n+1)$$

$$f(x) = \frac{1}{\cosh x} = \frac{1}{\left(\frac{e^x + e^{-x}}{2}\right)}$$

$$= \frac{2}{e^x + e^{-x}}$$

$$f'(x) = \frac{0(e^x + e^{-x}) - 2(e^x - e^{-x})}{(e^x + e^{-x})^2}$$

$$= \frac{-2\overset{\nearrow\infty}{e^x} + 2\overset{\nearrow 0}{e^{-x}}}{(e^x + e^{-x})^2} < 0$$

f is decreasing

$\boxed{\text{Convergent by Alt. Series Test}}$

§11.7 #31 $\displaystyle\sum_{k=1}^{\infty} \frac{5^k}{3^k+4^k}$

Comparison Test.

$a_n = \dfrac{5^k}{3^k+4^k}$, $b_k = \dfrac{5^k}{4^k}$

Limit Comp Test

$$\lim_{k\to\infty} \frac{a_n}{b_k} = \lim_{k\to\infty}\left(\frac{\cancel{5^k}}{3^k+4^k}\right)\cdot\frac{4^k}{\cancel{5^k}}$$

$$= \lim_{k\to\infty}\left(\frac{4^k}{3^k+4^k}\right)\frac{1/4^k}{1/4^k}$$

$$= \lim_{k\to\infty}\frac{1}{\frac{3^k}{4^k}+1} = \lim_{k\to\infty}\frac{1}{\underbrace{\left(\frac{3}{4}\right)^k}_{\to 0}+1}$$

$$= \frac{1}{0+1} = 1$$

$$\boxed{\sum\left(\frac{5}{4}\right)^k \text{ is divergent Geom Series } r=\frac{5}{4}>1}$$

Divergent

§11.8 #21 $\sum_{n=1}^{\infty} \frac{n}{b^n}(x-a)^n$; $b>0$

Ratio Test $\lim_{n\to\infty}\left|\frac{n+1}{b^{n+1}}(x-a)^{n+1}\cdot\frac{b^n}{n(x-a)^n}\right|$

$$=\lim_{n\to\infty}\frac{n+1}{n}\cdot\frac{b^n}{b^{n+1}}\left|\frac{(x-a)^{n+1}}{(x-a)^n}\right|$$

$$=\lim_{n\to\infty}\underbrace{\left(\frac{n+1}{n}\right)}_{\to 1}\frac{1}{b}|x-a|$$

$$=\frac{|x-a|}{b}<1$$

$$\frac{|x-a|}{b}<1$$

$$|x-a|<b$$

$R=b$

$$-b<x-a<b$$

$$\boxed{\begin{array}{c}a-b<x<a+b\\ \text{int. of conv.}\end{array}}$$

endpts

- $x=a+b$

- $\sum_{n=1}^{\infty}\frac{n}{b^n}(x-a)^n=\sum_{n=1}^{\infty}\frac{n}{b^n}(\cancel{a}+b-\cancel{a})^n$

$$=\sum_{n=1}^{\infty}n\frac{\cancel{b^n}}{\cancel{b^n}}=\sum_{n=1}^{\infty}n \quad \text{div}$$

- $x=a-b$

$\sum_{n=1}^{\infty}\frac{n}{b^n}((\cancel{a}-b)-\cancel{a})^n=\sum_{n=1}^{\infty}\frac{n(-b)^n}{b^n}=\sum_{n=1}^{\infty}\frac{(-1)^n n\cancel{b^n}}{\cancel{b^n}}$

$\lim_{n\to\infty}(-1)^n n$ ~~DNE~~ does not exist. Divergent by Test for Div.

§11.8 # 17 $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$

Ratio test

$$\lim_{n\to\infty} \left| \frac{3^{n+1}(x+4)^{n+1}}{\sqrt{n+1}} \cdot \frac{\sqrt{n}}{3^n (x+4)^n} \right|$$

$$= \lim_{n\to\infty} \frac{3^{n+1}}{3^n} \frac{\sqrt{n}}{\sqrt{n+1}} \cdot \left| \frac{(x+4)^{n+1}}{(x+4)^n} \right|$$

$$= \lim_{n\to\infty} 3 \quad \underbrace{\sqrt{\frac{n}{n+1}}}_{\to 1} \quad |x-4|$$

$$= 3|x+4| < 1$$

$$|x+4| < \frac{1}{3} \qquad \boxed{R = \frac{1}{3}}$$

$$-\frac{1}{3} < x+4 < \frac{1}{3}$$

$$-4 - \tfrac{1}{3} < x < \cancel{4 + \tfrac{1}{3}} \; -4 + \tfrac{1}{3}$$

$$\cancel{\tfrac{11}{3} < x < \tfrac{13}{3}} \qquad -\frac{13}{3} < x < -\frac{11}{3}$$

Endpoints

- $x = -\frac{13}{3}$ $\displaystyle\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n(-4\tfrac{1}{3}+4)^n}{\sqrt{n}}$

$$= \sum_{n=1}^{\infty} \frac{3^n(-\tfrac{1}{3})^n}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{(-1)^n}{\sqrt{n}}$$

Alt. Series Test

$$b_n = \frac{1}{\sqrt{n}}$$

- $b_{n+1} < b_n$

$$\frac{1}{\sqrt{n+1}} < \frac{1}{\sqrt{n}}$$

$$\sqrt{n} < \sqrt{n+1}$$

$$n < n+1$$

$$0 < 1 \checkmark$$

- $\lim\limits_{n\to\infty} b_n = \lim\limits_{n\to\infty} \frac{1}{\sqrt{n}} = 0$

Conv. by A.S.T

- $x = \frac{-11}{3}$

$$\sum_{n=1}^{\infty} \frac{3^n \cancel{x^n}^{(x+4)^n}}{\sqrt{n}} = \sum_{n=1}^{\infty} \frac{3^n\left(-4+\frac{1}{3}+4\right)^n}{\sqrt{n}}$$

$$= \sum \frac{3^n\left(\frac{1}{3}\right)^n}{\sqrt{n}} = \sum \frac{1}{\sqrt{n}}$$

div p-series $p = \frac{1}{2}$

Radius Conv. $R = \frac{1}{3}$

Interval Conv.

$$\frac{-13}{3} \leq x < \frac{-11}{3}$$

$$\left[\frac{-13}{3}, \frac{-11}{3}\right)$$