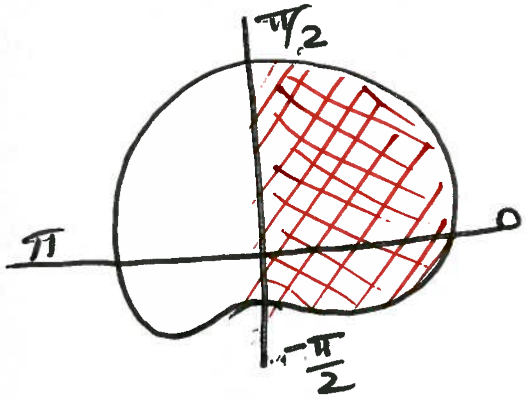
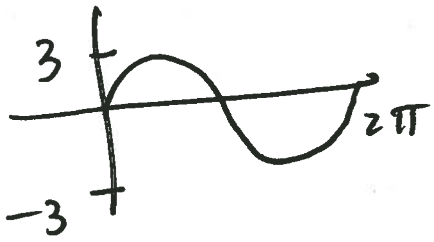
§10.4#7 Find the area of the shaded region.



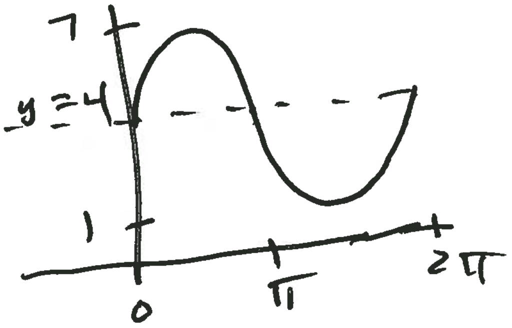
$r = 4 + 3\sin\theta$

$y = 3\sin x$



$$A = \int_a^b \frac{1}{2} r^2\, d\theta$$

$y = 4 + 3\sin x$



$$A = \int_{-\frac{\pi}{2}}^{\pi/2} \frac{1}{2}(4 + 3\sin\theta)^2\, d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\pi/2} (16 + 24\sin\theta + 9\sin^2\theta)\, d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\pi} 16 + 24\sin\theta + \frac{9}{2}(1 - \cos 2\theta)\, d\theta$$

$$= \frac{1}{2}\int_{-\frac{\pi}{2}}^{\pi/2} \left[\frac{41}{2} + 24\sin\theta - \frac{9}{2}\cos 2\theta\right] d\theta$$

$$= \frac{1}{2}\left[\frac{41}{2}\theta\right]_{-\pi/2}^{\pi/2} + \frac{1}{2}24\left[-\cos\theta\right]_{-\pi/2}^{\pi/2} - \frac{1}{2}\frac{9}{2}\cdot\frac{1}{2}\left[\sin 2\theta\right]_{-\pi/2}^{\pi/2}$$

$$= \frac{1}{2}\cdot\frac{41}{2}\left[\pi/2 - -\frac{\pi}{2}\right] + \bar{1}2\left[\overset{=0}{\cos\pi/2} - \overset{=0}{\cos -\frac{\pi}{2}}\right] - \frac{9}{8}\left[\overset{=0}{\sin 2\cdot\frac{\pi}{2}} - \overset{=0}{\sin\frac{2\pi}{2}}\right.$$

$$= \frac{41}{4}\left[\pi\right] = \frac{41\pi}{4}$$

§ 10.3 # 43 $r^2 = 9\sin 2\theta$,

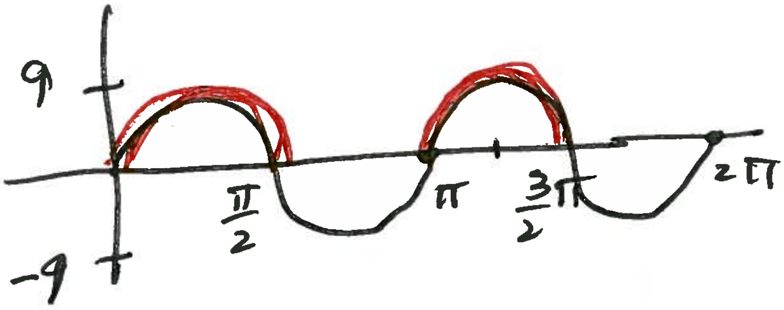
always positive ~~y = sin~~

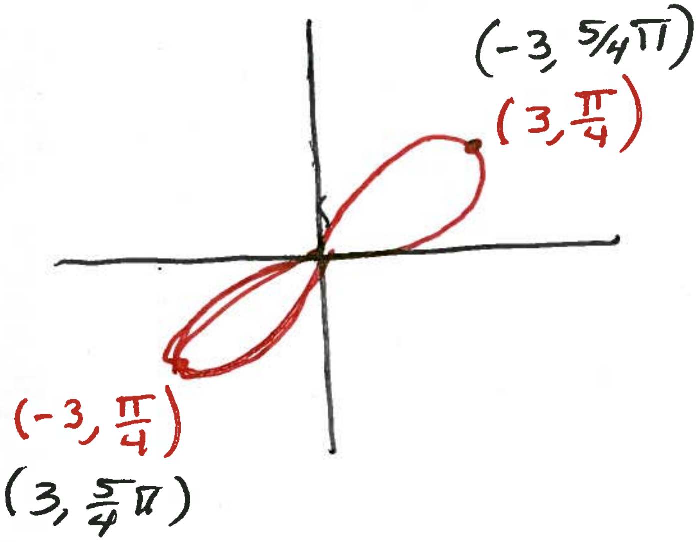
$r = \pm 3\sqrt{\sin 2\theta}$

sin2θ must be pos.

$y = 9\sin 2x$

$P = \frac{2\pi}{2} = \pi$

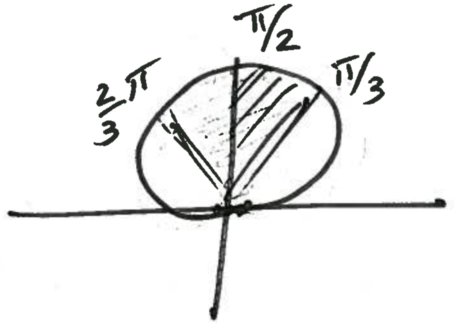




θ	r
0	0
$\frac{\pi}{4}$	±3
$\frac{\pi}{2}$	0
π	0
$\frac{5}{4}\pi$	±3
$\frac{3}{2}\pi$	0

§10.4 #3 $r = \sin\theta, \quad \frac{\pi}{3} \le \theta \le \frac{2\pi}{3}$

Find the area of the region that is bounded by the curve.



$$A = \frac{1}{2}\int_a^b r^2\,d\theta$$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{2\pi/3} (\sin\theta)^2\,d\theta$$

$r = \sin\theta$

$r^2 = r\sin\theta$

$x^2 + y^2 = y$

$x^2 + y^2 - y = 0$

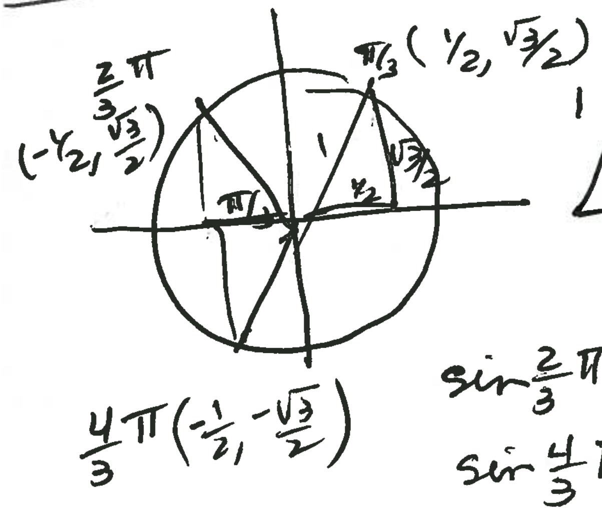
$x^2 + (y^2 - y + \frac{1}{4}) = \frac{1}{4}$

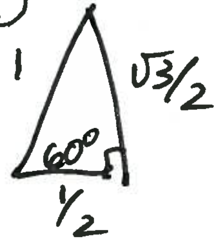
$x^2 + (y - \frac{1}{2})^2 = (\frac{1}{2})^2$

$$= \frac{1}{2}\int_{\frac{\pi}{3}}^{2/3\pi} \frac{1}{2}(1 - \cos 2\theta)\,d\theta$$

$$= \frac{1}{4}\left[\theta - \frac{1}{2}\sin 2\theta\right]_{\pi/3}^{2/3\pi}$$

$$= \frac{1}{4}\left[\frac{2}{3}\pi - \frac{1}{2}\sin 2(\frac{2}{3}\pi)\right] - \frac{1}{4}\left[\frac{\pi}{3} - \frac{1}{2}\sin 2(\frac{\pi}{3})\right]$$





$\sin\frac{2}{3}\pi = \frac{\sqrt{3}}{2}$

$\sin\frac{4}{3}\pi = -\frac{\sqrt{3}}{2}$

$$= \frac{1}{4}\left[\frac{2}{3}\pi - \frac{1}{2}\left(-\frac{\sqrt{3}}{2}\right)\right] - \frac{1}{4}\left[\frac{\pi}{3} - \frac{1}{2}\cancel{\sin}\left(\frac{\sqrt{3}}{2}\right)\right]$$

$$= \frac{2\pi}{12} + \frac{\sqrt{3}}{4\cdot 4} - \frac{\pi}{12} + \frac{\sqrt{3}}{16}$$

$$= \frac{\pi}{12} + \frac{2\sqrt{3}}{16} = \boxed{\frac{\pi}{12} + \frac{\sqrt{3}}{8}}$$

$$= \frac{1}{4}\left[\theta - \frac{1}{2}\sin 2\theta\right]_{\pi/3}^{2/3\pi}$$

$$= \frac{1}{4}\left[\frac{2}{3}\pi - \frac{\pi}{3}\right] - \frac{1}{8}\left[\sin 2\cdot\frac{2}{3}\pi - \sin 2\cdot\frac{\pi}{3}\right]$$

$$= \frac{1}{4}\left[\frac{\pi}{3}\right] - \frac{1}{8}\left[-\frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}\right]$$

$$= \frac{\pi}{12} - \frac{1}{8}\left[-\sqrt{3}\right]$$

$$= \frac{\pi}{12} + \frac{\sqrt{3}}{8}$$

§ 11.1 Sequences

<u>EXAMPLE</u> Determine whether $a_n = (-1)^n$ is convergent or divergent

(If the limit exists, the sequence is said to be convergent. If the limit does not exist, we say that the limit is divergent).

<u>SOLUTION</u>

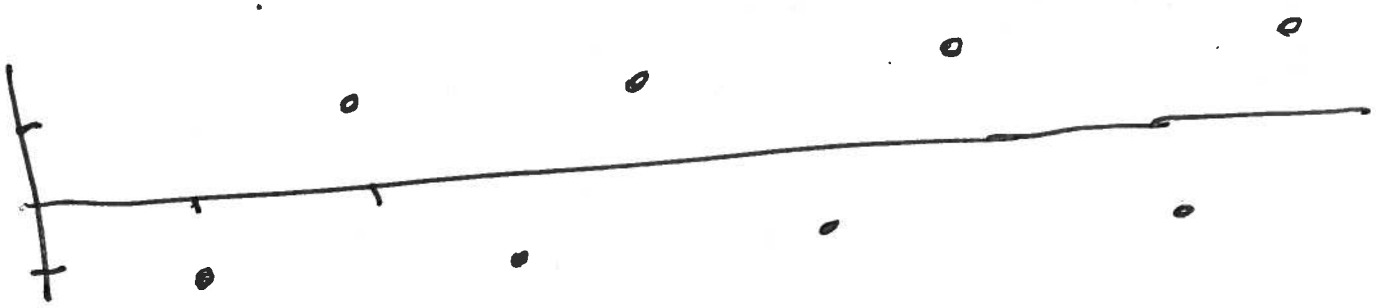
$a_1 = (-1)^1 = -1$

$a_2 = (-1)^2 = 1$

$a_3 = (-1)^3 = -1$

$a_4 = (-1)^4 = 1$

$\vdots$

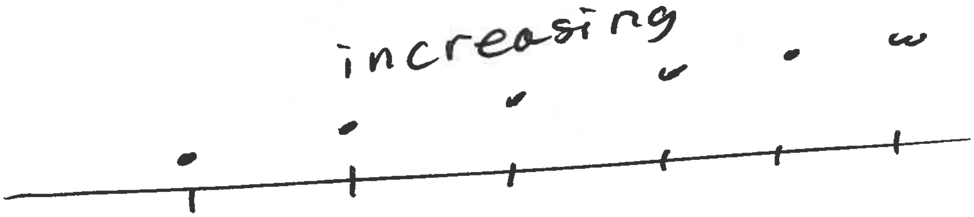


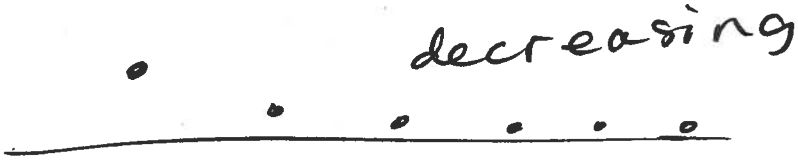
<u>Divergent.</u>

A sequence $\{a_n\}$ is called <u>increasing</u> if $a_n < a_{n+1}$ for all $n \geq 1$.

A sequence is called <u>decreasing</u> is ~~a_{n+1}~~ $a_n > a_{n+1}$ for all $n \geq 1$.

It is called <u>monotonic</u> if it is either increasing or decreasing.





<u>EXAMPLE</u> Show that $a_n = \frac{n}{n^2+1}$ is decreasing.

<u>Method 1</u> show $a_n > a_{n+1}$.

$$\frac{n}{n^2+1} > \frac{(n+1)}{(n+1)^2+1}$$

$$n[(n+1)^2+1] > (n+1)(n^2+1)$$

$$n[n^2+2n+1+1] > n^3+n+n^2+1$$

$$n^3+2n^2+2n > n^3+n^2+n+1$$

$$(\cancel{n^3}+2n^2+2n)-(\cancel{n^3}+n^2+n+1) > 0$$

$$n^2+n-1 > 0$$

$$n^2+n > 1$$ yes for $n \geq 1$

Method 2: Let $f(x) = \frac{x}{x^2+1}$, $x \in \mathbb{R}$ is a member of the set

Then $f(n) = \frac{n}{n^2+1} = a_n$,

~~$n \in \mathbb{N}$~~ ~~posi~~.

Show f is decreasing.
$x \geq 1$

$n \in \mathbb{N}$ ↑ positive integers.

$$f'(x) = \frac{(1)(x^2+1) - (x)(2x)}{(x^2+1)^2}$$

$$= \frac{x^2+1-2x^2}{(x^2+1)^2}$$

$$= \frac{-x^2+1}{\boxed{(x^2+1)^2}} \leftarrow$$

Then denom is alsways positive.

$-x^2+1 < 0$

$1 < x^2$ true for $x > 1$

So $f'(x) < 0$ for $x > 1$

So f is decreasing.

Therefor a_n is decreasing.

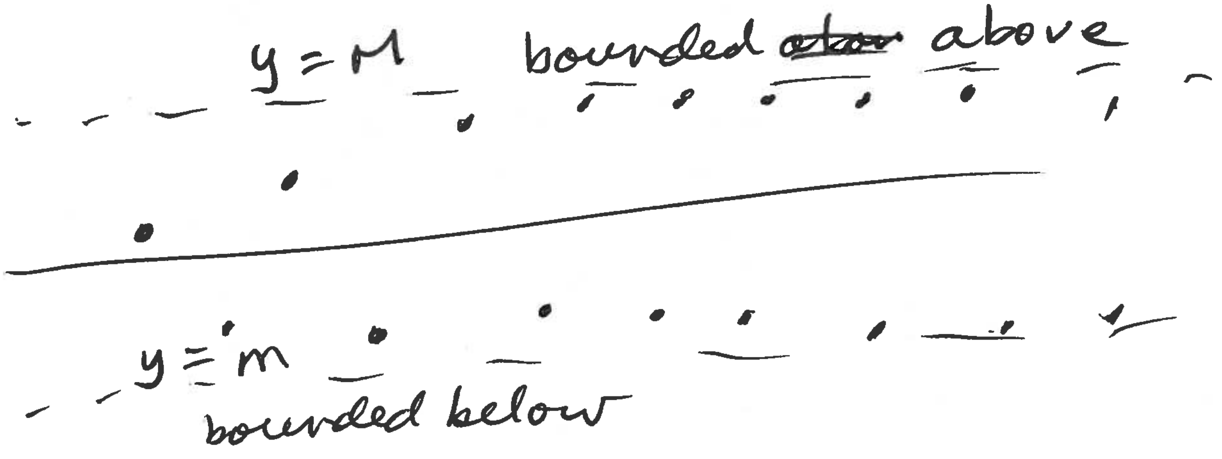
__Definition__ A sequence $\{a_n\}$ is __bounded above__ if $\overset{\exists}{\text{there}}$ is a number M $\underset{\text{s.t.}}{\text{such that}}$

$$a_n \leq M \quad \underset{\forall}{\text{for all}}\ n \geq 1$$

It is __bounded__ __below__ if ~~∃~~ there exists a number m such that

$$m \leq a_n \quad \text{for all } n \geq 1$$

If it is bounded above and below, then $\{a_n\}$ is called __bounded__.



Monotonic Sequence Theorem

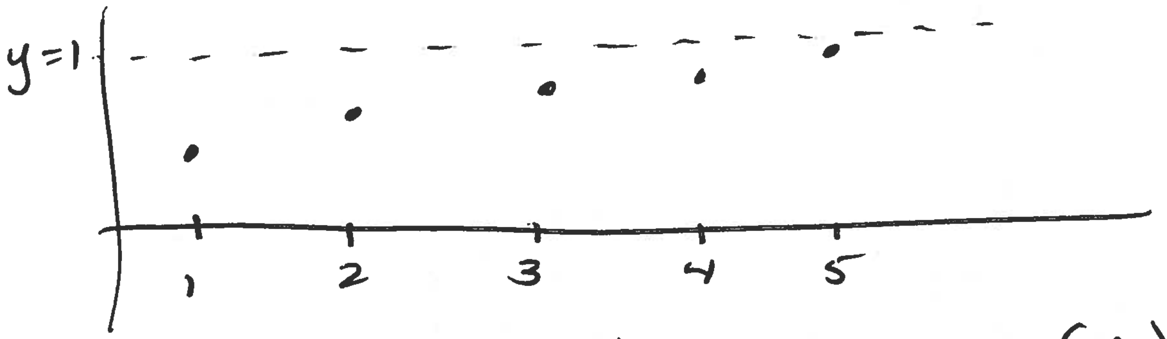
Every bounded, monotonic sequence is convergent.

Example $a_n = \frac{n}{n+1}$

$$a_1 = \frac{1}{1+1} = \frac{1}{2}$$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$



$$\lim_{n\to\infty} \frac{n}{n+1} = \lim_{n\to\infty} \left(\frac{n}{n+1}\right) \frac{1/n}{1/n}$$

$$= \lim_{n\to\infty} \frac{1}{1+\underset{\to 0}{(1/n)}} = 1$$

Example: Investigate the sequence $\{a_n\}$ defined by the recurrence relation

$$a_1 = 2, \quad a_{n+1} = \frac{1}{2}(a_n + 6), \quad n = 1, 2, 3, \ldots$$

SOLUTION

$$a_1 = 2$$

$$a_2 = \frac{1}{2}(a_1 + 6) = \frac{1}{2}(2+6) = 4$$

$$a_3 = \frac{1}{2}(a_2 + 6) = \frac{1}{2}(4+6) = 5$$

$$a_4 = \frac{1}{2}(5+6) = \frac{11}{2} = 5.5$$

$$a_5 = \frac{1}{2}\left(\frac{11}{2} + 6\right) = \frac{23}{4} = 5.75$$

$$a_6 = \frac{1}{2}(5.75 + 6) = 5.875$$

~~Prove $\{a_n\}$ is increasing using Mathematic induction.~~

~~show $a_{n+1} > a_n$ for all n.~~

~~Base case n=1 show $a_2 > a_1$~~

~~$\frac{1}{2}(2+6) > 2$~~

~~$4 > 2$ ✓~~