

§9.5 #17

Linear Differential Equations Solve the initial value problem.

$$\frac{dv}{dt} - 2tv = 3t^2e^{t^2}, \quad v(0) = 5$$

$$v' + P(t)v = Q(t)$$

$$P(t) = -2t$$

$$I(t) = e^{\int P(t) dt} = e^{\int (-2t) dt} = e^{-t^2}$$

Multiply through by integrating factor, $I(t)$.

$$e^{-t^2} \frac{dv}{dt} - 2te^{-t^2}v = 3t^2e^{t^2}e^{-t^2}$$

$$(e^{-t^2}v)' = 3t^2$$

$$e^{-t^2}v = \int 3t^2 dt + C$$

$$e^{-t^2}v = t^3 + C$$

$$v = e^{t^2}(t^3 + C)$$

$$v = t^3e^{t^2} + Ce^{t^2}$$

Solve for C . When $t = 0$, $v = 5$.

$$5 = (0)^3e^{(0)^3} + Ce^{(0)^2}$$

$$5 = C$$

$$\text{Answer: } v = t^3e^{t^2} + 5e^{t^2}$$