

§ 9.3 Separable DE's

Quiz Tomorrow
§ 9.3, 9.5

#16 $xy' + y = y^2, \quad y(1) = -1$

$$x \frac{dy}{dx} + y = y^2$$

$$x \frac{dy}{dx} = y^2 - y$$

$$\frac{dy}{dx} = \frac{y^2 - y}{x}$$

$$\int \frac{\textcircled{\text{I}}}{y^2 - y} dy = \int \frac{\textcircled{\text{II}}}{x} dx$$

Ⓘ $\int \frac{1}{y^2 - y} dy$

Partial Fractions

$$\frac{1}{y(y-1)} = \frac{A}{y} + \frac{B}{y-1}$$

$$1 = A(y-1) + By$$

$$y=1$$

$$1 = B \cdot 1, \quad B=1$$

$$y=0$$

$$1 = A(0-1)$$

$$A=-1$$

$$\int \left(\frac{-1}{y} + \frac{1}{y-1} \right) dy = -\ln|y| + \ln|y-1|$$

Ⓣ $\int \frac{1}{x} dx = \ln|x|$

$$- \ln|y| + \ln|y-1| = \ln|x| + C$$

$$\ln \left| \frac{y-1}{y} \right| = \ln|x| + C$$

$$e^{\ln \left| \frac{y-1}{y} \right|} = e^{\ln|x| + C}$$

$$\left| \frac{y-1}{y} \right| = e^{\ln|x|} \cdot e^C$$

$$\left| \frac{y-1}{y} \right| = e^C |x|$$

$$\frac{y-1}{y} = \pm e^C x$$

$$\text{Let } k = \pm e^C$$

$$\frac{y-1}{y} = kx$$

Solve for k

$$y(1) = -1$$

when $x=1$, $y=-1$

$$\frac{(-1)-1}{(-1)} = k(1)$$

$$\frac{-2}{-1} = k, \quad k=2$$

$$\frac{y-1}{y} = 2x$$

$$y-1 = 2xy$$

$$y - 2xy = 1$$

$$y(1-2x) = 1$$

$$\boxed{y = \frac{1}{1-2x}}$$