§8.2 #16 The given curve is rotated about the y-axis.
Find the area of the resulting surface.

$$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \le x \le 2$$

SOLUTION

$$A = \int 2\pi x \, ds$$

$$ds = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

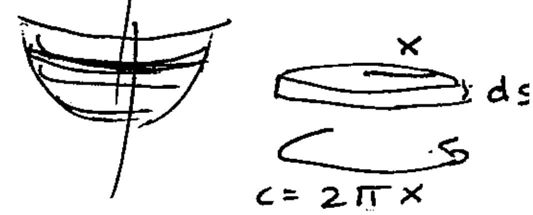
$$A = \int 2\pi x \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

Find $\frac{dy}{dx}$.

$$\frac{dy}{dx} = \frac{1}{4}(2x) - \frac{1}{2}\frac{1}{x}$$

$$= \frac{1}{2}x - \frac{1}{2x}$$

Aside



$c = 2\pi x$

- $A = \int 2\pi x \, ds$

$$ds^2 = dx^2 + dy^2$$

$$ds = \sqrt{dx^2 + dy^2}$$

$$= \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$A = \int_1^2 2\pi x \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{1 + \left(\frac{x^2}{4} - 2\left(\frac{x}{2}\right)\left(\frac{1}{2x}\right) + \frac{1}{4x^2}\right)}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}}\, dx$$

$$= 2\pi \int_1^2 x \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}}\, dx = 2\pi \int x \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2}\, dx$$

$$= 2\pi \int_1^2 x \left(\frac{x}{2} + \frac{1}{2x}\right) dx = 2\pi \int_1^2 \left(\frac{x^2}{2} + \frac{1}{2}\right) dx$$

$$= 2\pi \left[\frac{x^3}{2\cdot 3} + \frac{x}{2}\right]_1^2 = 2\pi \left[\frac{2^3}{6} + \frac{2}{2}\right] - 2\pi \left[\frac{1^3}{6} + \frac{1}{2}\right]$$

$$= 2\pi \left[\frac{8}{6} + 1 - \frac{1}{6} - \frac{1}{2}\right] = 2\pi \left[\frac{8}{6} + \frac{6}{6} - \frac{1}{6} - \frac{3}{6}\right] = 2\pi\left(\frac{10}{6}\right) = \frac{10}{3}\pi$$

$$A = \frac{10}{3}\pi$$