§8.1 #9 $\quad y = \frac{x^5}{6} + \frac{1}{10x^3}, \quad 1 \le x \le 2$

Find the length of the curve.

$$L = \int ds$$

$$ds = \sqrt{1 + (y')^2}\,dx$$

or

$$ds = \sqrt{1 + (x')^2}\,dy$$

Find y': $\quad y = \frac{x^5}{6} + \frac{x^{-3}}{10}$

$$y' = 5\frac{x^4}{6} + \frac{-3x^{-4}}{10}$$

$$L = \int_1^2 \sqrt{1 + \left(\frac{5}{6}x^4 - \frac{3}{10x^4}\right)^2}\,dx$$

$$= \int_1^2 \sqrt{1 + \frac{25}{36}x^8 - \cancel{2}\left(\frac{5}{\cancel{6}_2}\cancel{x^4}\right)\left(\frac{\cancel{3}}{\cancel{10}\cancel{x^4}}\right) + \frac{9}{100x^8}}\,dx$$

$$= \int_1^2 \sqrt{1 + \frac{25}{36}x^8 - \frac{1}{2} + \frac{9}{100x^8}}\,dx$$

$$= \int_1^2 \sqrt{\frac{25}{36}x^8 + \frac{1}{2} + \frac{9}{100x^8}}\,dx$$

$$= \int_1^2 \sqrt{\left(\frac{5}{6}x^4 + \frac{3}{10x^4}\right)^2}\,dx$$

$$= \int_1^2 \left(\frac{5}{6}x^4 + \frac{3}{10}x^{-4}\right)dx$$

$$= \left[\frac{\cancel{5}}{6}\cdot\frac{x^5}{\cancel{5}} + \frac{\cancel{3}}{10}\,\frac{x^{-3}}{-\cancel{3}}\right]_1^2$$

$$= \left[\frac{2^5}{6} - \frac{2^{-3}}{10}\right] - \left[\frac{1^5}{6} - \frac{1^{-3}}{10}\right]$$

$$= \frac{32}{6} - \frac{1}{10}\cdot\frac{1}{8} - \frac{1}{6} + \frac{1}{10}\cdot\frac{8}{8}$$

$$= \frac{31}{6} + \frac{7}{80} = \frac{31\cdot 40}{6\cdot 40} + \frac{7\cdot 3}{80\cdot 3}$$

$$= \frac{1261}{240}$$

$$\begin{array}{r} 31 \\ 40 \\ \hline 1240 \\ +21 \end{array}$$