§8.1 #13 $y = \ln(\sec x)$, $0 \le x \le \frac{\pi}{4}$

Find arc length

$$y' = \frac{1}{\sec x} \cdot \sec x \tan x = \tan x$$

$$L = \int_a^b \sqrt{1 + (y')^2}\, dx = \int_0^{\pi/4} \sqrt{1 + \tan^2 x}\, dx$$

$$= \int_0^{\pi/4} \sqrt{\sec^2 x}\, dx = \int_0^{\pi/4} \sec x\, dx$$

$$= \ln|\sec x + \tan x| \Big]_0^{\pi/4}$$

$$= \ln|\sec \pi/4 + \tan \pi/4| - \ln|\sec 0 + \tan 0|$$

$$= \ln(\sqrt{2} + 1) - \ln(1 + 0)$$

$$= \ln(\sqrt{2} + 1)$$