

§7.8 #7. Determine whether each integral is convergent or divergent. Evaluate those that are convergent.

$$\int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw$$

SOLUTION: • First solve the improper integral, $\int (2-w)^{-1/2} dw$.

$$u = 2 - w$$

$$du = -dw$$

$$-du = dw$$

$$\begin{aligned} \int (2-w)^{-1/2} dw &= - \int u^{-1/2} du = -\frac{u^{1/2}}{1/2} = -2u^{1/2} \\ &= -2(2-w)^{1/2} = -2\sqrt{2-w} + C \end{aligned}$$

$$\begin{aligned} \bullet \int_{-\infty}^{-1} \frac{1}{\sqrt{2-w}} dw &= \lim_{t \rightarrow -\infty} \int_t^{-1} (2-w)^{-1/2} dw \\ &= \lim_{t \rightarrow -\infty} \left[-2\sqrt{2-w} \right]_t^{-1} \\ &= \lim_{t \rightarrow -\infty} \left[\left(-2\sqrt{2 - (-1)} \right) - \left(-2\sqrt{2-t} \right) \right] \\ &= -2\sqrt{3} + \lim_{t \rightarrow -\infty} 2\sqrt{2-t} = \infty. \quad \text{The integral is divergent.} \end{aligned}$$