§7.8 #49 $\displaystyle\int_0^\infty \frac{x}{x^3+1}\,dx$

Use Comparison Test to determine whether convergent or divergent.

If we throw away the lower degree terms we have $\frac{x}{x^3} = \frac{1}{x^2}$.
So let's compare to $\frac{1}{x^2}$, which has convergent integral.

$\bullet$ $0 \le \frac{x}{x^3+1} \le \frac{x}{x^3} = \frac{1}{x^2}$ for $x \geq 1$

note $x^3 \le x^3+1$ for $x \geq 1$

this implies that $\frac{1}{x^3+1} \le \frac{1}{x^3}$ on $[1, \infty)$

$\Rightarrow \quad \frac{x}{x^3+1} \le \frac{x}{x^3} = \frac{1}{x^2}$

$\bullet$ $\displaystyle\int_1^\infty \frac{1}{x^2}\,dx = \lim_{t\to\infty} \int_1^t x^{-2}\,dx = \lim_{t\to\infty}\left[-\frac{1}{x}\right]_1^t$

$\displaystyle = \lim_{t\to\infty}\left[\underbrace{-\frac{1}{t}}_{\to 0} - \left(-\frac{1}{1}\right)\right] = 1$

convergent.

$\therefore$ By Comp. Test Convergent.