

§10.2#65 Find the surface area generated by rotating the given curve about the y-axis

$$x = 3t^2, y = 2t^3, 0 \leq t \leq 5$$

SOLUTION

$$S = \int 2\pi x ds$$

$$= \int 2\pi x \sqrt{(x')^2 + (y')^2} dt$$

$$x' = 6t, y' = 6t^2$$

$$S = \int_0^5 2\pi (3t^2) \sqrt{(6t)^2 + (6t^2)^2} dt$$

$$= 6\pi \int_0^5 t^2 \sqrt{(6t)^2 [1 + t^2]} dt$$

$$= 6\pi \int_0^5 t^2 \cdot 6t \sqrt{1 + t^2} dt$$

$$= 36\pi \int_0^5 t^3 \sqrt{1 + t^2} dt$$

$$= 36\pi \int_0^5 t^2 \cdot t \sqrt{1 + t^2} dt$$

$$= 36\pi \cdot \frac{1}{2} \int_1^{26} (u-1) u^{1/2} du$$

$$= 18\pi \int_1^{26} (u^{3/2} - u^{1/2}) du$$

$$= 18\pi \left[\frac{2}{5} u^{5/2} - \frac{2}{3} u^{3/2} \right]_1^{26}$$

$$= 36\pi \left[\frac{u^2 \sqrt{u}}{5} - \frac{u \sqrt{u}}{3} \right]_1^{26}$$

$$= 36\pi \left[\frac{(26)^2 \sqrt{26}}{5} - \frac{26 \sqrt{26}}{3} \right] - 36\pi \left[\frac{1}{5} - \frac{1}{3} \right]$$

$$= 36\pi \cdot 26 \sqrt{26} \left[\frac{26}{5} - \frac{1}{3} \right] - 36\pi \left(-\frac{2}{15} \right)$$

$$\begin{aligned} u &= t^2 + 1 \\ du &= 2t dt \\ \frac{1}{2} du &= t dt \\ t^2 &= u - 1 \\ t=0, u &= 1 \\ t=5, u &= 26 \end{aligned}$$

§ 10.2 #65
continued

$$= \frac{12}{36} \pi \cdot 26 \sqrt{26} \frac{73}{155} + \frac{12}{36} \pi \left(\frac{2}{155} \right)$$

$$\begin{array}{r} 1 \\ 26 \\ \hline 3 \\ 78 \\ \hline 15 \\ \hline 73 \end{array}$$

$$= \frac{12}{5} \pi (26 \sqrt{26} \cdot 73 + 2)$$

$$= \frac{24}{5} \pi (13 \cdot 73 \sqrt{26} + 1)$$

$$= \frac{24}{5} \pi (949 \sqrt{26} + 1)$$

$$\begin{array}{r} 73 \\ 13 \\ \hline 219 \\ 730 \\ \hline 949 \end{array}$$