§10.2 #5

Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.

$$x = e^{\sqrt{t}}, \quad y = t - \ln t^2; \quad t = 1$$

SOLUTION

First find the slope.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{y'(t)}{x'(t)}$$

$$\begin{aligned}
x(t) &= e^{t^{1/2}} \\
x'(t) &= e^{t^{1/2}} \left(t^{1/2}\right)' = e^{t^{1/2}} \left(\frac{1}{2} t^{-1/2}\right) \\
x'(t) &= \frac{e^{\sqrt{t}}}{2\sqrt{t}} \\
y(t) &= t - \ln t^2 = t - 2\ln t \\
y'(t) &= 1 - \frac{2}{t} \\
\frac{dy}{dx} &= \frac{y'(t)}{x'(t)} = \frac{(1 - 2/t)}{(e^{\sqrt{t}}/2\sqrt{t})} \\
\text{Put in} \quad & t = 1. \\
m &= \frac{(1 - 2/1)}{(e^{\sqrt{1}}/2\sqrt{1})} = -\frac{2}{e} \\
x(1) &= e^{\sqrt{1}} = e, \quad y(1) = 1 - 2\ln 1 = 1 - 2(0) = 1 \\
y - y_1 &= m(x - x_1) \\
y - 1 &= -\frac{2}{e}(x - e) \\
y - 1 &= -\frac{2}{e}x + 2 \\
y &= -\frac{2}{e}x + 3
\end{aligned}$$