

§10.2 #45 Sketch the graph and find its length.

$$x = e^t \cos t, \quad y = e^t \sin t$$

$$0 \leq t \leq \pi$$

SOLUTION: Use graphing calculator & to sketch graph.

$$L = \int ds = \int \sqrt{(x')^2 + (y')^2} dt$$

$$x' = e^t \cos t + e^t(-\sin t)$$

$$y' = e^t \sin t + e^t \cos t$$

$$L = \int_0^{\pi} \sqrt{(e^t \cos t - e^t \sin t)^2 + (e^t \cos t + e^t \sin t)^2} dt$$

$$= \int_0^{\pi} \sqrt{(e^t)^2 [\cos^2 t - 2 \cos t \sin t + \sin^2 t + \cos^2 t + 2 \sin t \cos t + \sin^2 t]} dt$$

$$= \int_0^{\pi} e^t \sqrt{2} dt$$

$$= 2e^t \Big|_0^{\pi} = 2e^{\pi} - 2e^0 = 2e^{\pi} - 2$$