Math 185 Chapter 11 Review

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = \dfrac{3^{n+2}}{5^n}$

   (b) $a_n = \dfrac{\cos^2 n}{2^n}$

   (c) $a_n = \dfrac{(-1)^n n^2}{5n^2+2n+1}$

2. *Geometric Series:* Determine whether the geometric series is convergent or divergent. If it is convergent, then find its sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} 3\cdot\left(-\frac{2}{5}\right)^{n-1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} 3\cdot\left(-\frac{5}{2}\right)^{n-1}$

3. The following series are divergent. State why they are divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+5n+1}{3n^2+2n+2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (1.001)^n$

4. *Telescoping Series:* Determine whether the geometric series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

5. *The Integral Test:* Use the Integral Test to determine whether the series is convergent or divergent. $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

6. The Comparison Test: Determine whether the series $\sum a_n$ converges or diverges. When using the Comparison Test, follow these steps:

   - Give a series $\sum b_n$ to which the series $\sum a_n$ is being compared.
   - State why $\sum b_n$ is either convergent or divergent.
   - Prove an appropriate inequality comparing the two series, that is, $a_n \le b_n$ or $a_n \ge b_n$ <u>or</u> Show that $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is a positive (nonzero) number.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n-1}{n4^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

7. Give two example of series $\sum a_n$ such that $\lim_{n\to\infty} a_n = 0$, but the series is divergent.

8. *The Alternating Series Test:* Test the series for convergence or divergence. When using the Alternating Series Test, show that all requirements of the hypothesis are satisfied.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n-1}{2n+1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3+4}$

9. Alternating Series Test: Show that the series is convergent. How many terms of the series do we need to add in order to find the sum to the indicated accuracy?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6} \qquad |\text{error}| < 0.00005$$

10. §*11.6 Absolute Convergence and the Ratio and Root Tests* Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

(a) $\sum_{n=0}^{\infty}(-1)^n 2^n$

(b) $\sum_{n=2}^{\infty}(-1)^{n+1}\frac{n}{n^2+1}$

(c) $\sum_{n=4}^{\infty}\left(\frac{n^2+1}{2n^2+1}\right)^n$

11. §11.8 Find the radius of convergence and interval of convergence of the series.

(a) $\sum_{n=1}^{\infty}\frac{(-1)^n n^2}{2^n}x^n$

(b) $\sum_{n=1}^{\infty}\frac{x^n}{n!}$

(c) $\sum_{n=1}^{\infty}\frac{n!x^n}{3^n}$

12. §11.9 Find a power series representation for the function and determine the interval of convergence.

$$f(x) = \frac{2}{3-x}$$

13. §11.9 Find a power series representation for the function and determine the radius of convergence.

(a) $f(x) = \ln(5-x)$

(b) $f(x) = \frac{x^3}{(x-2)^2}$

Table 1: Some important Maclaurin series that we have derived.

- $\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n = 1+x+x^2+x^3+\ldots$ $R = 1$
- $e^x = \sum_{n=0}^{\infty}\frac{x^n}{n!} = 1+\frac{x}{1!}+\frac{x^2}{2!}+\frac{x^3}{3!}+\ldots$ $R = \infty$
- $\sin x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!} = x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots$ $R = \infty$
- $\cos x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!} = 1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\ldots$ $R = \infty$
- $\tan^{-1} x = \sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{2n+1} = x-\frac{x^3}{3}+\frac{x^5}{5}-\frac{x^7}{7}+\ldots$ $R = 1$
- $(1+x)^k = \sum_{n=0}^{\infty}\binom{k}{n}x^n = 1+kx+\frac{k(k-1)}{2!}x^2+\frac{k(k-1)(k-2)}{3!}x^3+\ldots$ $R = 1$

Maclaurin Series $f(x) = \sum_{n=0}^{\infty}\frac{f^n(0)}{n!}x^n$

Taylor Series $f(x) = \sum_{n=0}^{\infty}\frac{f^n(a)}{n!}(x-a)^n$

14. §11.10 Find the Taylor Series for $f(x)$ centered at the given value of a.

$$f(x) = 1/x, \quad a = -3$$

15. §11.10 Use a MacLaurin series in Table 1 to obtain the McLaurin series for the given function.

$$f(x) = x\cos(\frac{1}{2}x^2)$$

16. §11.10 Use the binomial series to expand the function as power series. State the radius of convergence.

$$f(x) = \frac{1}{(2+x)^3}$$

Solutions

1. (a) Convergent. 0
   (b) Convergent. 0
   (c) Divergent

2. (a) Convergent. 15/7
   (b) Divergent.

3. (a) Test for Divergence.
   (b) Harmonic Series
   (c) Geometric Series, $r = 1.001 > 1$ <u>or</u> Test for Divergence, $\lim_{n\to\infty}(1.001)^n = \infty$

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5. Convergent.

6. (a) Convergent.
   (b) Convergent.
   (c) $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$

   SOLUTION: Divergent.

7. $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

8. (a) Divergent by Test for Divergence.
   (b) Convergent.

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10. (a) Divergent by the Ratio Test and the Test for Divergence.
    (b) Conditionally Convergent.
    (c) Convergent by Root Test

11. (a) $R = 2, (-2, 2)$
    (b) $R = \infty, (-\infty, \infty)$
    (c) $R = 0, \{0\}$

12. $2\sum_{n=0}^{\infty} \frac{1}{3^{n+1}} x^n, \quad (-3, 3)$

13. (a) $\ln 5 - \sum_{n=1}^{\infty} \frac{x^n}{n5^n}, \quad R = 5$
    (b) $\sum_{n=3}^{\infty} \frac{n-2}{2^{n-1}} x^n, \quad R = 2$

14. $\sum_{n=0}^{\infty} (-1)^n \frac{1 \cdot 3 \cdot 5 \ldots (2n-1)}{2^n 3^{2n+1} \cdot n!} (x-9)^n, \quad R = 9$

15. $\sum_{n=0}^{\infty} (-1)^n \frac{1}{2^{2n}(2n)!} x^{4n+1}, \quad R = \infty$

16. $\sum_{n=0}^{\infty} (-1)^n \frac{(n+1)(n+2)}{2^{n+4}} x^n, \quad R = 2$