Math 185 Chapter 11 Review

1. Determine whether the sequence converges or diverges. If it converges, find the limit.

   (a) $a_n = \dfrac{3^{n+2}}{5^n}$

   (b) $a_n = \dfrac{\cos^2 n}{2^n}$

   (c) $a_n = \dfrac{(-1)^n n^2}{5n^2 + 2n + 1}$

2. *Geometric Series:* Determine whether the geometric series is convergent or divergent. If it is convergent, then find its sum.

   (a) $\displaystyle\sum_{n=1}^{\infty} 3 \cdot \left(-\frac{2}{5}\right)^{n-1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} 3 \cdot \left(-\frac{5}{2}\right)^{n-1}$

3. The following series are divergent. State why they are divergent.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2 + 5n + 1}{3n^2 + 2n + 2}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} (1.001)^n$

4. *Telescoping Series:* Determine whether the geometric series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty} \frac{3}{n(n+3)}$$

5. *The Integral Test:* Use the Integral Test to determine whether the series is convergent or divergent. $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$

6. The Comparison Test: Determine whether the series $\sum a_n$ converges or diverges. When using the Comparison Test, follow these steps:

   - Give a series $\sum b_n$ to which the series $\sum a_n$ is being compared.
   - State why $\sum b_n$ is either convergent or divergent.
   - Prove an appropriate inequality comparing the two series, that is, $a_n \le b_n$ or $a_n \ge b_n$ or Show that $\lim_{n\to\infty} \frac{a_n}{b_n} = L$, where L is a positive (nonzero) number.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2n^3 + 1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} \frac{n - 1}{n4^n}$

   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2 + 1}}$

7. Give two example of series $\sum a_n$ such that $\lim_{n\to\infty} a_n = 0$, but the series is divergent.

8. *The Alternating Series Test:* Test the series for convergence or divergence. When using the Alternating Series Test, show that all requirements of the hypothesis are satisfied.

   (a) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{3n - 1}{2n + 1}$

   (b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n^2}{n^3 + 4}$

9. Alternating Series Test: Show that the series is convergent. How many terms of the series do we need to add in order to find the sum to the indicated accuracy?

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n^6} \qquad |\text{error}| < 0.00005$$

Solutions

1. (a) Convergent. 0
   (b) Convergent. 0
   (c) Divergent

2. (a) Convergent. 15/7
   (b) Divergent.

3. (a) Test for Divergence.
   (b) Harmonic Series
   (c) Geometric Series, $r = 1.001 > 1$ or Test for Divergence, $\lim_{n\to\infty}(1.001)^n = \infty$

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5. Convergent.

6. (a) Convergent.
   (b) Convergent.
   (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{\sqrt{n^2+1}}$
   SOLUTION: Divergent.

7. $\sum_{n=1}^{\infty} \frac{1}{n}$ and $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

8. (a) Divergent by Test for Divergence.
   (b) Convergent.

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