

§4.7 Optimization Problems

HW §4.7 # 2-6, 9, 11, 17-19, 22, 23

EXAMPLE Suppose that $x+y=100$. Find values for x and y such that the product xy is greatest.

SOLUTION

FORMULAS

$$P = xy$$

$$x + y = 100$$

Eliminate a variable

$$x + y = 100$$

$$y = -x + 100$$

$$P = xy$$

$$P = x(-x + 100)$$

$$P = -x^2 + 100x$$

Find the max

find
critnums \nearrow

$$P' = -2x + 100$$

$$-2x + 100 = 0$$

$$-2x = -100$$

$$x = \frac{-100}{-2} = 50$$

$$\boxed{x = 50}$$

Solve for y

$$y = -x + 100$$

$$y = -50 + 100$$

$$y = 50$$
$$\boxed{y = 50}$$

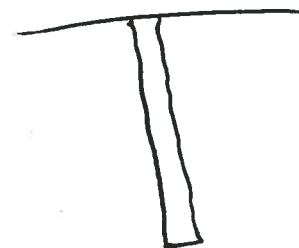
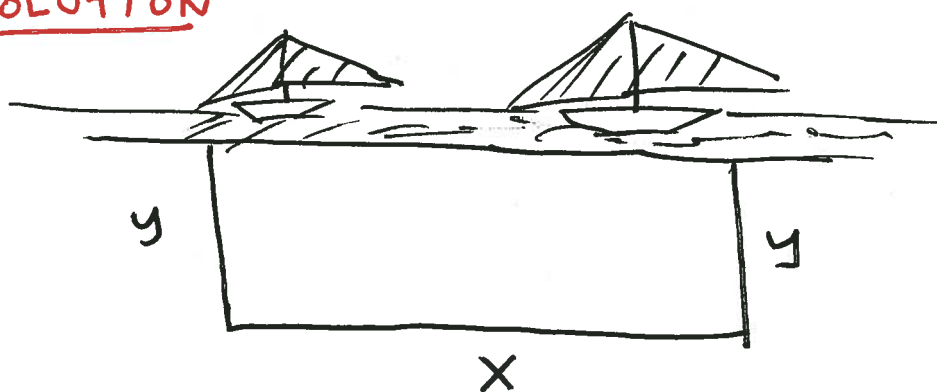
EXAMPLE A farmer has 2400ft

of fencing and wants to fence off a rectangular field that borders a straight river.

He needs no fence along the river.

What are the dimensions of the field that has the largest area.

SOLUTION



FORMULAS: $A = xy$

$$x + 2y = 2400$$

Eliminate a Variable

$$x = -2y + 2400$$

$$A = xy$$

$$A = (-2y + 2400)y$$

$$A = -2y^2 + 2400y$$

Find the max: $A' = -2 \cdot 2y + 2400 = 0$

$$-4y + 2400 = 0$$

$$4y = 2400$$

$$\boxed{y = 600}$$

Solve for x:

$$x = -2y + 2400$$

$$x = -2(600) + 2400$$

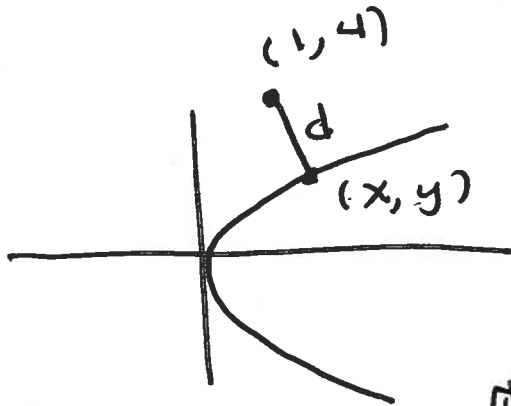
$$\boxed{x = 1200}$$

EXAMPLE Find the point on

the parabola $y^2 = 2x$ that is closest to the point $(1, 4)$.

SOLUTION

$$x = \frac{y^2}{2}$$



FORMULAS

$$D = d^2 = (x-1)^2 + (y-4)^2$$

~~$$d = \sqrt{(x-1)^2 + (y-4)^2}$$~~

$$D = (x-1)^2 + (y-4)^2$$
$$y^2 = 2x$$

Eliminate a variable

$$x = \frac{y^2}{2}$$

$$D = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$$

Critnum

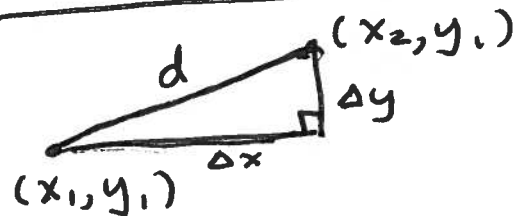
$$D' = 2\left(\frac{y^2}{2} - 1\right) \frac{2y}{2} + 2(y-4) \cdot 1$$

$$= \left(\frac{2y^2}{2} - 2\right)y + 2y - 8$$

$$= y^3 - 2y + 2y - 8$$

$$y^3 - 8 = 0, \quad y^3 = 8, \quad \boxed{y=2}$$

• DISTANCE FORMULA



$$d^2 = (\Delta x)^2 + (\Delta y)^2$$

$$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$$

Solve for x

$$x = \frac{y^2}{2}$$

$$x = \frac{(2)^2}{2}, \quad \boxed{x=2}$$

The point (2, 2).