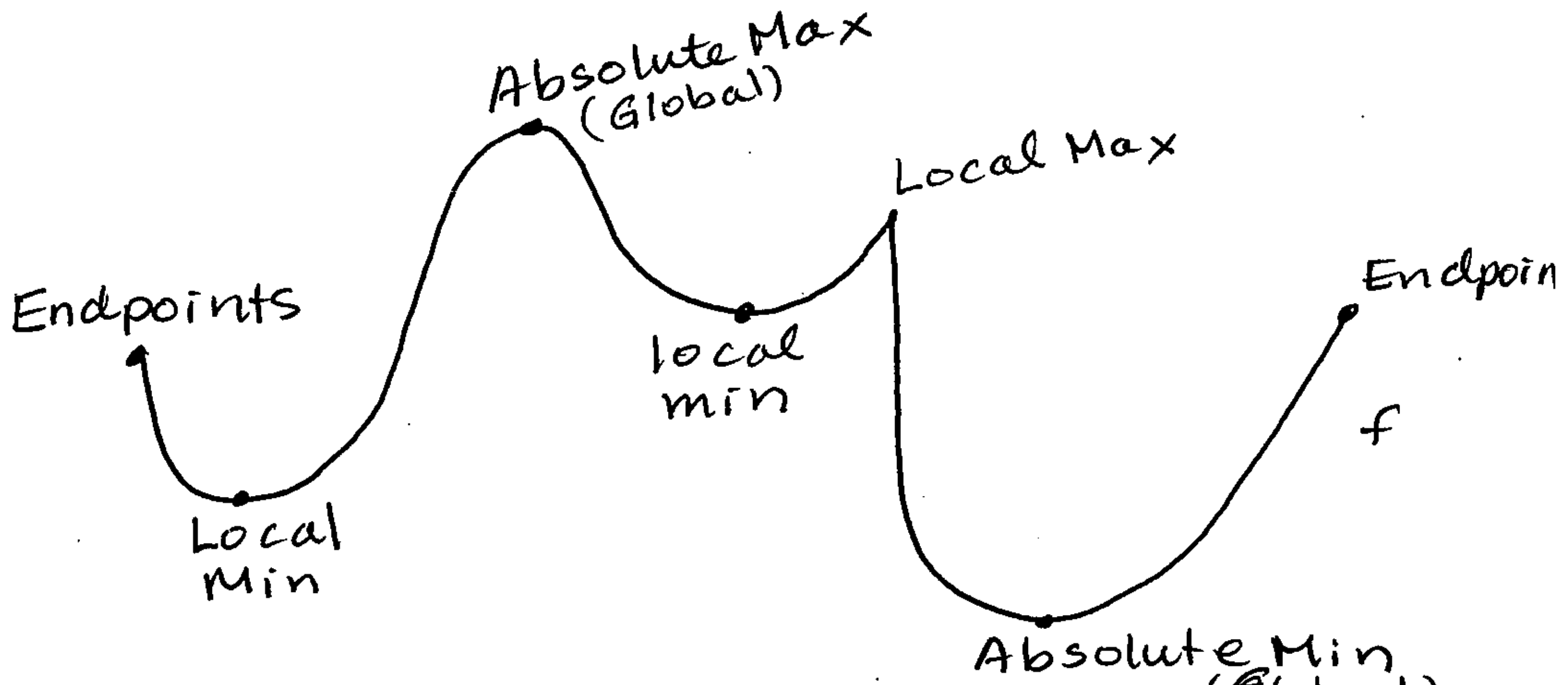
Chapter 4 Applications of Derivatives.

§4.1 Max and Min Values on a closed interval.

HW §4.1 # 29-44, 47-63



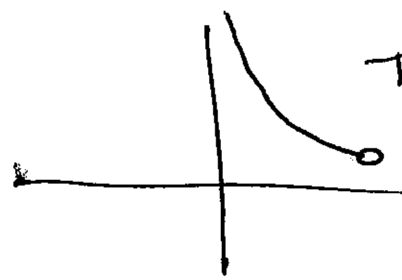
f is continuous on a closed interval $[a, b]$.

Goal: To find the absolute max and min of a continuous function on a closed interval.

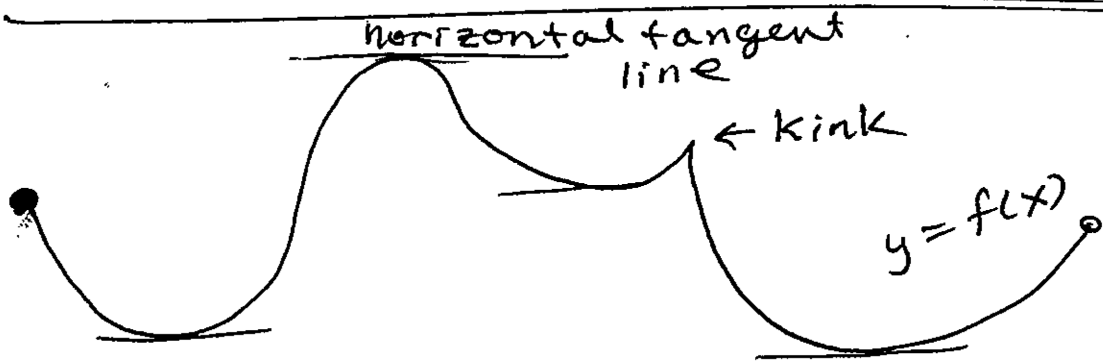
Absolute max's and min's are called extreme values.

The Extreme Value Theorem:
If f is continuous on a closed interval $[a,b]$, then f attains an absolute max and min on the interval.

Example: $y = \frac{1}{x}$, $0 < x < 1$



There is no abs. max, no abs. min.
Notice that $y = \frac{1}{x}$ is continuous on the open interval $(0,1)$, not the closed interval.



Where should we look for the extreme values?

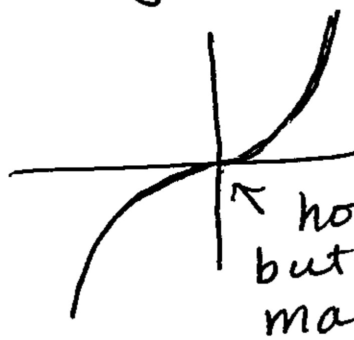
Answer:
- endpoints
- where the tangent line is horizontal, $f'(x) = 0$
- where the derivative does not exist, $f'(x)$ DNE

Fermat's Theorem: If f has a local max or min at c, then $f'(c)=0$.

<u>Note</u>: It's possible that $f'(c)=0$, but we do not have a local max or min.

Example $y=x^3$, $y'=3x^2$

$y'=0$ when $x=0$



horizontal tangent, but not local max or min.

A <u>critical number</u> of a function f is a number c in the domain of f such that $f'(c)=0$ or $f'(c)$ does not exist.

Example: Find the absolute max and absolute min values of $f(x) = x^3 - 3x^2 + 1$ on $[-\frac{1}{2}, 4]$.

SOLUTION

1. Find the critical numbers of f.

$$f'(x) = 3x^2 - 6x$$

Solve $f'(x) = 0$

$$3x^2 - 6x = 0$$

$$3x(x-2) = 0$$

$$\boxed{x = 0, \quad x = 2} \text{ critnums}$$

2. Plot the critnums and endpoints.

endpoint $x = -\frac{1}{2}$, $f(-\frac{1}{2}) = (-\frac{1}{2})^3 - 3(-\frac{1}{2})^2 + 1$

$$= -\frac{1}{8} - 3\left(\frac{1}{4}\right) + 1$$

$$= -\frac{1}{8} - \frac{6}{8} + \frac{8}{8} = \frac{1}{8}$$

critnum $x = 0$, $f(0) = (0)^3 - 3(0)^2 + 1 = 1$

$x = 2$ $f(2) = (2)^3 - 3(2)^2 + 1 = 8 - 12 + 1 = -3$ min

endpt $x = 4$ $f(4) = (4)^3 - 3(4)^2 + 1 = 64 - 48 + 1 = 17$ max

Abs. Min $(2, -3)$

Abs. Max $(4, 17)$

This is called the closed interval method.

Example Find the critical numbers $f(x) = x^{3/5}(4-x)$.

SOLUTION • Step 1 : Find $f'(x)$

$$f(x) = 4x^{3/5} - x^{3/5}x^1$$

$$f(x) = 4x^{3/5} - x^{8/5}$$

$$f'(x) = 4 \cdot \frac{3}{5} x^{-2/5} - \frac{8}{5} x^{3/5}$$

$$= \frac{12}{5x^{2/5}} - \frac{8x^{3/5}}{5}\left(\frac{x^{2/5}}{x^{2/5}}\right)$$

$$= \frac{12}{5x^{2/5}} - \frac{8x^{3/5}x^{2/5}}{5x^{2/5}}$$

$$= \frac{12-8x}{5x^{2/5}}$$

Step 2: Solve: $f'(x) = 0$

$$\frac{12-8x}{5x^{2/5}} = 0$$

A fraction equals zero when the numerator is zero.

$$12 - 8x = 0$$

$$12 = 8x$$

$$\frac{12}{8} = x$$

$$\boxed{x = \frac{3}{2}}$$ critnum.

Step3 : Determine if $f'(x)$ is undefined for some values of x.

$$f'(x) = \frac{12-8x}{5x^{2/5}}$$

Undefined when denom is zero.

$$5x^{2/5} = 0$$

$$\boxed{x=0}$$ crit num.