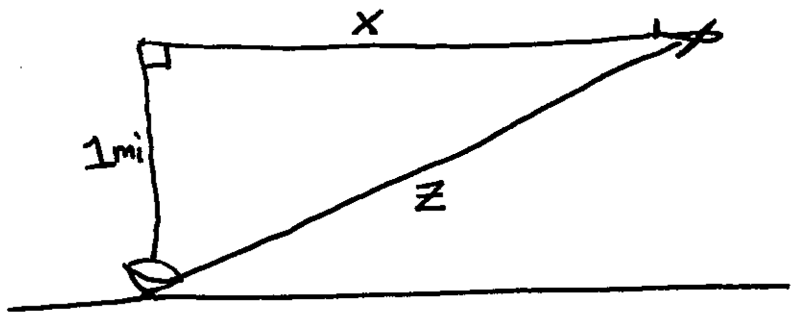
§3.9 #11 A plane flying horizontally at an altitude of 1 mi and a speed of 500 mi/h passes directly over a radar station. Find the rate at which the distance from the plane to the station is increasing when it is 2 mi away from the station.

SOLUTION



<u>Find</u>: $\frac{dz}{dt}$ when $z = 2$ mi

<u>Facts</u>: $\frac{dx}{dt} = 500$ mi/h

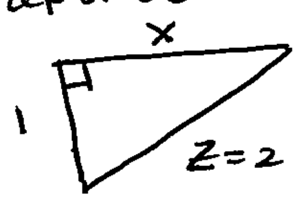
<u>Formulas</u>: $x^2 + 1^2 = z^2$

<u>Derivative</u>: $2x\,x' + 0 = 2z\,z'$

$$\cancel{2}x\,x' = \cancel{2}z\,z'$$

<u>FILL-IN</u>: $x' = 500$, $z = 2$, $x = ?$
$x = \sqrt{3}$

snapshot



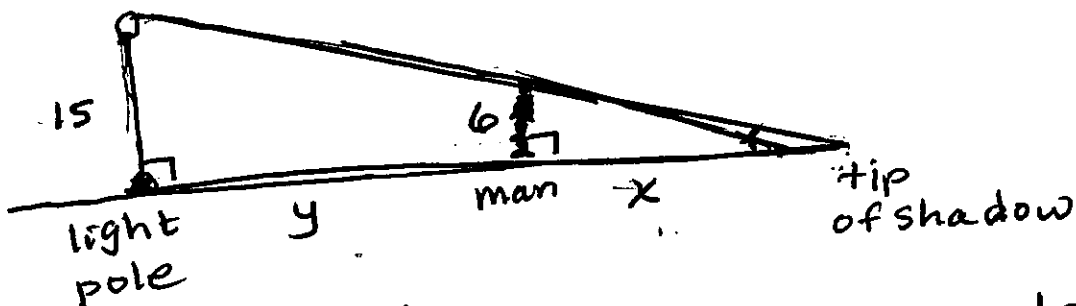
$x^2 + 1^2 = 2^2$
$x^2 + 1 = 4$
$x^2 = 3$
$x = \sqrt{3}$

$x\,x' = z\,z'$
$(\sqrt{3})(500) = 2z'$
$\frac{500\sqrt{3}}{2} = z'$

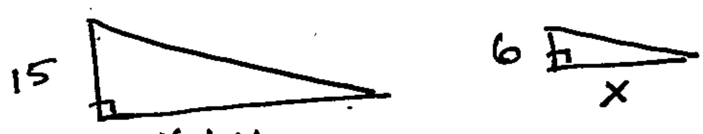
$$\boxed{\frac{dz}{dt} = 250\sqrt{3} \text{ mi/h when } z = 2 \text{ mi}}$$

§3.9 #13 A street light is mounted at the top of a 15 ft tall pole. A man 6 ft tall walks away from the pole with a speed of 5 ft/s along a straight path. How fast is the tip of his shadow moving when he is 40 ft from the pole?

SOLUTION



- x is the length of shadow
- y is the distance from the man to the pole.
- We have similar triangles.



Similar Δ's have proportional sides.

Facts: $dy/dt = 5$ ft/s

Find $\frac{d}{dt}(x+y)$

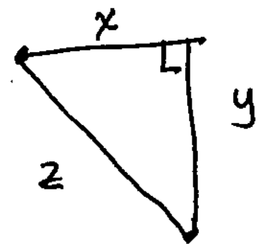
$$\frac{15}{6} = \frac{x+y}{x}$$

$$15x = 6(x+y)$$
$$15x = 6x + 6y$$
$$9x = 6y$$
$$y = \frac{9}{6}x = \frac{3}{2}x$$
$$x = 2/3\, y$$

$$\frac{d}{dt}(x+y) = \frac{d}{dt}\left(\frac{2}{3}y + y\right)$$
$$= \frac{d}{dt}(5/3\, y) = \frac{5}{3}\frac{dy}{dt} = \frac{5(5)}{3}$$
$$= \frac{25}{3} \text{ ft/s}$$

§3.9 #15 Two cars start moving from the same point. One travels south at 60 mi/h and the other travels west at 25 mi/h. At what rate is the distance between the cars increasing two hours later?

SOLUTION



Fact: $\frac{dx}{dt} = 25$ mi/h

$\frac{dy}{dt} = 60$ mi/h

FORMULA: $x^2 + y^2 = z^2$

Find $\frac{dz}{dt}$ when $t = 2$ h.

DERIVATIVE

$2xx' + 2yy' = 2zz'$

FILL-IN when $t = 2$ h,

$\frac{\text{dist}}{\text{time}} = \text{velocity}$, dist = (velocity)(time)

$x = (25 \text{ mi/h})(2\text{ h}) = 50$ mi

$y = (60 \text{ mi/h})(2\text{ h}) = 120$ mi

$z = \sqrt{x^2+y^2} = \sqrt{50^2+120^2}$

$xx' + yy' = zz'$

$z' = \frac{xx' + yy'}{z} = \frac{(50)(25) + (120)(60)}{\sqrt{50^2+120^2}}$

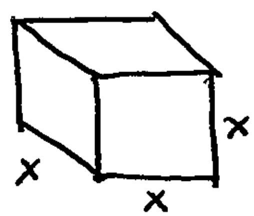
≈ 65

§3.10 #33 The edge of a cube was found to be 30 cm with a possible error in measurement of 0.1 cm. Use differentials to estimate the maximum possible error, relative error, and percentage error in computing

(a) the volume of the cube

(b) the surface area of the cube.

<u>SOLUTION</u>



$V = x^3$

$A = 6x^2$ ← six faces, each face has area of x^2.

$x = 30$ cm.

$dx = .1$ cm

(a) $dV = V'(x)\,dx$

$V'(x) = 3x^2$

$V'(30) = 3(30)^2 = 3(900) = 2700$ cm³

$dV = V'(30)\,dx = (2700)(.1) = 270$ cm³

$\boxed{dV = 270 \text{ cm}^3 \text{ possible error in volume}}$

<u>Relative error.</u> $\frac{dV}{V} = \frac{270}{V(30)} = \frac{270}{(30)^3} = \frac{270}{27000}$

$= \frac{1}{100} = .01$

<u>percentage error in volume</u> 1%

b) $dA = A'(x)\,dx$ $\quad A(x) = 6x^2$

$x = 30$

$A'(x) = 12x$ $\quad dx = .1$

$A'(30) = 12(30) = 360\text{cm}^2$

$dA = A'(30)\,dx = (360)(.1) = 36\text{ cm}^2$

possible error in Area: $dA = 36\text{cm}^2$

relative error: $\dfrac{dA}{A} = \dfrac{36}{6(30)^2} = \dfrac{\cancel{6}\cdot\cancel{6}^{\,2}}{\cancel{6}(\cancel{9}_3 00)}$

$= \dfrac{2}{300} = \dfrac{1}{150}$

$= 0{,}00\overline{6}$

percentage error $= .\overline{6}\,\% \approx .67\%$