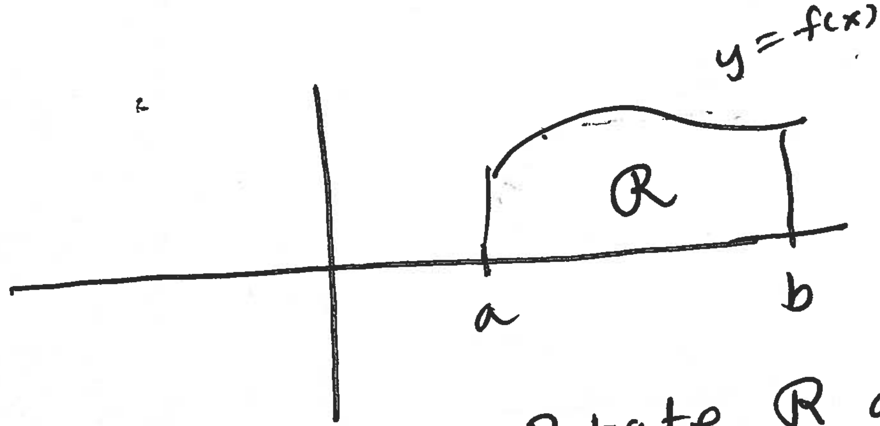
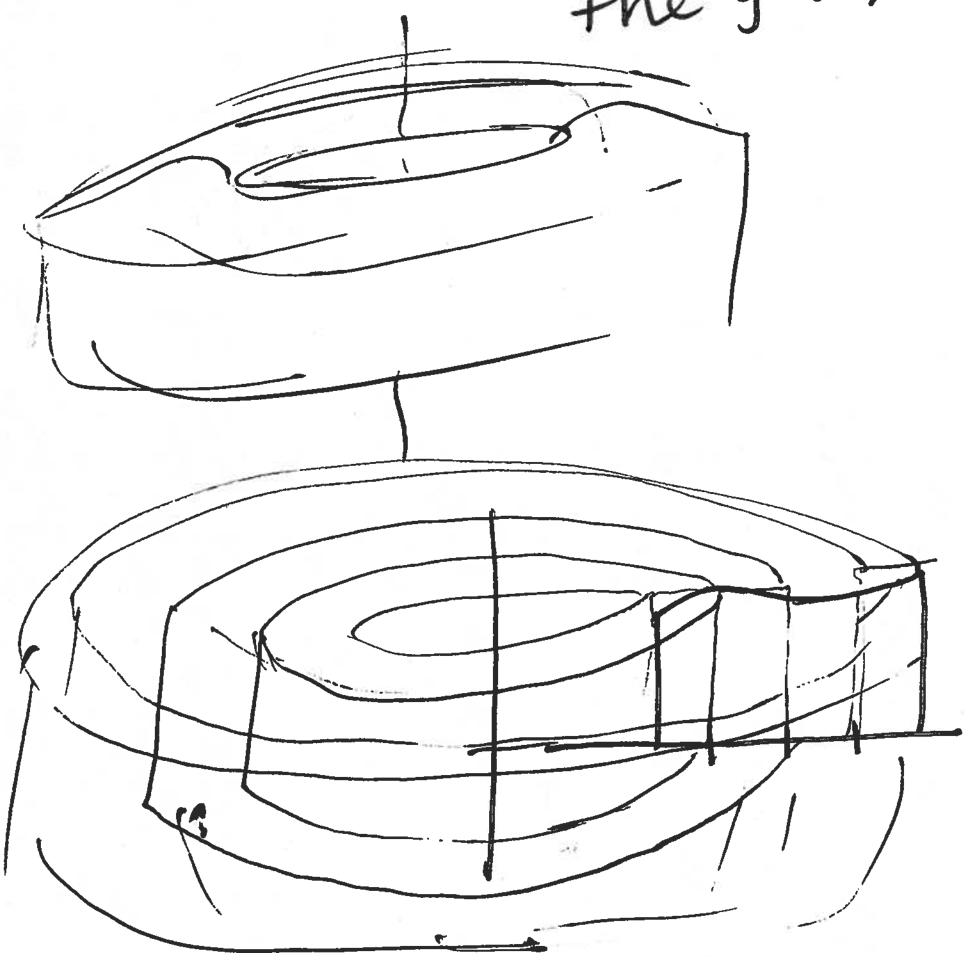
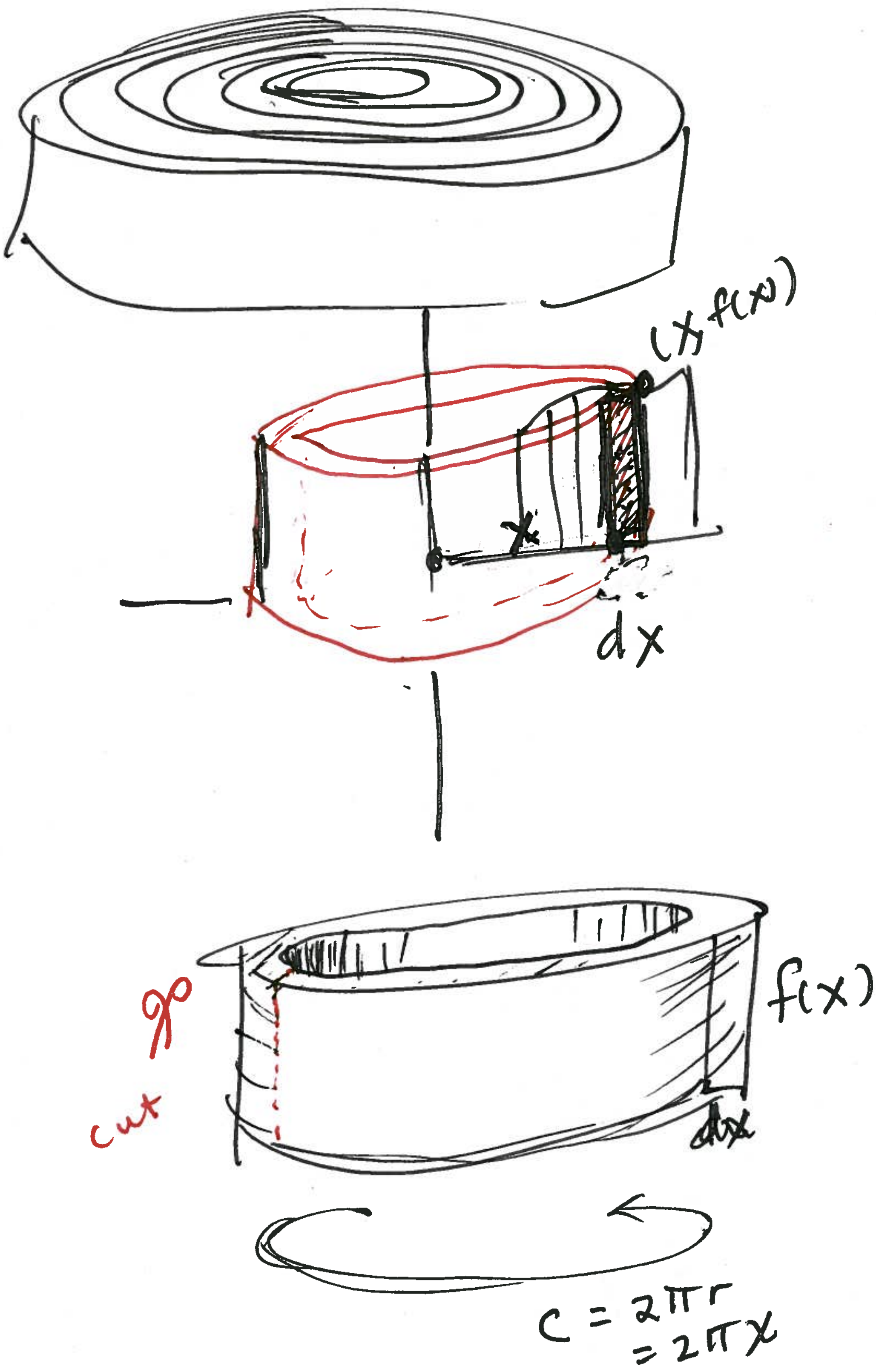
§6.3 The Shell Method

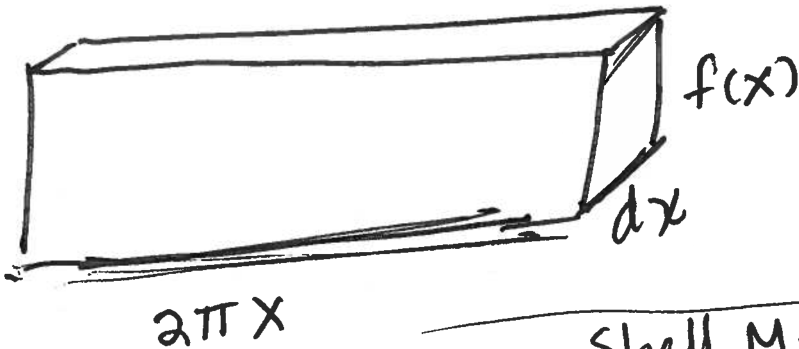
HW §6.3 #3–7



Rotate R about the y-axis.




(x, f(x))
x
dx
f(x)
cut
dx
C = 2πr
= 2πx



Volume = l × w × h

Shell Method.

$$V = \int_a^b 2\pi x\, f(x)\, dx$$

rotation about y-axis

EXAMPLE Find the volume of the solid obtained by rotating about the y-axis the region bounded by $y = 2x^2 - x^3$ and $y = 0$.

SOLUTION

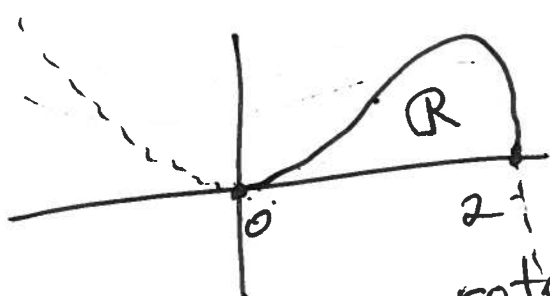
x-intercepts

$2x^2 - x^3 = 0$

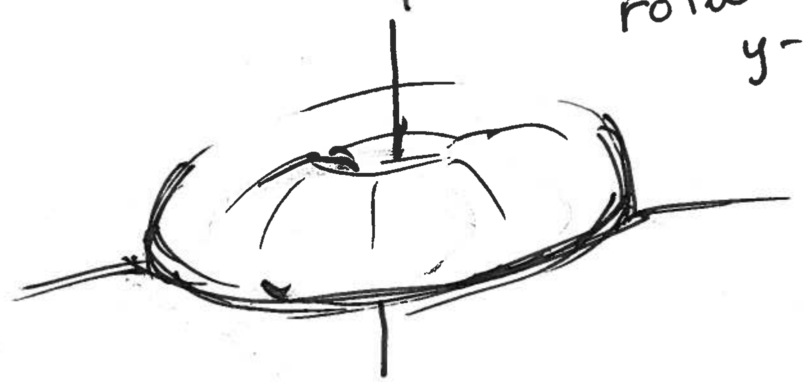
$x^2(2 - x) = 0$

$x = 0,\ x = 2$

x	y
0	0
1	1
2	0



rotate about y-axis



$$V = \int_a^b 2\pi x f(x)\, dx$$

$$= \int_0^2 2\pi x\,(2x^2 - x^3)\, dx$$

$$= 2\pi \int_0^2 (2x^3 - x^4)\, dx$$

$$= 2\pi \left[2\frac{x^4}{4} - \frac{x^5}{5}\right]_0^2$$

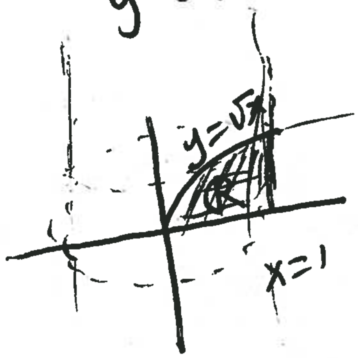
$$= 2\pi \left[\frac{2(2)^4}{4} - \frac{(2)^5}{5}\right]$$

$$= 2\pi \cdot 2^5 \left[\frac{1}{4} - \frac{1}{5}\right]$$

$$= 2^6\pi \left(\frac{5}{20} - \frac{4}{20}\right) = 64\pi\left(\frac{1}{20}\right)$$

$$= \frac{32}{10}\pi = \frac{16}{5}\pi \checkmark$$

EXAMPLE Let $\mathcal{R}$ be the region bounded by $y=\sqrt{x}$, $y=0$, and $x=1$. $\mathcal{R}$ is rotated about the y-axis. Find the volume.



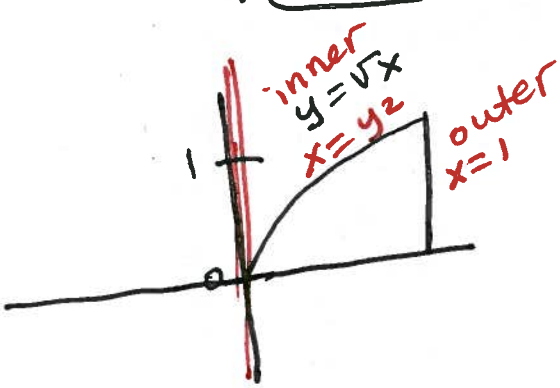
Shell Method

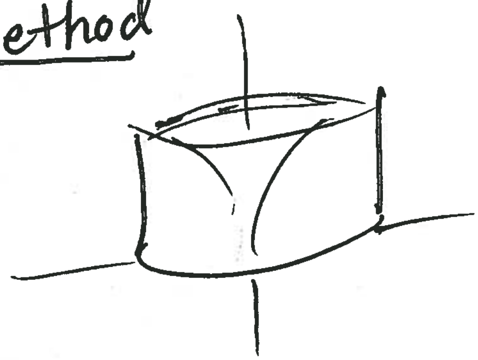
$$V=\int_a^b 2\pi x\, f(x)\,dx$$

$$=\int_0^1 2\pi x\sqrt{x}\,dx = 2\pi\int_0^1 x^{3/2}\,dx$$

$$=2\pi\left[\frac{x^{5/2}}{5/2}\right]_0^1 = 2\pi\cdot\frac{2}{5}=\frac{4}{5}\pi$$

Washer ~~Med~~ Method





$L=0$

$$V = \int_a^b \left[\pi(f(y)-L)^2 - \pi(g(y)-L)\right] dy$$

$$= \int_0^1 \left[\pi(1-0)^2 - \pi(y^2-0)^2\right] dy$$

$$= \pi \int_0^1 (1-y^4)\, dy = \pi\left(y - \frac{y^5}{5}\right)\Big|_0^1$$

$$= \pi\left(1 - \frac{1}{5}\right) = \frac{4}{5}\pi \quad \square$$

§6.4 Work

Work is defined as
Force times distance.

$$W = Fd$$

- Force is constant
- Force = mass × acceleratio

$$F = ma$$

Newton's Second Law.

EXAMPLE: How much work is done in lifting a 1.2-kg book off the floor to put it on a desk that 0.7 m high? Use the fact that the acceleration due to gravity is $g = 9.8\ m/s^2$.

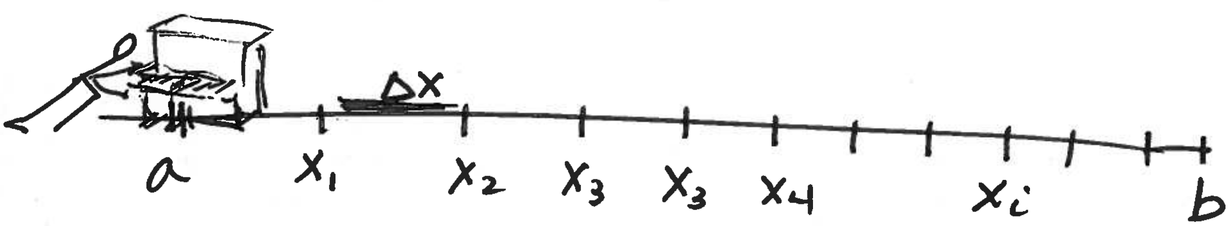
SOLUTION $F = ma = (1.2\ kg)(9.8\ m/s^2)$
$= 11.76\ N$ ← newtons

$$W = Fd = (11.76\ N)(.7\ m)$$

$\approx 8.2\ J$ ← Joules

Now we assume that force is a continuous function of distance. Force $= F(x)$.

we move an object from a to b.



~~The fore~~ If the interval $[X_{i-1}, X_i]$ is really small, then force is almost constant on the interval, say $F(X_i)$. Then the work required on the ith interval is about

$$W_i = F(X_i)\,\Delta X$$

Let's add the work of all intervals.

$$W_1 + W_2 + W_3 + \cdots + W_n$$

$$= F(X_1)\Delta X + F(X_2)\Delta X + \cdots F(X_n)\Delta X$$

$$= \sum_{i=1}^{n} F(X_i)\Delta X \qquad \text{Let } n \to \infty$$

Define

$$\text{Work} = \lim_{n\to\infty} \sum_{i=1}^{n} F(X_i)\Delta X = \int_a^b F(x)\,dx$$

$$W = \int_a^b F(x)\,dx$$

EXAMPLE When a particle is located a distance x feet from the origin, a force of x^2+2x pounds acts on it. How much work is done in moving it from $x=1$ to $x=3$?

$$W = \int_1^3 (x^2+2x)\,dx$$

$$= \left.\frac{x^3}{3} + \frac{\cancel{2}x^2}{\cancel{2}}\right|_1^3$$

$$= \left(\frac{3^3}{3} + 3^2\right) - \left(\frac{1^3}{3} + 1^2\right)$$

$$= \frac{50}{3} \text{ ft-lb.}$$