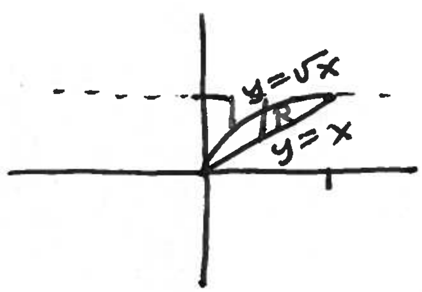
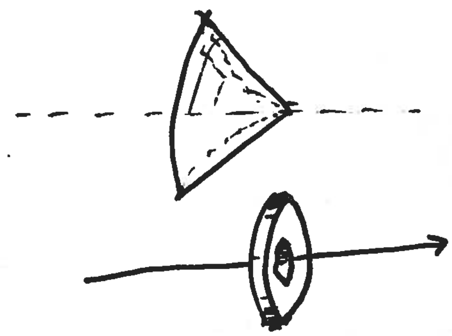
§6.2 Continued

EXAMPLE The region $\mathcal{R}$ is bounded by $y = x$ and $y = \sqrt{x}$. $\mathcal{R}$ is rotated about the line $y = 1$. Find the volume.





$$V = \int_a^b \Big[\pi(\underbrace{f(x) - L}_{\text{outer}})^2 - \pi(\underbrace{g(x) - L}_{\text{inner}})^2\Big]dx$$

$$= \int_0^1 \left(\pi(x-1)^2 - \pi(\sqrt{x}-1)^2\right) dx$$

$$= \pi \int_0^1 \left((x-1)^2 - (\sqrt{x}-1)^2\right) dx$$

$$= \pi \int_0^1 \left((x^2 - 2x + \cancel{1}) - (x - 2x^{1/2} + \cancel{1})\right) dx$$

$$= \pi \int_0^1 \left(x^2 - 2x + \cancel{1} - x + 2x^{1/2} - \cancel{1}\right) dx$$

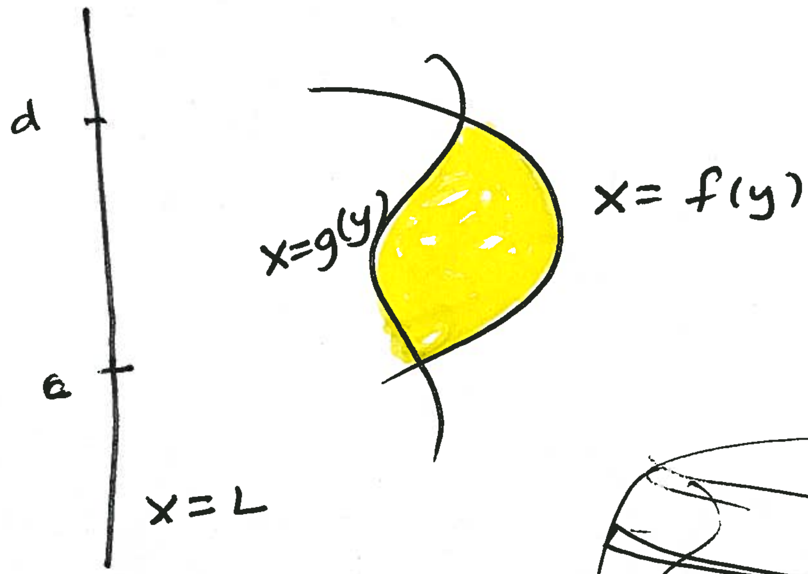
$$= \pi \int_0^1 \left(x^2 - 3x + 2x^{1/2}\right) dx$$

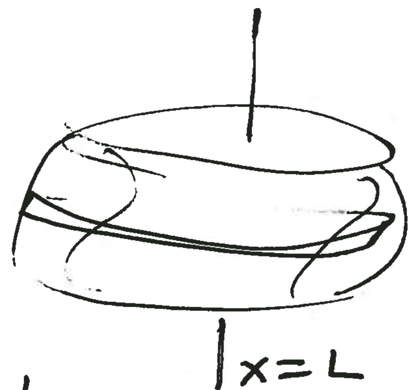
$$= \pi \left[\frac{x^3}{3} - 3\frac{x^2}{2} + 2\frac{x^{3/2}}{3/2}\right]_0^1$$

$$= \pi \left[\frac{1}{3} - \frac{3}{2} + 2 \cdot \frac{2}{3}\right] - 0 = \pi\left[\frac{5}{3} - \frac{3}{2}\right]$$

$$= \pi\left[\frac{10}{6} - \frac{9}{6}\right] = \frac{\pi}{6}$$

Rotations about Vertical Lines

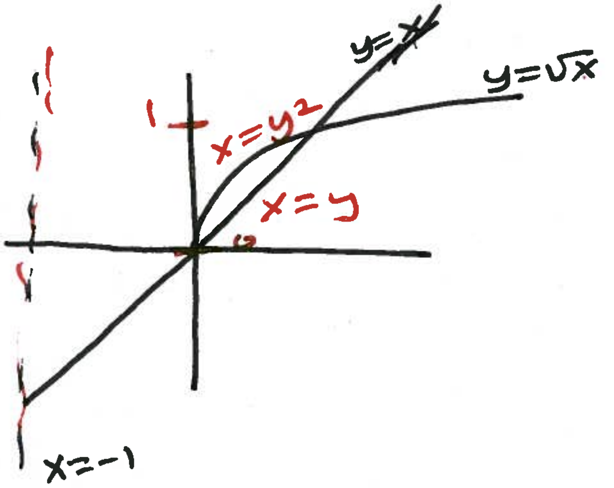


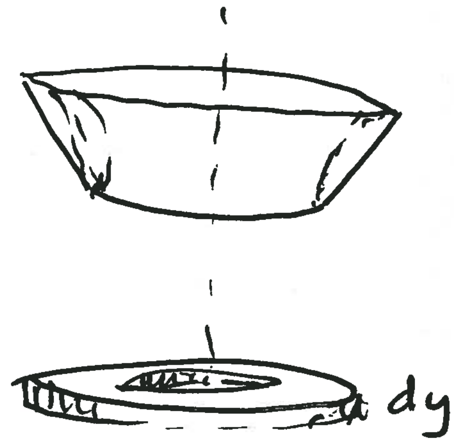


Rotation about the line $x=L$.

$$V = \int_c^d \Big(\underbrace{\pi (f(y)-L)^2}_{\text{outer}} - \underbrace{\pi \big(g(y)-L\big)^2}_{\text{inner}} \Big) dy$$

EXAMPLE The region $\mathcal{R}$ is bounded by $y=\sqrt{x}$ and $y=x$. $\mathcal{R}$ is rotated about the line $x=-1$. Find the volume.





$L=-1$

$$V=\int_c^d \left[\pi(\underbrace{f(y)}_{\text{outer}}-L)^2-\pi(\underbrace{g(y)}_{\text{inner}}-L)^2\right]dy$$

$$=\int_{y=0}^{y=1}\left(\pi(\underbrace{y}_{\text{outer}}-\,-1)^2-\pi(\underbrace{y^2}_{\text{inner}}-\,-1)^2\right)dy$$

$$=\pi\int_0^1\left[(y+1)^2-(y^2+1)^2\right]dy$$

$$=\pi\int_0^1\left[(y^2+2y+1)-(y^4+2y^2+1)\right]dy$$

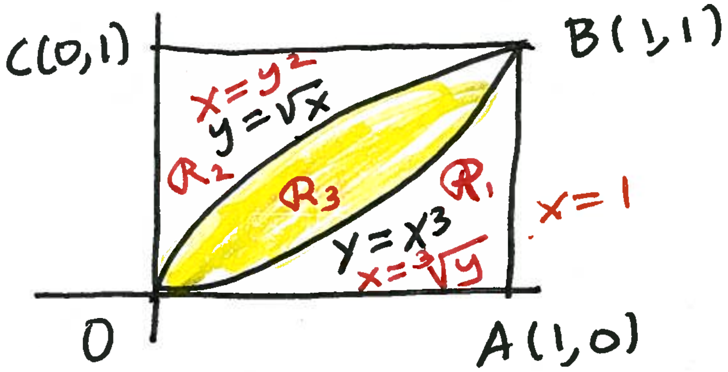
$$=\pi\int_0^1\left[y^2+2y+\cancel{1}-y^4-2y^2-\cancel{1}\right]dy$$

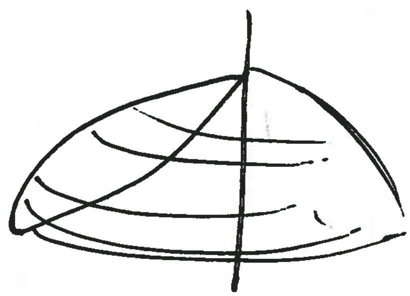
$$=\pi\int_0^1\left[-y^4-y^2+2y\right]dy$$

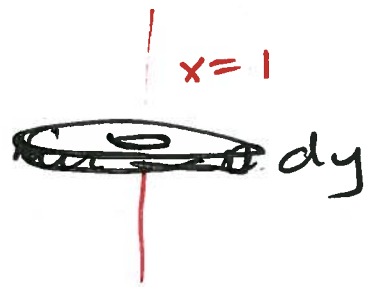
$$=\pi\left[\frac{-y^5}{5}-\frac{y^3}{3}+\frac{2y^2}{2}\right]_0^1=\pi\left(\frac{-1}{5}-\frac{1}{3}+1\right)$$

$$=\pi\left(\frac{-3}{15}-\frac{5}{15}+\frac{15}{15}\right)=\frac{7}{15}\pi$$

§6.2 #29 R_3 about AB







$$V=\int_0^1 \pi\underbrace{(y^2-1)^2}_{\text{outer}}-\pi\underbrace{(\sqrt[3]{y}-1)^2}_{\text{inner}}\,dy$$

$$=\pi\int_0^1 (y^4-2y^2+1)-((y^{1/3})^2-2y^{1/3}+1)\,dy$$

$$=\pi\int_0^1 (y^4-2y^2-y^{2/3}+2y^{1/3})\,dy$$

$$=\pi\left[\frac{y^5}{5}-\frac{2y^3}{3}-\frac{y^{5/3}}{5/3}+2\frac{y^{4/3}}{4/3}\right]_0^1$$

$$=\pi\left[\frac{1}{5}-\frac{2}{3}-\frac{3}{5}+2\cdot\frac{3}{4}\right]$$

$$=\pi\left[-\frac{2}{5}-\frac{2}{3}+\frac{3}{2}\right]=\pi\left[\frac{-12}{30}-\frac{20}{30}+\frac{45}{30}\right]$$

$$=\frac{13\pi}{30}$$