§5.5 Evaluate the definite integral.

#59) $\int_1^2 \frac{e^{1/x}}{x^2}\,dx$

$u = \frac{1}{x} = x^{-1}$

$du = -1 \cdot x^{-2}\,dx$

$du = -\frac{1}{x^2}\,dx$

$-du = \frac{1}{x^2}\,dx$

x	$u = 1/x$
1	1
2	1/2

$\int_1^2 e^{\overset{u}{1/x}} \cdot \underbrace{\frac{1}{x^2}\,dx}_{-du}$

$= -\int_1^{1/2} e^u\,du$

$= \int_{1/2}^{1} e^u\,du$

$= \left[e^u\right]_{1/2}^{1}$

$= e^1 - e^{1/2}$

$= e - \sqrt{e}$

§5.5
\# 67

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$$\int_e^{e^4} \frac{1}{\sqrt{\ln x}} \cdot \frac{1}{x}\,dx$$

$$\int \frac{1}{\sqrt{u}}\,du = \int u^{-1/2}\,du$$

$$= \frac{u^{1/2}}{1/2} = 2\sqrt{u}$$

$$= 2\sqrt{\ln x}\,\Big|_e^{e^4}$$

$$= 2\left[\sqrt{\ln e^4} - \sqrt{\ln e}\right] = 2\left[\sqrt{4} - \sqrt{1}\right]$$

$$= 2(2-1) = 2$$

§5.5 #65

$$\int_1^2 \underbrace{x}_{u+1} \sqrt{\underbrace{x-1}_{u}}\, \underbrace{dx}_{du}$$

$u = x-1$, $u+1 = x$
$du = dx$

x	$u = x-1$
1	$1-1=0$
2	$2-1=1$

$$\int_0^1 (u+1)\sqrt{u}\, du$$

$$= \int_0^1 (u+1)\, u^{1/2}\, du$$

$$= \int_0^1 \left(u^{3/2} + u^{1/2}\right) du$$

$$= \frac{u^{5/2}}{5/2} + \frac{u^{3/2}}{3/2} \Big|_0^1$$

$$= \frac{2}{5}(1)^{5/2} + \frac{2}{3}(1)^{3/2} - 0$$

$$= \frac{2}{5} + \frac{2}{3} = \frac{6}{15} + \frac{10}{15} = \frac{16}{15}$$

§5.5 #58 $\int_0^1 x e^{-x^2} dx$

$u = -x^2$

$du = -2x\,dx$

$-\frac{1}{2} du = x\,dx$

$$\int_0^1 e^{\overset{u}{-x^2}} \cdot \underbrace{x\,dx}_{-\frac{1}{2}du}$$

x	$u = -x^2$
0	0
1	$-(1)^2 = -1$

$$-\frac{1}{2}\int_0^{-1} e^u\,du$$

$$= \frac{1}{2}\int_{-1}^{0} e^u\,du$$

$$= \frac{1}{2}\left[e^u\right]_{-1}^{0} = \frac{1}{2}\left(e^0 - e^{-1}\right)$$

$$= \frac{1}{2}\left(1 - \frac{1}{e}\right)$$

§5.5 #21

$$\int \frac{\cos\sqrt{t}}{\sqrt{t}}\,dt$$

$u = \sqrt{t} = t^{1/2}$

$du = \frac{1}{2} t^{-1/2} dt$

$2\,du = \frac{1}{\sqrt{t}}\,dt$

$= \int \cos\sqrt{t} \cdot \frac{1}{\sqrt{t}}\,dt$

$= 2\int \cos u\,du$

$= 2\sin u + C$

$= 2\sin\sqrt{t} + C$

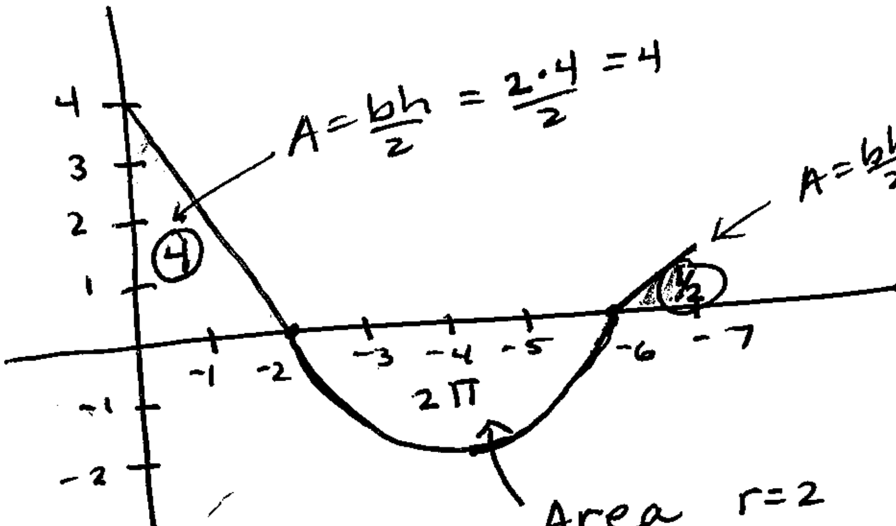
§5.2

#34 Evaluate the integrals

a) $\int_0^2 g(x)\,dx = 4$

b) $\int_2^6 g(x)\,dx = -2\pi$

c) $\int_0^7 g(x)\,dx = 4 - 2\pi + \frac{1}{2} = \frac{9}{2} - 2\pi$



$A = \frac{bh}{2} = \frac{2 \cdot 4}{2} = 4$

$A = \frac{bh}{2} = \frac{1 \cdot 1}{2} = \frac{1}{2}$

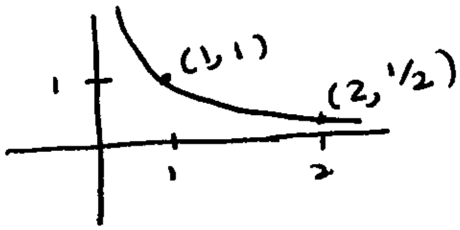
Area $r = 2$

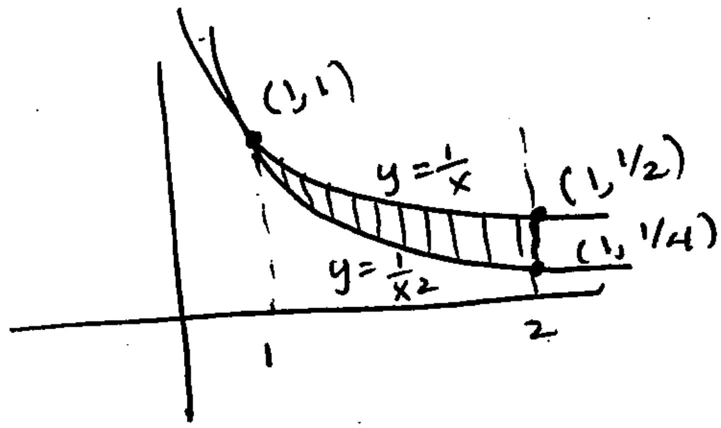
half of circle $\frac{1}{2}(\pi r^2) = \frac{1}{2}\pi(2)^2 = 2\pi$

§6.1 # 9 Find the area between the curves. Sketch the graph.

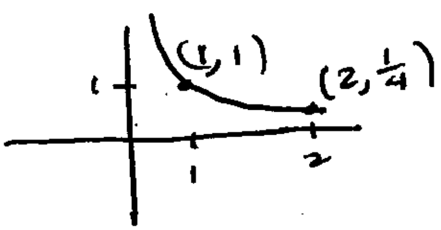
$y = \frac{1}{x}, \quad y = \frac{1}{x^2}, \quad x = 2$

$y = \frac{1}{x}$





$y = \frac{1}{x^2}$



$$A = \int_1^2 \left(\frac{1}{x} - \frac{1}{x^2}\right) dx$$

$$= \int_1^2 \left(\frac{1}{x} - x^{-2}\right) dx$$

Points of intersection

$$= \left[\ln|x| - \frac{x^{-1}}{-1}\right]_1^2 = \left[\ln|x| + \frac{1}{x}\right]_1^2$$

$\frac{1}{x} = \frac{1}{x^2}$

$x^2 = x$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 1$

$$= \left(\ln|2| + \frac{1}{2}\right) - \left(\overbrace{\ln|1|}^{=0} + \frac{1}{1}\right)$$

$$= \ln 2 + \frac{1}{2} - 1 = \ln 2 - \frac{1}{2}$$

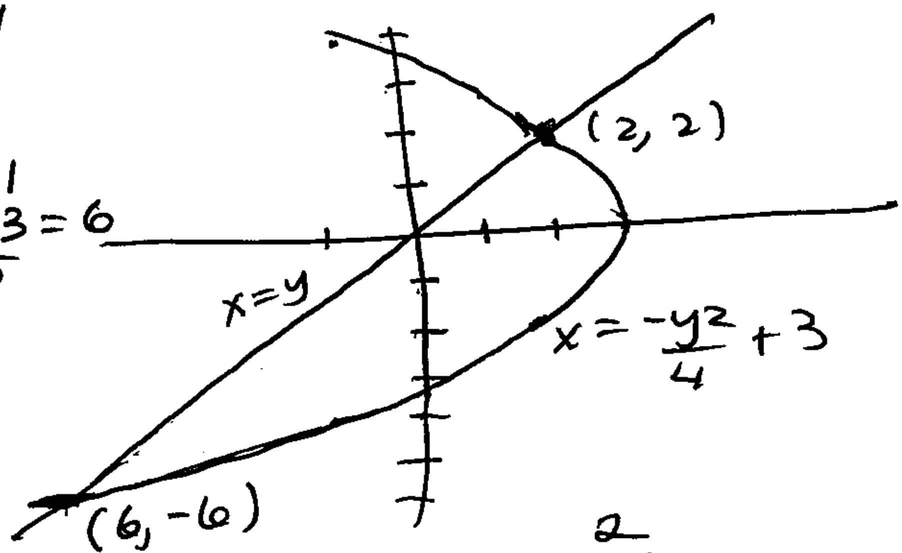
§6.1 #20 $\quad 4x + y^2 = 12, \qquad x = y$

$$4x = -y^2 + 12$$

$$x = \frac{-y^2}{4} + 3$$

$$(x-3) = \frac{-y^2}{4}$$

y	$x = \frac{-y^2}{4} + 3$
0	3
± 1	$-\frac{1}{4} + 3 = \frac{11}{4}$
± 2	$-1 + 3 = 2$
± 4	$-\frac{16}{4} + 3 = -1$
± 6	$-\frac{36}{4} + 3 = 6$
± 8	$-\frac{64}{8} + 3 = -5$



Points of intersection

$$\frac{-y^2}{4} + 3 = y$$

$$-y^2 + 12 = 4y$$

$$0 = y^2 + 4y - 12$$

$$0 = (y+6)(y-2)$$

$$y = -6,\ y = 2$$

$$A = \int_{-6}^{2} \left[\left(\frac{-y^2}{4} + 3 \right) - (y) \right] dy$$

$$= \left[\frac{-y^3}{4 \cdot 3} + 3y - \frac{y^2}{2} \right]_{-6}^{2}$$

$$= \left(\frac{-2^3}{12} + 3(2) - \frac{2^2}{2} \right) - \left(\frac{-(-6)^3}{12} + 3(-6) - \frac{(-6}{2} \right.$$

$$= \left(\frac{-8}{12} + 6 - 2 \right) - \left(18 - 18 - 18 \right)$$

$$= \frac{-2}{3} + 4 + 18 = 22 - \frac{2}{3} = \frac{66 - 2}{3}$$

$$= \frac{64}{3}$$