§6.1 #8

$y = x^2 - 2x, \quad y = x + 4$

$y = (x^2 - 2x + 1) - 1$

$y = (x-1)^2 - 1$

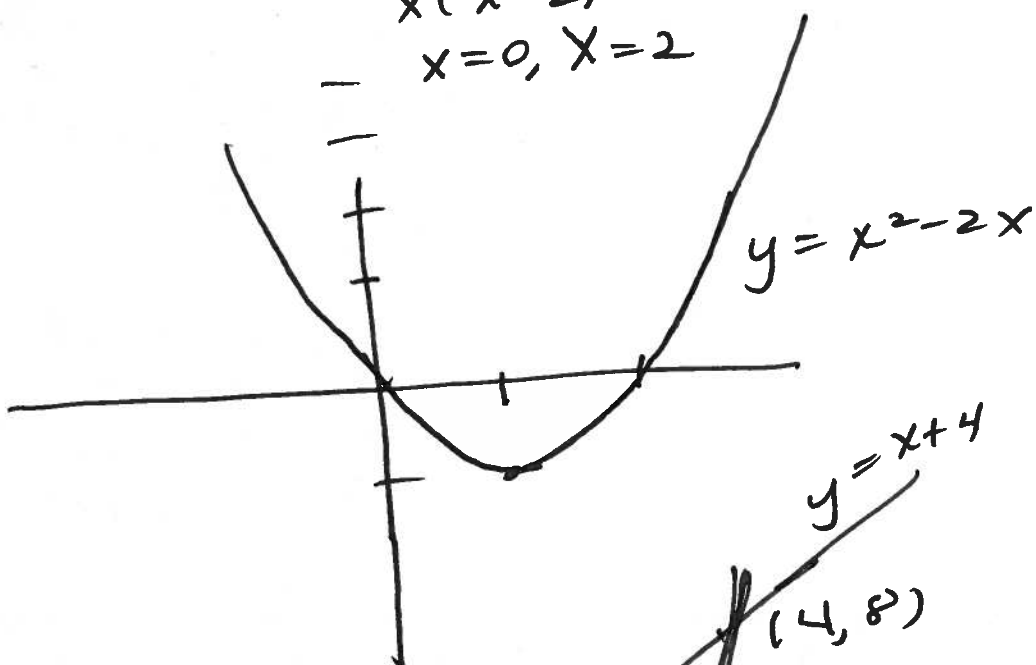
vertex $(1, -1)$

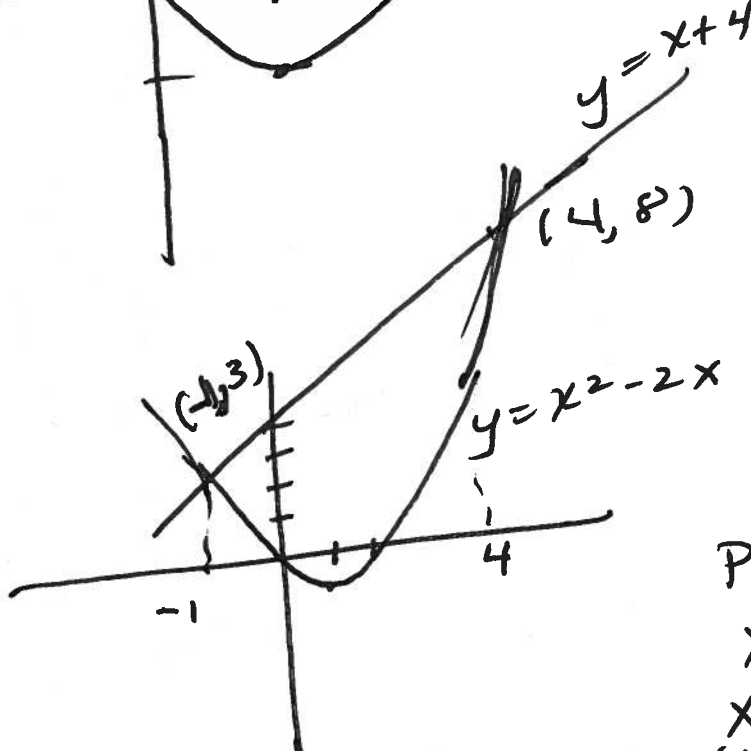
<u>x-int</u>

$x^2 - 2x = 0$

$x(x-2) = 0$

$x = 0, \ x = 2$





Pts of Intersectio

$x^2 - 2x = x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$x = -1, \ x = 4$

$$A = \int_{-1}^{4} \left[(x+4) - (x^2 - 2x) \right] dx$$

$$= \int_{-1}^{4} \left(-x^2 + 3x + 4 \right) dx$$

$$= \frac{-x^3}{3} + 3\frac{x^2}{2} + 4x \Big]_{-1}^{4}$$

$$= \left[\frac{-4^3}{3} + \frac{3}{2}(4)^2 + 4 \cdot 4 \right]$$

$$- \left[\frac{-(-1)^3}{3} + \frac{3}{2}(-1)^2 + 4(-1) \right]$$

$$= \left(-\frac{64}{3} + \frac{3}{\cancel{2}}(\cancel{16}^{\,8}) + 16 \right) - \left(\frac{1}{3} + \frac{3}{2} - 4 \right)$$

$$= \frac{-64}{3} + 24 + 16 - \frac{1}{3} - \frac{3}{2} + 4$$

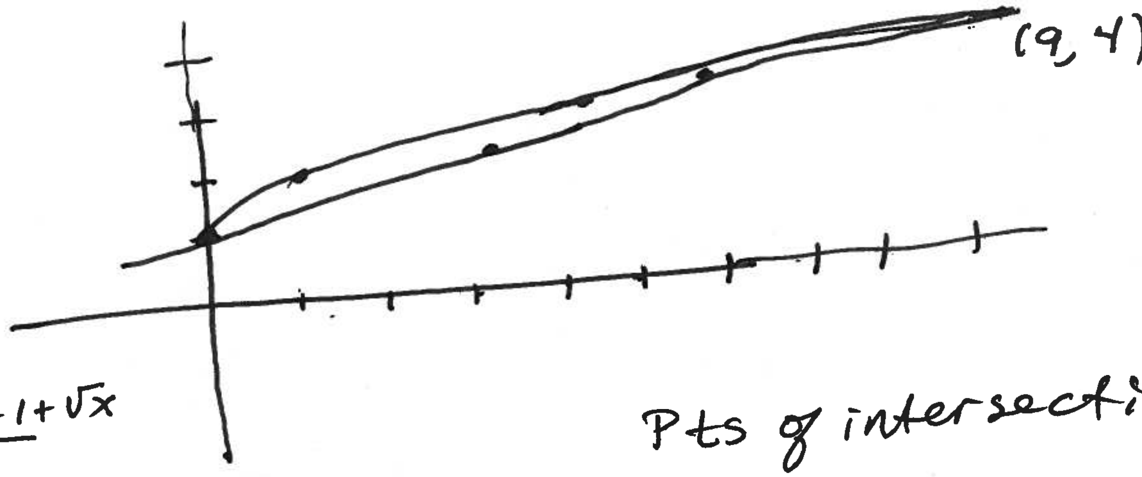
$$= \frac{-65}{3} - \frac{3}{2} + 44$$

$$= \frac{-130}{6} - \frac{9}{6} + \frac{264}{6}$$

$$= \frac{264 - 139}{6} = \frac{125}{6}$$

$$\begin{array}{r} 2\cancel{6}^{5}4 \\ 139 \\ \hline 125 \end{array}$$

§6.1 #10 $y = 1+\sqrt{x}$, $y = \dfrac{(3+x)}{3} = \dfrac{x}{3}+1$



x	$y=1+\sqrt{x}$
0	1
1	2
4	3
9	4

Pts of intersection

$$1+\sqrt{x} = \frac{x}{3}+1$$

$$\sqrt{x} = \frac{x}{3}$$

$$x = \frac{x^2}{9}$$

$$9x = x^2$$

$$x^2 - 9x = 0$$

$$x(x-9) = 0$$

$$x=0, \ x=9$$

Area

$$A = \int_0^9 \left((1+x^{1/2}) - \left(\frac{x}{3}+1\right) \right) dx$$

$$= \int_0^9 \left(x^{1/2} - \frac{x}{3} \right) dx$$

$$= \left[\frac{x^{3/2}}{3/2} - \frac{x^2}{3\cdot 2} \right]_0^9$$

$$= \left(\frac{2}{3} x^{3/2} - \frac{x^2}{6} \right)_0^9$$

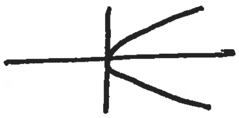
$$= \frac{2}{3}(9)^{3/2} - \frac{(9)^2}{6} - 0$$

$$= \frac{2}{3}\cdot 27 - \frac{81}{6} = 18 - \frac{27}{2} = \frac{36-27}{2} = \frac{9}{2}$$

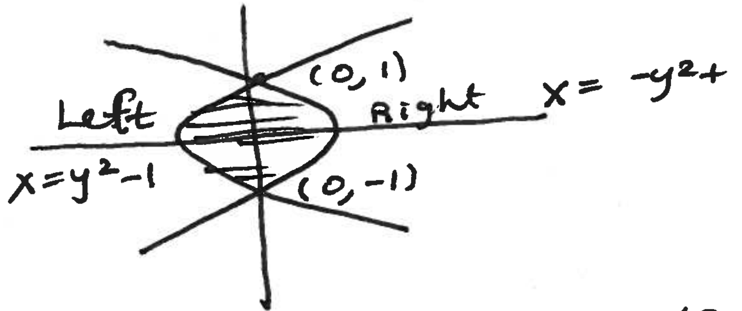
§6.1 #21 $x = 1 - y^2$, $x = y^2 - 1$

$x = -y^2 + 1$

$x = y^2$



$x = -y^2$



intercepts

$1 - y^2 = y^2 - 1$

$-2y^2 = -2$

$y^2 = 1$

$y = \pm 1$

Right − Left

$$\int_{-1}^{1} \left((-y^2+1) - (y^2-1)\right) dy$$

$$= 2\int_{0}^{1} (-2y^2 + 2)\, dy$$

$$= 2(-2) \int_{0}^{1} (y^2 - 1)\, dy$$

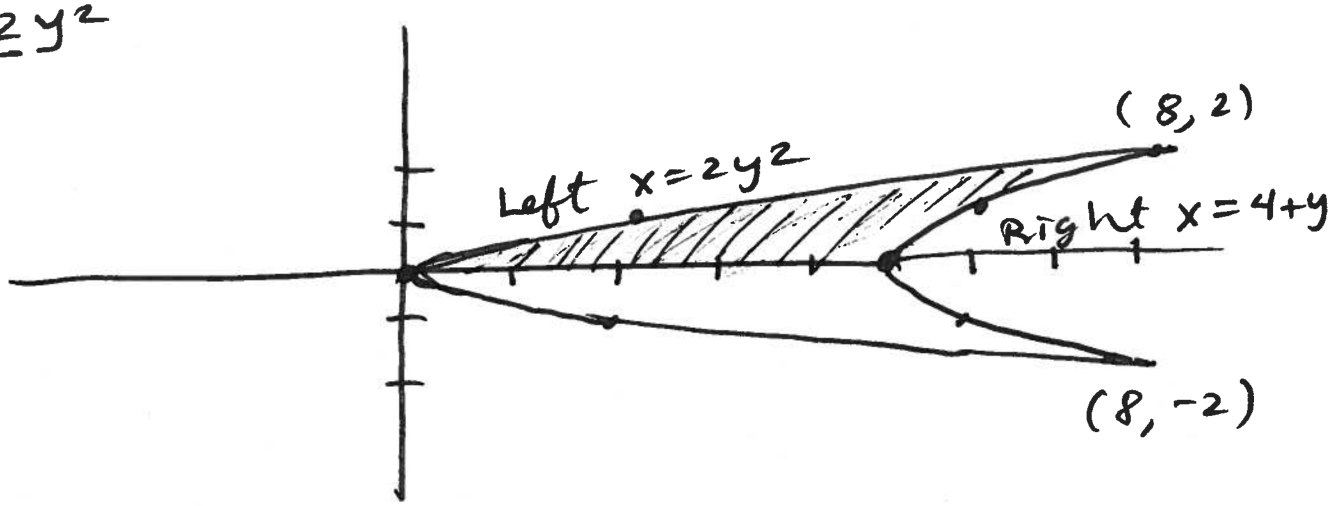
$$= -4 \left[\frac{y^3}{3} - y \right]_0^1 =$$

$$= -4\left(\frac{1^3}{3} - 1\right) - (0)$$

$$= -4\left(\frac{-2}{3}\right) = \frac{8}{3}$$

§6.1 #19 $x = 2y^2, \quad x = 4 + y^2$

y	$x = 2y^2$
0	0
1	2
2	8
-1	2
-2	8



y	$x = 4 + y^2$
0	4
±1	5
±2	8

Points of intersection

$$2y^2 = 4 + y^2$$
$$y^2 = 4$$
$$y = \pm 2$$

$$A = \int_{-2}^{2} \left((4+y^2) - (2y^2)\right) dy$$
$$= 2\int_0^2 (-y^2 + 4)\, dy$$
$$= 2\left(-\frac{y^3}{3} + 4y\right)\Big|_0^2$$
$$= 2\left(-\frac{8}{3} + 8\right) = 2\left(\frac{-8+24}{3}\right)$$
$$= 2\left(\frac{16}{3}\right) = \frac{32}{3}$$

Example Use Part 1 of the Fundamental Theorem of Calculus to find the derivative.

a) $g(x) = \int_1^x \sqrt{\sin t}\, dt$

$g'(x) = \sqrt{\sin x}$

b) $h(x) = \int_x^2 e^{t^2}\, dt = -\int_2^x e^{t^2} dt$

$h'(x) = -e^{x^2}$

c) $h(x) = \int_2^{\cos x} \sqrt{t^5+1}\; dt$

$h'(x) = \sqrt{(\cos x)^5 + 1} \cdot \underbrace{(-\sin x)}_{\frac{d}{dx}(\cos x)}$

$= -\sin x \sqrt{\cos^5 x + 1}$

§5.5 Example.

Evaluate

$$\int \tan x \, dx = \int \frac{\sin x}{\cos x} \, dx$$

$u = \cos x$

$du = -\sin x \, dx$

$-du = \sin x \, dx$

$$= \int \left(\frac{1}{\underbrace{\cos x}_{u}}\right) \underbrace{\sin x \, dx}_{-du}$$

$$= -\int \frac{1}{u} \, du$$

$$= -\ln|u| + C$$

$$= -\ln|\cos x| + C$$

$$= (-1) \ln|\cos x| + C$$

$$= \ln|\cos x|^{-1} + C$$

$$= \ln\left|\frac{1}{\cos x}\right| + C$$

$$\boxed{\int \tan x \, dx = \ln|\sec x| + C}$$

§5.5 # 67 $\displaystyle\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}}$

$u = \ln x$

$du = \frac{1}{x}\,dx$

x	$u = \ln x$
e	$\ln e = 1$
e^4	$\ln e^4 = 4$

$$\int_e^{e^4} \left(\frac{1}{\sqrt{\ln x}}\right) \cdot \frac{1}{x}\,dx$$

$$= \int_1^4 \frac{1}{\sqrt{u}}\,du$$

$$= \int_1^4 u^{-1/2}\,du$$

$$= \frac{u^{1/2}}{1/2}\Bigg|_1^4 = 2\left[\sqrt{u}\right]_1^4 = 2\left[\sqrt{4} - \sqrt{1}\right]$$

$$= 2(2-1)$$

$$= 2$$

§5.5
#44

$$\int \frac{x^2}{\sqrt{1-x}}\,dx \qquad x^2 \overset{?}{=} (1-u)^2, \quad \sqrt{\underbrace{1-x}_{u}}, \quad \underbrace{dx}_{-du}$$

$u = 1-x \Rightarrow u-1 = -x$
$du = -dx \qquad -u+1 = x$
$-du = dx \qquad x = 1-u$

$$-\int \frac{(1-u)^2}{\sqrt{u}}\,du$$

$$= -\int \frac{(1-2u+u^2)}{u^{1/2}}\,du$$

$$= -\int \left(\frac{1}{u^{1/2}} - \frac{2u}{u^{1/2}} + \frac{u^2}{u^{1/2}}\right)du$$

$$= -\int (u^{-1/2} - 2u^{1/2} + u^{3/2})\,du$$

$$= -\left[\frac{u^{1/2}}{1/2} - \frac{2u^{3/2}}{3/2} + \frac{u^{5/2}}{5/2}\right]$$

$$= -2u^{1/2} + 2\cdot\frac{2}{3}\,u\,u^{1/2} + -\frac{2}{5}\,u^2 u^{1/2}$$

$$= -2u^{1/2} + \frac{4}{3}u^{3/2} - \frac{2}{5}u^{5/2}$$

$$= -2(1-x)^{1/2} + \frac{4}{3}(1-x)^{3/2} - \frac{2}{5}(1-x)^{5/2} + C$$

§5.5 #46 $\int x^3 \sqrt{x^2+1}\, dx$

$$\int x^2 \sqrt{x^2+1} \cdot x\,dx$$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2} du = x\,dx$

$x^2 = u-1$

$$\frac{1}{2}\int (u-1)\sqrt{u}\, du$$

$$= \frac{1}{2}\int (u-1)\, u^{1/2}\, du$$

$$= \frac{1}{2}\int \left(u^{3/2} - u^{1/2}\right) du$$

$$= \frac{1}{2}\left(\frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2}\right) + C$$

$$= \frac{1}{2}\left(\frac{2}{5}(x^2+1)^{5/2} - \frac{2}{3}(x^2+1)^{3/2}\right) + C$$

$$= \frac{1}{5}(x^2+1)^{5/2} - \frac{1}{3}(x^2+1)^{3/2} + C$$