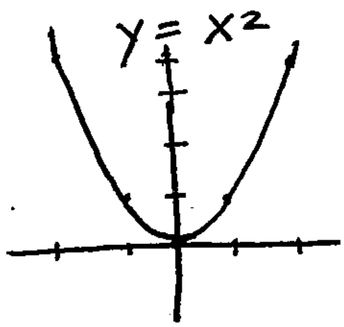
§6.1 Continued

<u>EXAMPLE</u> Find the area bounded by $y = x^2$ and $y = 2x - x^2$. Sketch the graph.

<u>SOLUTION</u>



$$y = -x^2 + 2x$$

complete the square

$$y = -(x^2 - 2x)$$

$$y = -(x^2 - 2x + 1) + 1$$

$$\left(\frac{-2}{2}\right)^2$$

$$y = -(x-1)^2 + 1$$

<u>or</u> $h = \frac{-b}{2a}$

$$h = \frac{-2}{2(-1)} = 1$$

$$k = f(1) = 1$$

$$y = a(x-h)^2 + k$$

<u>or</u>

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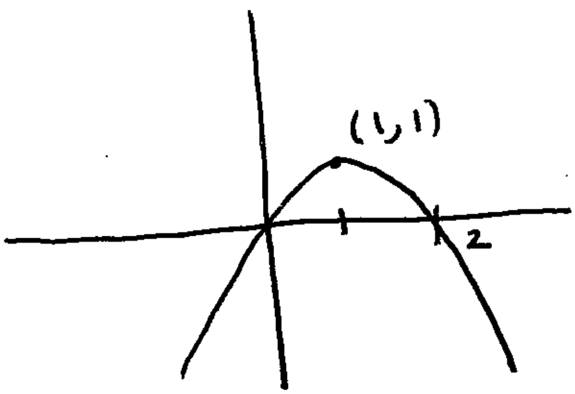
$$y' = -2x + 2 = 0$$

$$-2x = -2$$

$$x = 1$$

$$h = 1$$

$$k = f(1) = 1$$



<u>x-int</u> $-x^2 + 2x = 0$

$$x^2 - 2x = 0$$

$$x(x-2) = 0$$

$$x = 0,\ x = 2$$

Points of intersection

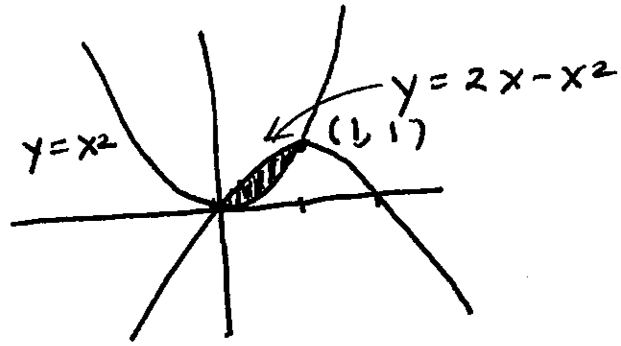
$y=x^2, \quad y=2x-x^2$

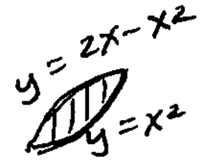
$x^2 = 2x - x^2$

$2x^2-2x=0$

$2x(x-1)=0$

$x=0, \quad x=1$





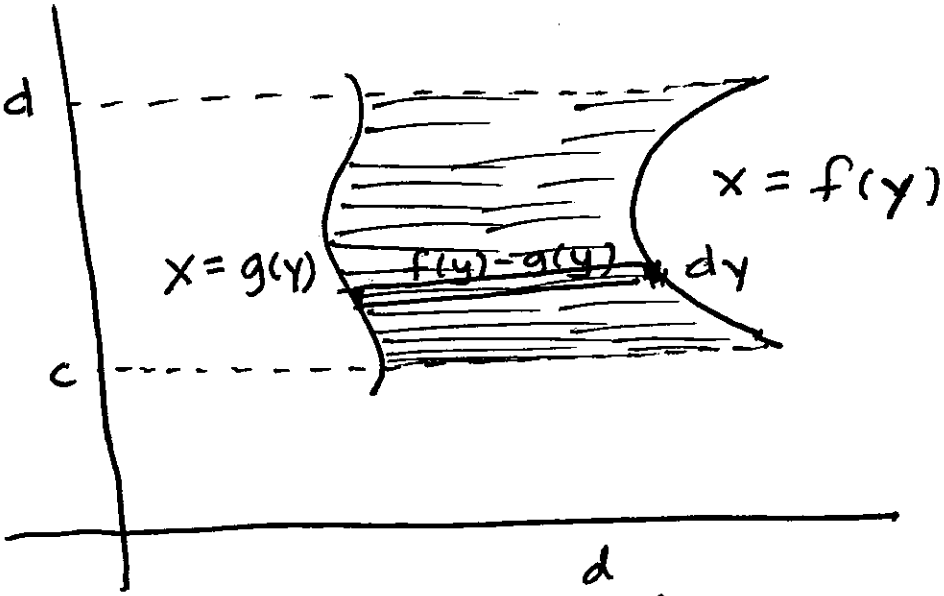
Area

$$A = \int_0^1 \left[\underset{\text{top}}{(2x-x^2)} - \underset{\text{bottom}}{(x^2)} \right] dx$$

$$= \int_0^1 (2x-2x^2)\,dx = \left[\cancel{2}\frac{x^2}{\cancel{2}} - 2\frac{x^3}{3} \right]_0^1$$

$$= (1)^2 - \frac{2}{3}(1)^3 - 0$$

$$= 1 - \frac{2}{3} = \frac{1}{3}$$



$$A = \int_c^d (f(y) - g(y))\,dy$$

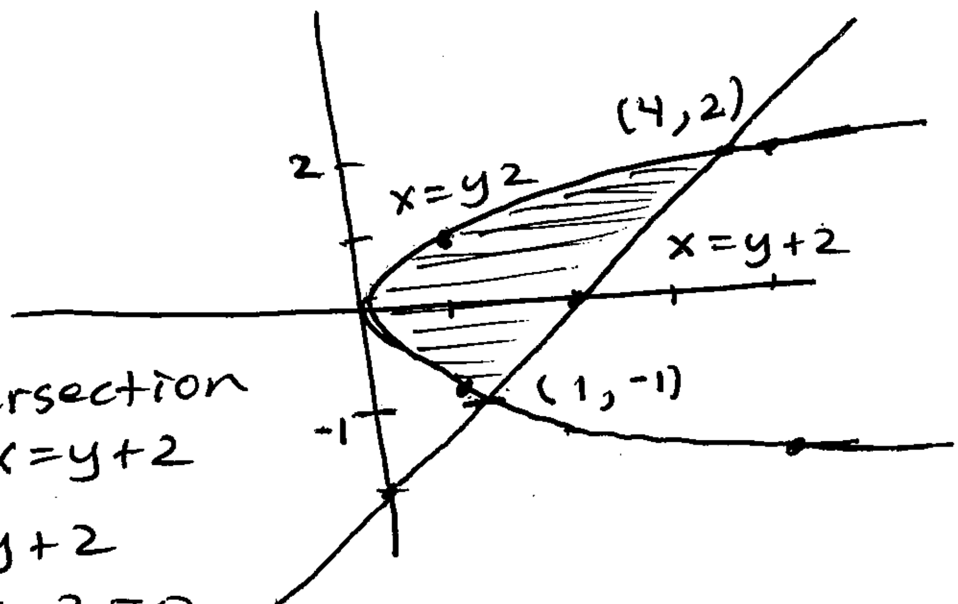
right − left

<u>EXAMPLE</u> Find the area between $y = x-2$ and the parabola $x = y^2$

<u>SOLUTION</u>

$y = x - 2$

$y + 2 = x$



Points of intersection

$x = y^2, \quad x = y + 2$

$y^2 = y + 2$

$y^2 - y - 2 = 0$

$(y-2)(y+1) = 0$

$y = 2, \ y = -1$

$$A = \int_{-1}^{2} \left[(y+2) - (y^2) \right] dy$$

$$= \left[\frac{y^2}{2} + 2y - \frac{y^3}{3} \right]_{-1}^{2}$$

$$= \left(\frac{(2)^2}{2} + 2(2) - \frac{(2)^3}{3} \right) - \left(\frac{(-1)^2}{2} + 2(-1) - \frac{(-1)^3}{3} \right)$$

$$= \left(2 + 4 - \frac{8}{3} \right) - \left(\frac{1}{2} - 2 + \frac{+1}{3} \right)$$

$$= \left(6 - \frac{8}{3} \right) - \quad \frac{1}{2} + 2 - \frac{1}{3}$$

$$= 8 - \frac{9}{3} - \frac{1}{2} = 8 - 3 - \frac{1}{2}$$

$$= 5 - \frac{1}{2} = \frac{10}{2} - \frac{1}{2} = \frac{9}{2}$$

$$\boxed{\text{Next Thurs. Test Ch 5 \& 6.1}}$$