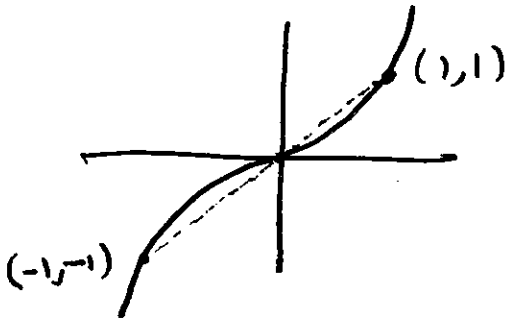
§5.5 Continued

Odd Functions

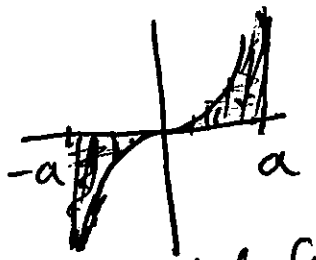
- A function f is odd if $f(-x) = -f(x)$
- Odd functions are symmetric about the origin.
- Example $f(x) = x^3$

$$f(-x) = (-x)^3 = -x^3 = -f(x)$$

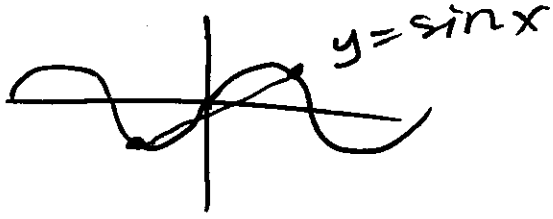


- If f is odd, then

$$\int_{-a}^{a} f(x)\,dx = 0$$

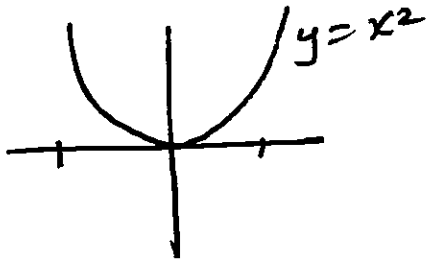


- Other examples of odd functions
  - $y = x^n$, n odd
  - $y = \sin x$
  - $y = \tan x$



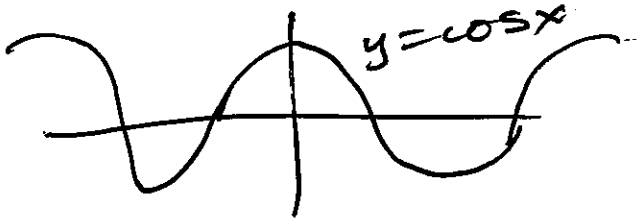
Even Functions

- A function, f, is even if
  $f(-x) = f(x)$
- Even functions are symmetric about the y-axis
- Example: $f(x) = x^2$
  $f(-x) = (-x)^2 = x^2$
  $= f(x)$



- If f is even, then
  $$\int_{-a}^{a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx$$

- Other examples of even functions: $y = x^n$, n even
  $y = \cos x$

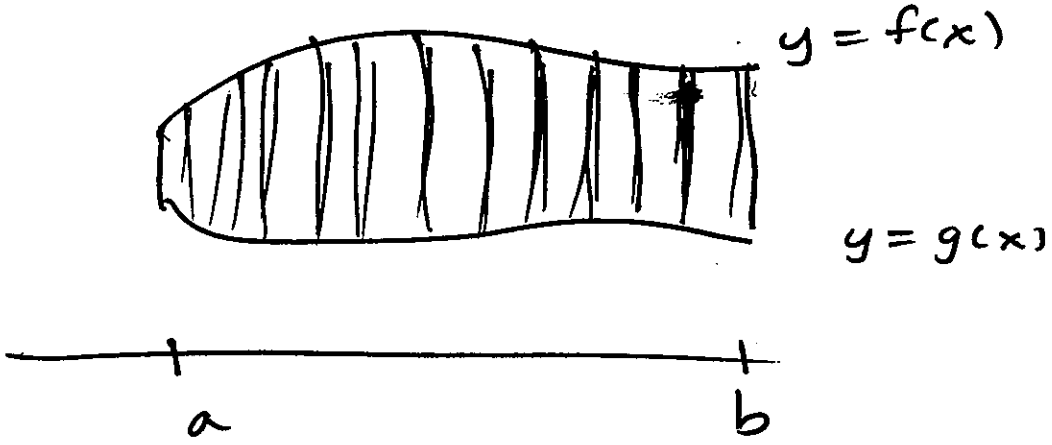


§6.1 Areas Between Curves

HW §6.1 #1-28

The area A of the region bounded by the curves $y = f(x)$, $y = g(x)$, and the lines $x = a$, $x = b$, where f and g are continuous and $f(x) \geq g(x)$ for all $a \leq x \leq b$

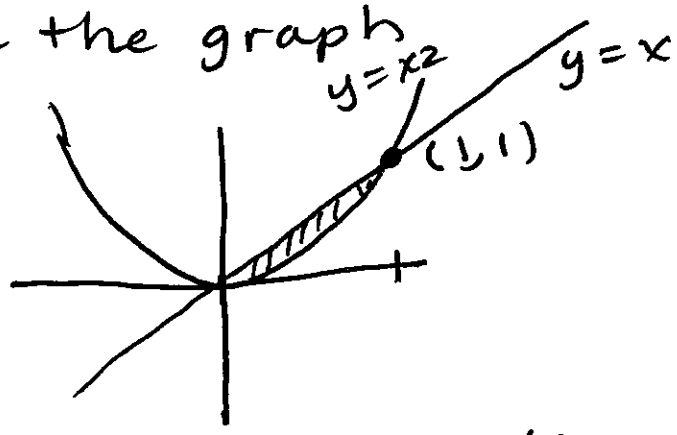
then $A = \int_a^b [\underbrace{f(x)}_{\text{top}} - \underbrace{g(x)}_{\text{bottom}}]\, dx$



EXAMPLE Find the area of the region bounded by the curves $y = x^2$, $y = x$.

SOLUTION

- Sketch the graph



- Points of intersection

$$y = x^2, \quad y = x$$

$$x^2 = x$$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0, \quad x - 1 = 0$$

$$x = 1$$

- Integrate

$$\int_0^1 (\underbrace{x}_{\text{top}} - \underbrace{x^2}_{\text{bottom}})\, dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1$$

$$= \frac{1^2}{2} - \frac{1^3}{3} = \frac{1}{2} - \frac{1}{3} = \frac{3}{6} - \frac{2}{6} = \boxed{\frac{1}{6}}$$

§5.3 #37 $\int_{1/2}^{\sqrt{3}/2} \frac{6}{\sqrt{1-t^2}}\,dt$ $\qquad \frac{d}{dt}\sin^{-1}t = \frac{1}{\sqrt{1-t^2}}$

$$= 6\left[\sin^{-1}(t)\right]_{1/2}^{\sqrt{3}/2}$$

$$= 6\left[\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) - \sin^{-1}\left(\frac{1}{2}\right)\right]$$

$$= 6\left[\pi/3 - \frac{\pi}{6}\right] = 6\left[\frac{\pi}{6}\right] = \pi$$