§5.4 The Indefinite Integral.

Hw §5.4 #1-44

The indefinite integral

$\int f(x)\,dx = F(x)$ means $F'(x) = f(x)$.

<u>Example:</u> Evaluate.

① $\int \sec^2 x\,dx = \tan x + C$

② $\int e^x\,dx = e^x + C$

③ $\int \frac{1}{1+x^2}\,dx = \tan^{-1} x + C$

§5.5 The Substitution Method.

HW §5.5 # 1–46, 51–70

EXAMPLE Find the derivative.

① $y = \sin(x^2)$

$y' = 2x\cos(x^2)$

② $y = (x^2+1)^{10}$

$y' = 10(x^2+1)^9 \cdot 2x$

$y' = 20x(x^2+1)^9$

EXAMPLE Evaluate the integral.

① $\int 2x\cos(x^2)\,dx = \sin(x^2) + C$

We know this from the previous example.

Let's use the substitution method.

$u = x^2$

$du = 2x\,dx$

$$\int \cos(\underbrace{x^2}_{u}) \cdot \underbrace{2x\,dx}_{du} = \int \cos u\,du = \sin u + C$$

$$= \sin(x^2) + C$$

(2)

$$\int 2x\,(x^2+1)^7\,dx \qquad u = x^2+1,\quad du = 2x\,dx$$

$$= \int \underbrace{(x^2+1)}_{u}{}^7 \cdot \underbrace{2x\,dx}_{du}$$

$$= \int u^7\,du$$

$$= \frac{u^8}{8} + C = \frac{(x^2+1)^8}{8} + C$$

(3) $\int x e^{(x^2)}\,dx$

$$u = x^2$$

$$du = 2x\,dx$$

$$\frac{1}{2}du = x\,dx$$

$$\int e^{(\overset{u}{x^2})} \cdot \underbrace{x\,dx}_{\frac{1}{2}du}$$

$$= \int e^u \cdot \frac{1}{2}\,du$$

$$= \frac{1}{2}\int e^u\,du = \frac{1}{2}e^u + C$$

$$= \frac{1}{2}e^{x^2} + C$$

④ $\int x\sec^2(x^2+1)\,dx = \int \sec^2(\underbrace{x^2+1}_{u})\,\underbrace{x\,dx}_{\frac{1}{2}du}$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \frac{1}{2}\int \sec^2 u\,du$

$= \frac{1}{2}\tan u + C$

$= \boxed{\frac{1}{2}\tan(x^2+1)+C}$

⑤ $\int x^2\sqrt{x^3+1}\,dx$

$u = x^3+1$

$du = 3x^2\,dx$

$\frac{1}{3}du = x^2\,dx$

$= \frac{1}{3}\int \sqrt{u}\,du$

$= \frac{1}{3}\int u^{1/2}\,du$

$= \frac{1}{3}\,\frac{u^{3/2}}{3/2} + C$

$= \frac{1}{3}\cdot\frac{2}{3}\,u^{3/2}+C$

$= \frac{2}{9}(x^3+1)^{3/2}+C$

⑥ $\int \frac{x}{x^2+1}\,dx$

$u = x^2+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$= \int \frac{1}{\underbrace{x^2+1}_{u}}\cdot\underbrace{x\,dx}_{1/2\,du}$

$= \frac{1}{2}\int \frac{1}{u}\,du = \frac{1}{2}\ln|u|+C$

$= \frac{1}{2}\ln|x^2+1|+C$

(7) $\int \underbrace{\cos^3 x}_{u^3} \underbrace{\sin x\,dx}_{-du}$

$u = \cos x$
$du = -\sin x\,dx$
$-du = \sin x\,dx$

$= \int (\cos x)^3 \sin x\,dx$

$= -\int u^3\,du$

$= -\frac{u^4}{4} + C = \boxed{-\frac{\cos^4 x}{4} + C}$

(8) $\int \frac{\cos(\ln x)}{x}\,dx$

$u = \ln x$
$du = \frac{1}{x}\,dx$

$= \int \cos(\underbrace{\ln x}_{u}) \cdot \underbrace{\frac{1}{x}\,dx}_{du}$

$= \int \cos u\,du = \sin u + C$
$= \sin(\ln x) + C.$

Quiz Thurs. §5.1–5.3

§5.1 #21 Determine a region whose area is equal to the given limit.

$$\lim_{n\to\infty} \sum_{i=1}^{n} \frac{\pi}{4n} \tan\left(\frac{i\pi}{4n}\right) = \int_a^b f(x)\,dx$$
$$= \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$$
$$a \le x \le b$$

SOLUTION

- $f(x) = \tan x$

$\Delta x = \frac{\pi}{4n}$, $\quad x_i = \frac{i\pi}{4n}$

$\Delta x = \frac{b-a}{n} = \frac{\pi}{4n}$ $\quad x_i = a + i\Delta x$

$\quad x_i = a + i\frac{\pi}{4n}$

$\neq \quad b - a = \frac{\pi}{4}$ $\quad \frac{i\pi}{4n} = a + \frac{i\pi}{4n}$

$a = 0$

$\boxed{\text{so } b = \frac{\pi}{4}}$ $\quad \boxed{a = 0}$

The region below $f(x) = \tan x$, $0 \le x \le \frac{\pi}{4}$.

§5.1 #19 $f(x) = x\cos x, \quad 0 \le x \le \frac{\pi}{2}$

Find an expression for the area under f as a limit.

$$\boxed{A = \int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x, \quad a \le x \le b}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} x_i \cos(x_i)\Delta x$$

$$= \boxed{\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{i\pi}{2n}\right)\cos\left(\frac{i\pi}{2n}\right)\frac{\pi}{2n}}$$

$f(x) = x\cos x$

$a = 0, \quad b = \frac{\pi}{2}$

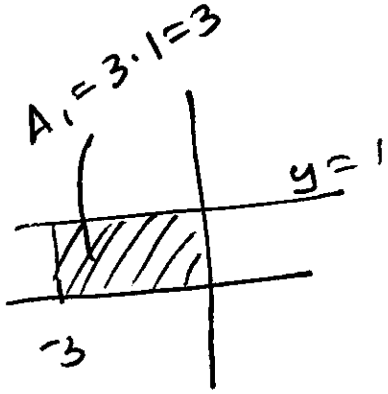
$$\boxed{\Delta x = \frac{b-a}{n}} = \frac{\pi/2 - 0}{n} = \frac{\pi}{2}\cdot\frac{1}{n} = \frac{\pi}{2n}$$

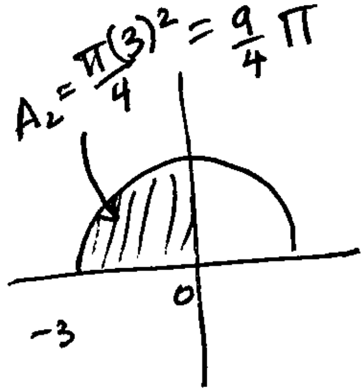
$$\boxed{x_i = a + i\Delta x}$$

$$x_i = 0 + i\frac{\pi}{2n} = \frac{i\pi}{2n}$$

§5.2 #37 Evaluate the integral by interpreting it in terms of area.

$$\int_{-3}^{0} \left(1+\sqrt{9-x^2}\right)dx = \int_{-3}^{0} 1\,dx + \int_{-3}^{0} \sqrt{9-x^2}\,dx$$





$y = \sqrt{9-x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

$x^2 + y^2 = 3^2$

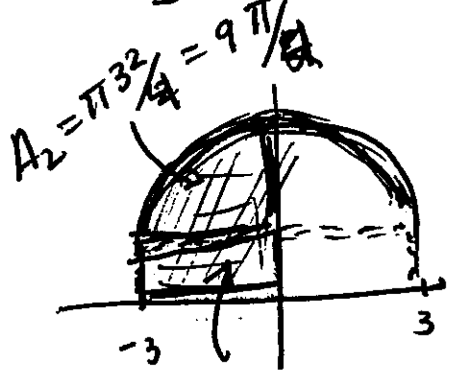
upper semi-circle

$r = 3$

Answer: $3 + \frac{9}{4}\pi$

or

$y = 1 + \sqrt{9-x^2}$



$A_1 = 3$

$A = 3 + \frac{9}{4}\pi$