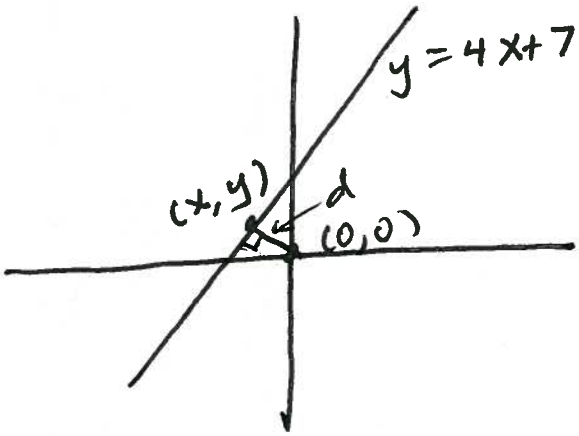
Test Thurs.
Chapter 4

§4.7 #17 Find the point on the line $y=4x+7$ that is closest to the origin.



FORMULAS

$D=d^2=(x-0)^2+(y-0)^2$

$D=x^2+y^2$

$y=4x+7$

Eliminate a variable

$D=x^2+y^2, \quad y=4x+7$

$D=x^2+(4x+7)^2$

Find critnums

$D'=2x+2(4x+7)\cdot 4=0$

$2x+32x+56=0$

$34x=-56$

$x=\frac{-56}{34}=\frac{-28}{17}$

Solve for y

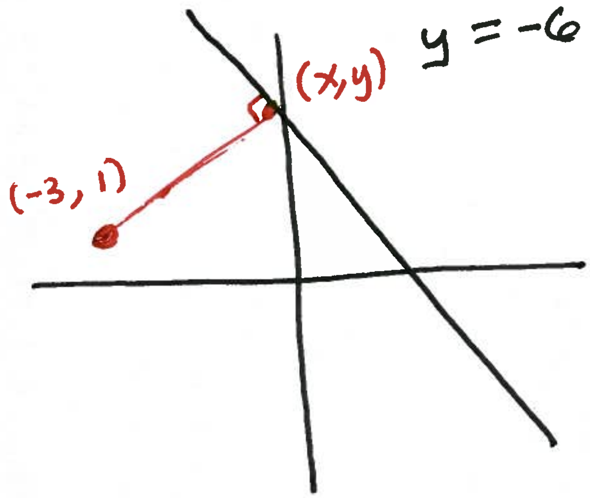
$y=4x+7$

$y=4\left(\frac{-28}{17}\right)+7$

$y=\frac{-112}{17}+\frac{119}{17}=\frac{7}{17}$

The point
$\left(\frac{-28}{17}, \frac{7}{17}\right)$

§4.7
#18 Find the point on the line $6x+y=9$ that is closest to the point $(-3,1)$.



$y=-6x+9$

$D=d^2=(x--3)^2+(y-1)^2$

$D=(x+3)^2+(y-1)^2$

$y=-6x+9$

Eliminate Variable

$D=(x+3)^2+((-6x+9)-1)^2$

$D=(x+3)^2+(-6x+8)^2$

Critnums

$D'=2(x+3)+2(-6x+8)(-6)$

$=2x+6+72x-96$

$=74x-90=0$

$74x=90$

$x=\frac{90}{74}$

$\boxed{x=\frac{45}{37}}$

Solve for y

$y=-6x+9$

$y=-6\left(\frac{45}{37}\right)+9$

$y=\frac{-270}{37}+\frac{333}{37}=\frac{67}{37}$

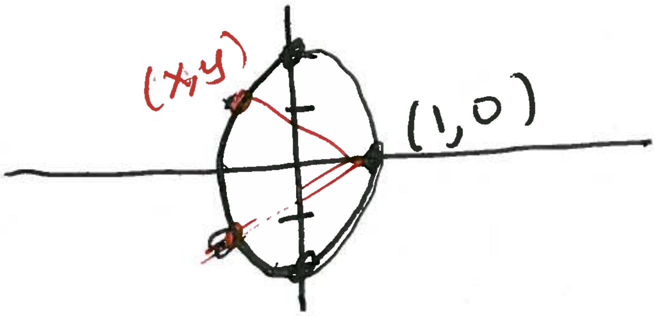
$\boxed{y=\frac{67}{37}}$

$\boxed{\left(\frac{45}{37},\frac{67}{37}\right)}$

§4.7

(19) Find the points on the ellipse $4x^2+y^2=4$ that are farthest away from the point $(1,0)$.

SOLUTION



$$D=d^2=(x-1)^2+(y-0)^2$$
$$D=(x-1)^2+y^2$$
$$4x^2+y^2=4$$

Eliminate a variable

$$y^2=4-4x^2$$
$$D=(x-1)^2+y^2$$
$$D=(x-1)^2+(4-4x^2)$$

Intercepts

$x=0$
$$4(0)^2+y^2=4$$
$$y^2=4$$
$$y=\pm 2$$
$$(0,\pm 2)$$

$y=0$
$$4x^2=4$$
$$x^2=1$$
$$x=\pm 1$$
$$(\pm 1,0)$$

Critnums

$$D'=2(x-1)+-8x$$
$$=2x-2-8x$$
$$=-6x-2=0$$
$$-6x=2$$
$$x=\frac{2}{-6}=-\frac{1}{3}$$
$$x=-\frac{1}{3}$$

Solve for y

$$y^2=4-4x^2$$
$$=4-4(-1/3)^2$$
$$=4-4(1/9)=\frac{36}{9}-\frac{4}{9}=\frac{32}{9}$$
$$y=\pm\sqrt{\frac{32}{9}}=\pm\frac{\sqrt{16}\sqrt{2}}{\sqrt{9}}=\pm\frac{4\sqrt{2}}{3}$$
$$\left(-1/2,\ \pm 4\sqrt{2}/2\right)$$

Second deriv. ~~test~~.

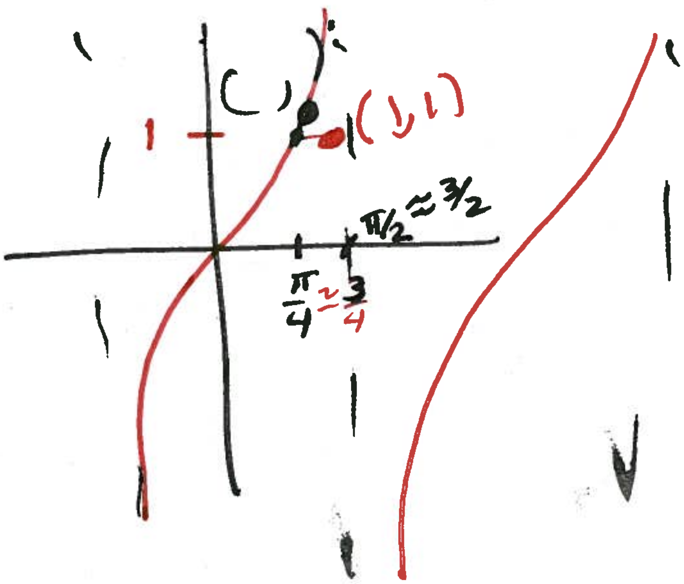
$D' = -6x-2$

$D'' = -6$ so D is concave down,

so $x = -1/3$ is a max.

§ 4.7 #20 (Graphing Calculator Problem).

Find, correct to two decimal places, the coordinates of the point on the curve $y = \tan x$ that is closest to the point $(1,1)$.



$D = d^2 = (x-1)^2 + (y-1)^2$

$y = \tan x$

Eliminate Variable

$D = (x-1)^2 + (\tan x - 1)^2$

Critnums

$D' = 2(x-1) + 2(\tan x - 1) \cdot \sec^2 x$

$2(x-1) + 2(\tan x - 1)\sec^2 x = 0$

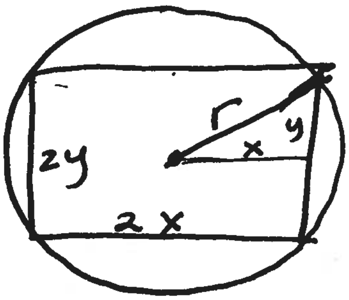
We need a computer to approximate the zeros.

$x \approx 0.82$

$y \approx \tan(.82) \approx 1.081$

$\boxed{(0.82, 1.1)}$

§4.7#21 Find the dimensions of the rectangle of largest area that can be inscribled in a circle of radius r.



$x^2+y^2=r^2$

Area: $A=(2x)(2y)$

$A=4xy$

$a=A^2=16x^2y^2$

$y^2=r^2-x^2$

Elim. Variable

$a=16x^2(r^2-x^2)$

$a=16r^2x^2-16x^4$

$a'=16r^2(2x)-64x^3$

$=32r^2x-64x^3$

$=32x(r^2-2x^2)$

$x=0,\quad r^2-2x^2=0$

$r^2=2x^2$

$x^2=\frac{r^2}{2}$

$x=\frac{r}{\sqrt{2}}=$

$=\frac{r\sqrt{2}}{2}$

Solve for y.

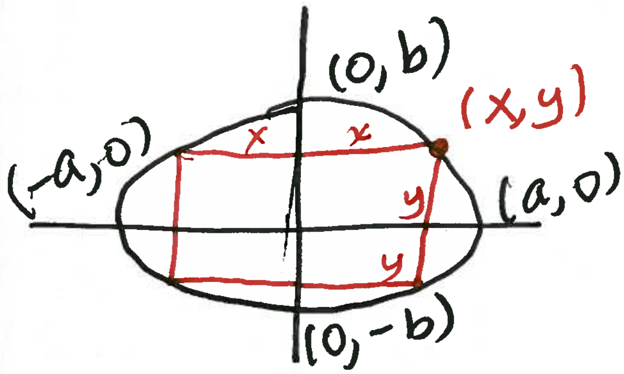
$y^2=r^2-x^2$

$y^2=r^2-\frac{r^2}{2}=\frac{r^2}{2}$

$y=\frac{r}{\sqrt{2}}=\frac{r\sqrt{2}}{2}$

Dimension 2x by 2y: $\boxed{(r\sqrt{2} \text{ by } r\sqrt{2})}$

§4.7 #22 Find the area of the largest rectangle that can be inscribed in the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.



$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

x-int.

$y=0$

$\frac{x^2}{a^2}=1$

$x^2=a$

$x=\pm a$

$(\pm a, 0)$

y-int. $x=0$

$\frac{y^2}{b^2}=1$

$y^2=b^2$

$y=\pm b$

$(0, \pm b$

Area: $A=(2x)(2y)$

$A=4xy$

$a=A^2=16x^2y^2$

$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$

$\frac{y^2}{b^2}=1-\frac{x^2}{a^2}$

$y^2=b^2-\frac{b^2x^2}{a^2}$

Elim. Variable

$a=16x^2y^2$

$a=16x^2\left(b^2-\frac{b^2x^2}{a^2}\right)$

$a=16b^2x^2-\frac{16b^2x^4}{a^2}$

critnums

$a'=16b^2(2x)-\frac{16b^2}{a^2}(4x^3)$

$32b^2x-\frac{64b^2}{a^2}x^3=0$

$32b^2x\left(1-\frac{2}{a^2}x^2\right)=0$

$x=0$, $1-\frac{2x^2}{a^2}=0$

min

$\frac{2x^2}{a^2}=1$, $x^2=\frac{a^2}{2}$

~~$x=$~~

$$x = \frac{a}{\sqrt{2}} = \frac{a\sqrt{2}}{2}$$

Solve for y

$$y^2 = b^2 - \frac{b^2 x^2}{a^2}$$

$$= b^2 - \frac{b^2}{a^2}\left(\frac{a^2}{2}\right) = \frac{b^2}{2}$$

$$y = \frac{b}{\sqrt{2}} = \frac{b\sqrt{2}}{2}$$

Dimensions $2x$ by $2y$

$= a\sqrt{2}$ by $b\sqrt{2}$.

$$\text{Area} = (2x) * (2y)$$

$$= (a\sqrt{2})(b\sqrt{2})$$

$$= 2ab$$

Quiz D

#2 $\lim_{x \to 0} \frac{e^{2x} \to e^0 = 1}{x^3 \to 0}$ DNE $\frac{1}{0}$

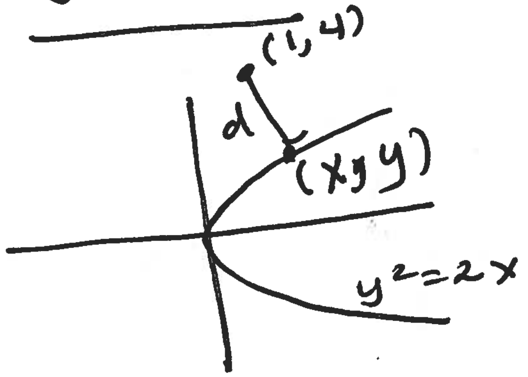
$\lim_{x \to 0^+} \frac{e^{2x} \to 1}{x^3 \to 0_{\text{small pos}}} = \infty$

$\lim_{x \to 0^-} \frac{e^{2x} \to 1}{x^3 \to 0_{\text{small neg}}} = -\infty$

EXAMPLE Find the point on the parabola $y^2 = 2x$ that closest to the point (1,4).

frois gras paté

SOLUTION



$$D = d^2 = (x-1)^2 + (y-4)^2$$

$$y^2 = 2x$$

$$x = \frac{y^2}{2}$$

Eliminate variable

$$D = \left(\frac{y^2}{2} - 1\right)^2 + (y-4)^2$$

Critnums

$$D' = 2\left(\frac{y^2}{2} - 1\right)\left(\frac{\cancel{2}y}{\cancel{2}}\right) + 2(y-4)$$

$$= y^3 - 2y + 2y - 8 = 0$$

$$y^3 - 8 = 0$$

$$y^3 = 8$$

$$\boxed{y = 2}$$

Solve for x

$$x = \frac{y^2}{2}$$

$$x = \frac{(2)^2}{2} = 2$$

$$\boxed{x = 2}$$

$\boxed{\text{The point } (2,2).}$

§4.9 #45 $f''(x) = x^{-2}$, $x > 0$,

$f(1) = 0$, $f(2) = 0$.

Find f.

$$f'(x) = \frac{x^{-1}}{-1} + C$$

$$f'(x) = -\frac{1}{x} + C = \cancel{-x^{-1} + C}$$

$$f(x) = -\ln|x| + Cx + D, \quad x > 0$$

$$f(x) = -\ln x + Cx + D$$

solve for C & D.

$$0 = f(1) = -\overset{=0}{\ln 1} + C(1) + D$$

$$0 = C + D, \quad \boxed{C + D = 0}$$

$$0 = f(2) = -\ln 2 + C(2) + D$$

$$0 = -\ln 2 + 2C + D$$

$$\boxed{2C + D = \ln 2}$$

$$-2(C + D) = (0) \cdot -2$$

$$2C + D = \ln 2$$

$$-2C + -2D = 0$$

$$2C + D = \ln 2$$

$$-D = \ln 2$$

$$\boxed{D = -\ln 2}$$

solve for C

$$C + D = 0$$

$$C - \ln 2 = 0$$

$$\boxed{C = \ln 2}$$

$$\boxed{f(x) = -\ln x + x\ln 2 - \ln 2}$$