

§ 4.9 Continued.

EXAMPLE Solve the initial value problem.

a) $f'(x) = \cos x + \sin x$, $f(0) = 3$.

$$f(x) = \sin x - \cos x + C$$

Solve for C

$$3 = f(0) = \sin 0 - \cos 0 + C$$

$$3 = 0 - 1 + C$$

$$C = 4$$

$$f(x) = \sin x - \cos x + 4$$

b) $f''(x) = e^x + \cos x$, $f'(0) = 5$, $f(0) = -3$

$$f'(x) = e^x + \sin x + C$$

Solve for C

$$5 = f'(0) = e^0 + \sin 0 + C$$

$$5 = 1 + 0 + C$$

$$C = 4$$

$$f'(x) = e^x + \sin x + 4$$

$$f(x) = e^x - \cos x + 4x + D$$

Solve for D

$$-3 = f(0) = e^0 - \cos 0 + 4(0) + D$$

$$-3 = 1 - 1 + D$$

$$-3 = D$$

$$f(x) = e^x - \cos x + 4x - 3$$

$$f(x) = \frac{4}{15} x^2 \sqrt{x} + \frac{-319}{45} x + 2$$

$$c) f''(x) = \sqrt{x}$$

$$f(0) = 2, f(9) = 3$$

$$f''(x) = x^{1/2}$$

$$f'(x) = \frac{x^{3/2}}{3/2} + C$$

$$f'(x) = \frac{2}{3} x^{3/2} + C$$

$$f(x) = \frac{2}{3} \frac{x^{5/2}}{5/2} + Cx + D$$

$$f(x) = \frac{2}{3} \cdot \frac{2}{5} x^{5/2} + Cx + D$$

$$f(x) = \frac{4}{15} x^2 x^{1/2} + Cx + D$$

$$f(x) = \frac{4}{15} x^2 \sqrt{x} + Cx + D$$

Solve for C and D.

$$2 = f(0) = \frac{4}{15} (0)^2 \sqrt{0} + C(0) + D$$

$$2 = D \quad \boxed{D=2}$$

$$3 = f(9) = \frac{4}{15} (9)^2 \sqrt{9} + C(9) + D$$

$$3 = \frac{4}{15} 81 \cdot 3 + 9C + 2$$

$$5 \cdot (3) = \left(\frac{324}{5} + 9C + 2 \right) 5$$

$$15 = 324 + 45C + 10$$

$$15 = 334 + 45C$$

$$-319 = 45C$$

$$C = \frac{-319}{45}$$

Rectilinear Motion.

$s(t)$ is the position of an object.

$$v(t) = s'(t) \quad \text{velocity}$$

$$a(t) = v'(t) \quad \text{acceleration} \\ = s''(t)$$

EXAMPLE: A particle moves in a straight line and has acceleration $a(t) = 6t + 4$.

Its initial velocity is

$$v(0) = -6 \text{ cm/s and its}$$

initial displacement is

$$s(0) = 9 \text{ cm. Find its position } s(t).$$

SOLUTION

$$s''(t) = a(t) = 6t + 4$$

$$s'(t) = \frac{6t^2}{2} + 4t + C$$

velocity

$$v(t) = 3t^2 + 4t + C$$

solve for C

$$-6 = v(0) = 3(0)^2 + 4(0) + C$$
$$C = -6$$

$$s'(t) = v(t) = 3t^2 + 4t - 6$$

$$s(t) = \frac{3t^3}{3} + \frac{4t^2}{2} - 6t + D$$

$$9 = s(0) = 0^3 + 2(0)^2 - 6(0) + D$$
$$D = 9$$

$$\boxed{s(t) = t^3 + 2t^2 - 6t + 9}$$