§4.9 Antiderivatives.

HW §4.9 #1-20, 23-46

<u>EXAMPLE</u>: The derivative of a function is given. Find a <u>possible</u> function.

a) $f'(x) = 3x^2$ $\qquad f(x) = ?$
$= x^3$

b) $f'(x) = 5e^{5x}$ $\qquad f(x) = ?$
$= e^{5x}$

c) $f'(x) = \sec^2 x$ $\qquad f(x) = ?$
$f(x) = \tan x$

$y = \tan x$ is an <u>antiderivative</u> of $y = \sec^2 x$.

If two functions f and g have the same derivative, then $f(x) = g(x) + C$ for some constant C.

The general antiderivative of

$y = \sec^2 x$ is $y = \tan x + C$.

EXAMPLE Find the general antiderivative, $F(x)$.

a) $f(x) = 5x^4$ Here, $F'(x) = f(x)$.

$F(x) = ?\ x^5 + C$

b) $f(x) = \sec x \tan x$

$F(x) = \sec x + C$

c) $f(x) = \sin x$

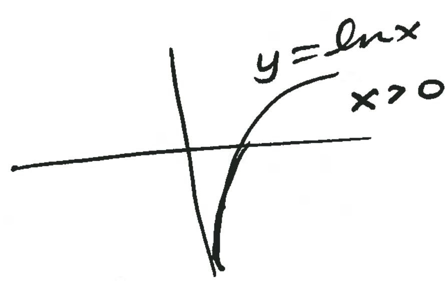
$F(x) = -\cos x + C$

d) $f(x) = \cos x$

$F(x) = \sin x + C$

e) $f(x) = \frac{1}{x}$

$F(x) = \ln|x| + C$



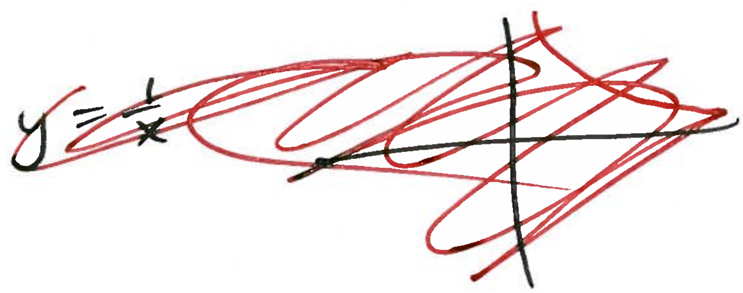
Show that $\frac{d}{dx} \ln|x| = \frac{1}{x}$

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

$$\ln|x| = \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \\ \text{undef} & \text{if } x = 0 \end{cases}$$

$$\frac{d}{dx}\ln|x| = \begin{cases} \frac{d}{dx}\ln x = \frac{1}{x} & \text{if } x > 0 \\ \frac{d}{dx}\ln(-x) = \frac{1}{(-x)}(-1) = \frac{1}{x} & \text{if } x > 0 \\ \text{undef} & \text{if } x = 0 \end{cases}$$

So $\frac{d}{dx}\ln|x| = \frac{1}{x}$ for $x \neq 0$



The Power Rule for Antiderivatives.

The antiderivative of $f(x) = x^n$ is $F(x) = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$

<u>Pf</u> $F'(x) = \frac{\cancel{(n+1)} x^n}{\cancel{(n+1)}} = x^n$

<u>EXAMPLE</u>: Find the general antiderivative.

① $f(x) = x^5$

$F(x) = \frac{x^6}{6} + C$

② $f(x) = x^9$

$F(x) = \frac{x^{10}}{10} + C$

③ $f(x) = x^{15}$

$F(x) = \frac{x^{16}}{16} + C$

④ $f(x) = \sqrt{x} = x^{1/2}$

$F(x) = \frac{x^{3/2}}{3/2} + C = \frac{2}{3} x^{3/2} + C$

⑤ $f(x) = \frac{1}{x^2} = x^{-2}$

$F(x) = \frac{x^{-1}}{-1} + C = \frac{-1}{x} + C$

⑥ $f(x) = \frac{1}{x} = x^{-1}$

$\cancel{F(x) = \frac{x^0}{0} + C}$

$F(x) = \ln|x| + C$

⑦ $f(x) = \frac{1}{(x+1)}$

$F(x) = \ln|x+1| + C$

⑧ $f(x) = \dfrac{x^4 - 3x^5 + x^9}{x^2}$

$f(x) = \dfrac{x^4}{x^2} - 3\dfrac{x^5}{x^2} + \dfrac{x^9}{x^2}$

$f(x) = x^2 - 3x^3 + x^7$

$F(x) = \dfrac{x^3}{3} - 3\dfrac{x^4}{4} + \dfrac{x^8}{8} + C$

Initial Value Problems

Example: Solve the initial value problem.

① $f'(x) = e^x + \sin x, \quad f(0) = 4.$

SOLUTION:

$f(x) = e^x - \cos x + C$

Solve for C.

$4 = f(0) = e^0 - \cos 0 + C$

$4 = 1 - 1 + C$

$4 = C \qquad C = 4$

$f(x) = e^x - \cos(x) + 4$

② $f'(x) = x^3 + 5e^x - 2\sin x, \quad f(0) = -4$

$$f(x) = \frac{x^4}{4} + 5e^x + 2\cos x + C$$

Solve for C.

$$-4 = f(0) = \frac{0^4}{4} + 5e^0 + 2\cos 0 + C$$

$$-4 = 5 + 2 + C$$

$$-4 = 7 + C$$

$$C = -11$$

$$f(x) = \frac{x^4}{4} + 5e^x + 2\cos x - 11$$

③ $f''(x) = e^x + \cos x, \quad f'(0) = 2, \ f(0) = 7$

$$f'(x) = e^x + \sin x + C$$

Solve for C

$$2 = f'(0) = e^0 + \sin 0 + C = 1 + C$$

$$2 = 1 + C$$

$$C = 1$$

$$f'(x) = e^x + \sin x + 1$$

$$f(x) = e^x + -\cos(x) + x + D$$

Solve for D

$$7 = f(0) = e^0 \overset{-\cos(0)}{\cancel{+\sin(0)}} + 0 + D$$

$$7 = 1 - 1 + 0 + D$$

$$D = 7$$

$$\boxed{f(x) = e^x - \cos(x) + x + 7}$$