Homework

(a) Critical Numbers: Find the derivative of f and the critical numbers of f.

(b) Increasing/Decreasing Chart: Make a chart showing where f' is positive or negative and f is increasing or decreasing.

(c) For large x, f behaves like its highest degree term. Evaluate the following.

- $\lim_{x\to\infty} f(x)$
- $\lim_{x\to-\infty} f(x)$

(d) Inflection Points: Find the second derivative of f and the inflection points of f.

(e) Concavity Chart: Make a chart showing where f'' is positive or negative and where f is concave up or concave down.

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points.

(g) Sketch and label the graph of f.

1. $f(x) = x^3 - 12x + 1$
2. $f(x) = x^4 - 4x - 1$
3. $f(x) = x^3 + x$
4. $f(x) = 2x^3 - 3x^2 - 12x$
5. $f(x) = 2 + 3x - x^3$
6. $f(x) = 200 + 8x^3 + x^4$
7. $f(x) = 3x^5 - 5x^3 + 3$
8. $f(x) = x^4 - x^2$
9. $f(x) = x^3 + 6x^2 + 9x$
10. $f(x) = 2 - 15x + 9x^2 - x^3$
11. $f(x) = 8x^2 - x^4$
12. $f(x) = x^4 + 4x^3$
13. $f(x) = x(x + 2)^3$
14. $f(x) = 2x^5 - 5x^2 + 1$
15. $f(x) = 20x^3 - 3x^5$

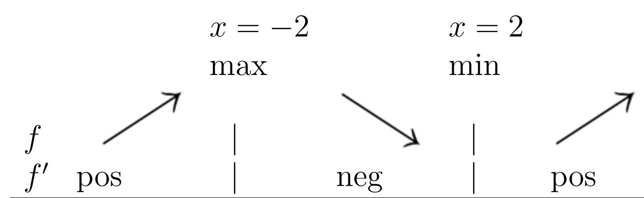
Solutions

1. $f(x) = x^3 - 12x + 1$

   (a) Critical Numbers.

$$\begin{aligned} f'(x) &= 3x^2 - 12 = 3(x-2)(x+2) = 0 \\ & \quad x = \pm 2 \end{aligned}$$

   (b) Increasing/Decreasing Chart



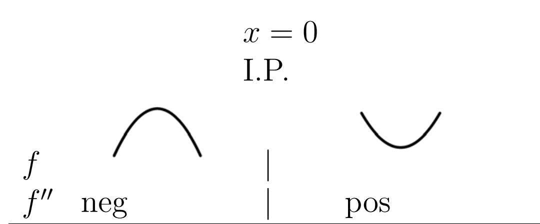
   (c) For large x, f behaves like its highest degree term, $y = x^3$

   - $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^3 = \infty$
   - $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^3 = -\infty$

   (d) Inflection Points

$$\begin{aligned} f'(x) &= 3x^2 - 12 \\ f''(x) &= 6x = 0 \\ & \quad x = 0 \end{aligned}$$

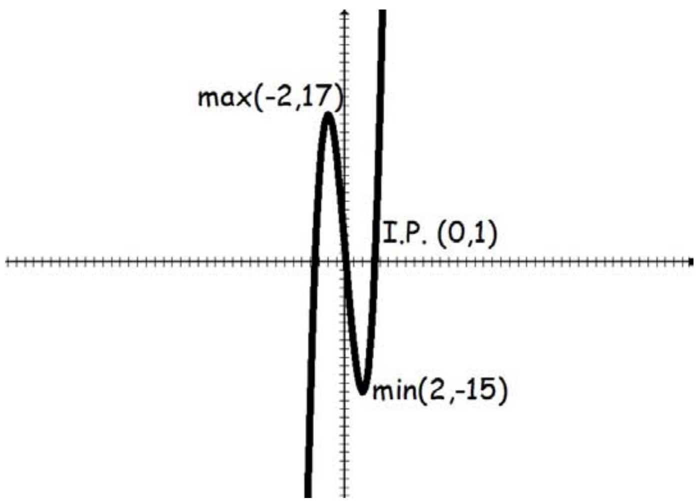
   (e) Concavity Chart



(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points.

	x	$f(x)$
max	-2	17
IP	0	1
min	2	-15

(g) Sketch and label the graph of f.



2. $f(x) = x^4 - 4x - 1$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x^4 - 4x - 1 \\ f'(x) &= 4x^3 - 4 = 4(x^3 - 1) = 4(x-1)(x^2+x+1) = 0 \\ & \quad x^3 - 1 = 0; \quad x^3 = 1; \quad x = 1 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = 1$ min	
f	↘	|	↗
f'	neg	|	pos

(c) For large x, f behaves like its highest degree term, $y = x^4$

- $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} x^4 = \infty$
- $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^4 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 4x^3 - 4 \\ f''(x) &= 12x^2 = 0 \\ & x = 0 \end{aligned}$$

(e) Concavity Chart

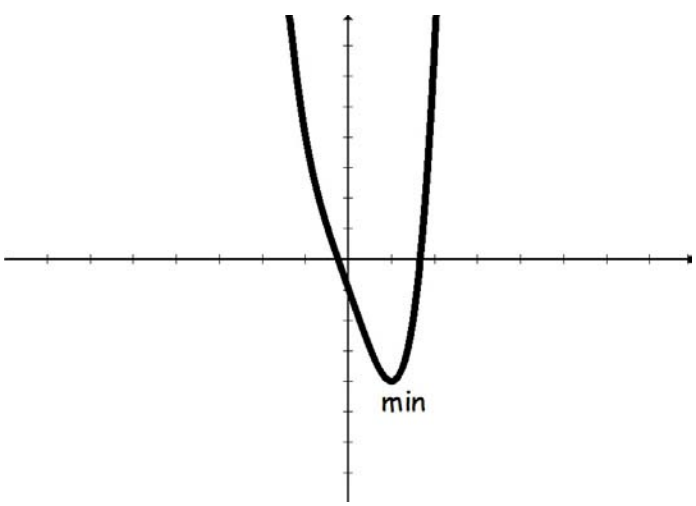
not IP
$x = 0$

f		|	
$f''(x) = 12x^2$	pos	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = x^4 - 4x - 1$.

	x	$f(x)$
min	1	-4
	0	-1

(g) Sketch and label the graph of f.



3. $f(x) = x^3 + x$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x^3 + x \\ f'(x) &= 3x^2 + 1 \end{aligned}$$

$3x^2 + 1 > 0$ for all x. No critical points

(b) Increasing/Decreasing Chart

f	↗
$f'(x) = 3x^2 + 1$	pos

(c) For large x, f behaves like its highest degree term, $y = x^3$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^3 = \infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^3 = -\infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 3x^2 + 1 \\ f''(x) &= 6x = 0 \\ & \quad x = 0 \end{aligned}$$

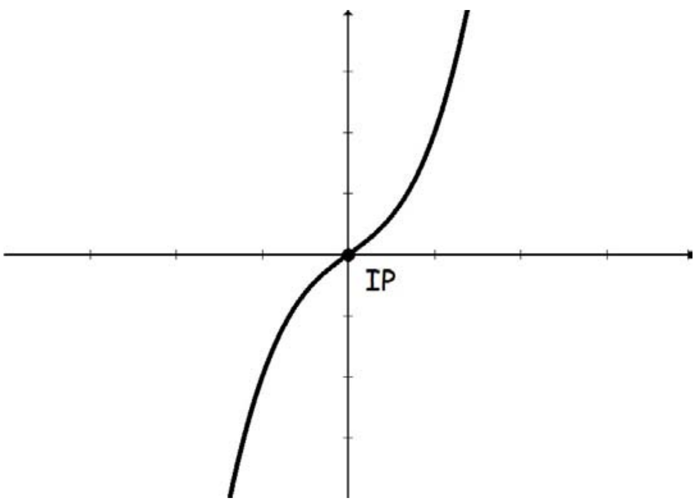
(e) Concavity Chart

		IP $x = 0$	
f	∩	|	∪
$f''(x) = 6x$	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = x^3 + x$.

	x	$f(x)$
IP	0	0

(g) Sketch and label the graph of f.



4. $f(x) = 2x^3 - 3x^2 - 12x$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= 2x^3 - 3x^2 - 12x \\ f'(x) &= 6x^2 - 6x - 12 = 6(x^2 - x - 2) = 6(x+1)(x-2) = 0 \\ & x = 2, \; x = -1 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = -1$ max		$x = 2$ min	
f	↗	|	↘	|	↗
$f'(x) = 6(x+1)(x-2)$	pos	|	neg	|	pos

(c) For large x, f behaves like its highest degree term, $y = 2x^3$

- $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} x^3 = \infty$
- $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^3 = -\infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 6x^2 - 6x - 12 \\ f''(x) &= 12x - 6 = 6(2x-1) = 0 \\ & \quad x = 1/2 \end{aligned}$$

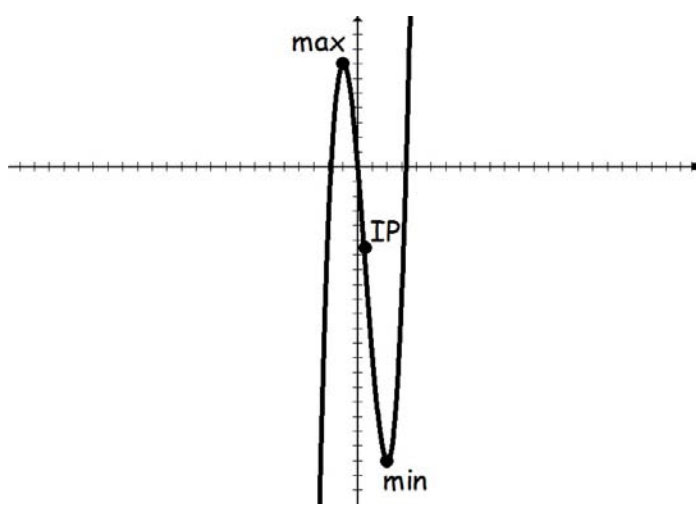
(e) Concavity Chart

		IP $x = 1/2$	
f		|	
$f''(x) = 6(2x-1)$	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = 2x^3 - 3x^2 - 12x$.

	x	$f(x)$
max	-1	7
IP	$1/2$	$-11/2$
min	2	-20

(g) Sketch and label the graph of f.



5. $f(x) = 2 + 3x - x^3 = -x^3 + 3x + 2$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= -x^3 + 3x + 2 \\ f'(x) &= -3x^2 + 3 = -3(x^2 - 1) = -3(x+1)(x-1) \\ & x = -1,\ x = 1 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = -1$ min		$x = 1$ max	
f	$\searrow$	|	$\nearrow$	|	$\searrow$
$f'(x) = -3(x+1)(x-1)$	neg	|	pos	|	neg

(c) For large x, f behaves like its highest degree term, $y = -x^3$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} -x^3 = -\infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} -x^3 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= -3x^2 + 3 \\ f''(x) &= -6x = 0 \\ & x = 0 \end{aligned}$$

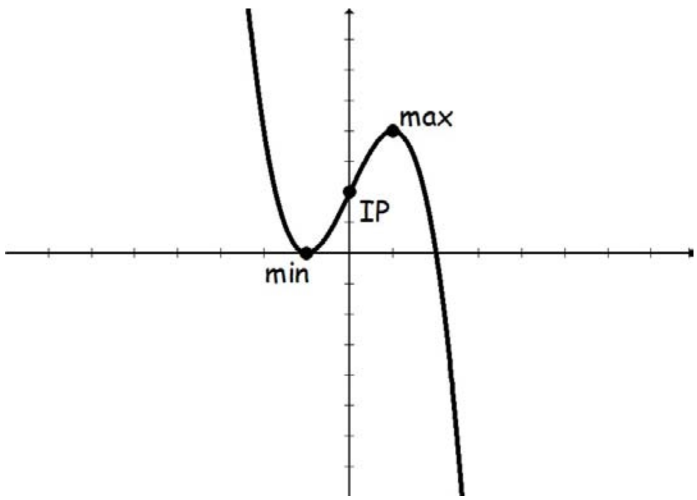
(e) Concavity Chart

		IP $x = 0$	
f	$\cup$	|	$\cap$
$f''(x) = -6x$	pos	|	neg

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = -x^3 + 3x + 2$.

	x	$f(x)$
min	-1	0
IP	0	2
max	1	4

(g) Sketch and label the graph of f.



6. $f(x) = 200 + 8x^3 + x^4 = x^4 + 8x^3 + 200$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x^4 + 8x^3 + 200 \\ f'(x) &= 4x^3 + 24x^2 = 4x^2(x+6) = 0 \\ & x = -6,\ x = 0 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = -6$ min		$x = 0$ neither	
f	↘	|	↗	|	↗
$f'(x) = 4x^2(x+6)$	neg	|	pos	|	pos

(c) For large x, f behaves like its highest degree term, $y = x^4$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^4 = \infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^4 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 4x^3 + 24x^2 \\ f''(x) &= 12x^2 + 48x = 12x(x+4) = 0 \\ & \quad x = -4, \quad x = 0, \end{aligned}$$

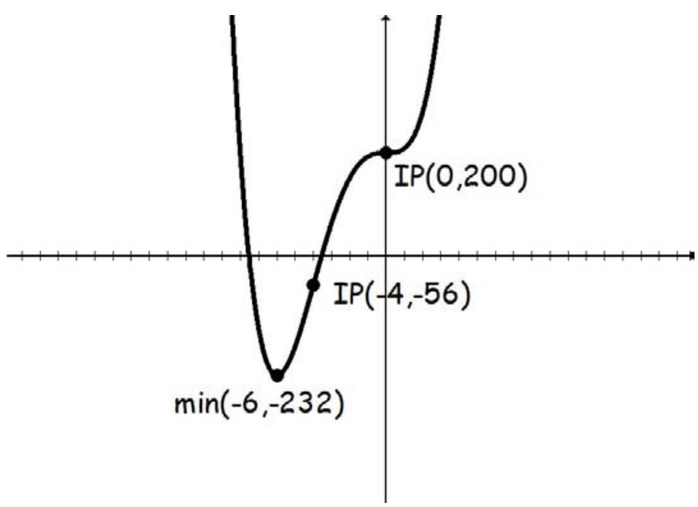
(e) Concavity Chart

		$x = -4$ IP		$x = 0$ IP	
f		|		|	
$f''(x) = 12x(x+4)$	pos	|	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = x^4 + 8x^3 + 200$.

	x	$f(x)$
min	-6	-232
IP	-4	-56
IP	0	200

(g) Sketch and label the graph of f.



7. $f(x) = 3x^5 - 5x^3 + 3$

   (a) Critical Numbers.

$$\begin{aligned} f(x) &= 3x^5 - 5x^3 + 3 \\ f'(x) &= 15x^4 - 15x^2 = 15x^2(x^2-1) = 15x^2(x+1)(x-1) = 0 \\ & x=-1,\ x=0\ x=1 \end{aligned}$$

   (b) Increasing/Decreasing Chart

		$x=-1$ max		$x=0$ neither		$x=1$ min	
f	↗	|	↘	|	↘	|	↗
$f'(x) = 15x^2(x+1)(x-1)$	pos	|	neg	|	neg	|	pos

   (c) For large x, f behaves like its highest degree term, $y = 3x^5$

   - $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 3x^5 = \infty$
   - $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} 3x^5 = -\infty$

   (d) Inflection Points

$$\begin{aligned} f'(x) &= 15x^4 - 15x^2 \\ f''(x) &= 60x^3 - 30x = 30x(2x^2-1) = 0 \\ & 30x = 0 \quad 2x^2 - 1 = 0 \\ & x = 0 \quad x = \pm\frac{\sqrt{2}}{2} \end{aligned}$$

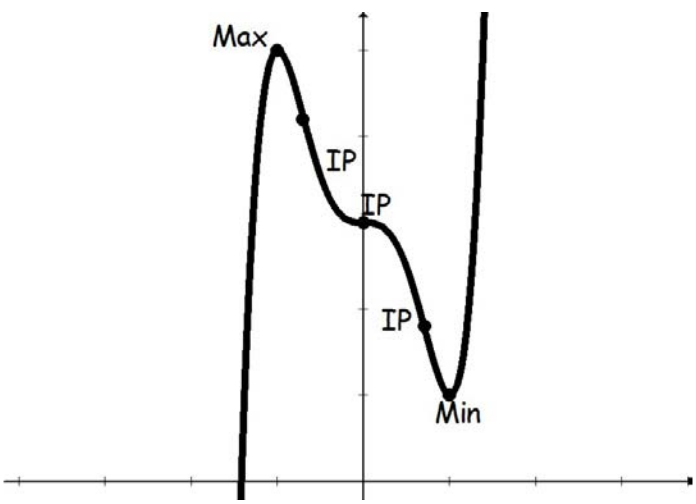
   (e) Concavity Chart

		$x=-\sqrt{2}/2$ IP		$x=0$ IP		$x=\sqrt{2}/2$ IP	
f	∩	|	∪	|	∩	|	∪
$f'(x) = 30x(2x^2-1)$	neg	|	pos	|	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = 3x^5 - 5x^3 + 3$.

	x	$f(x)$
max	-1	5
IP	$-\sqrt{2}/2$	≈ 4.2
IP	0	3
IP	$\sqrt{2}/2$	≈ 1.8
min	1	1

(g) Sketch and label the graph of f.



8. $f(x) = x^4 - x^2$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x^4 - x^2 \\ f'(x) &= 4x^3 - 2x = 2x(2x^2 - 1) = 0 \\ & x = 0, \quad 2x^2 - 1 = 0 \\ & x = 0, \quad x = \pm\sqrt{2}/2 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x=-\sqrt{2}/2$ min		$x=0$ max		$x=\sqrt{2}/2$ min	
f	↘	|	↗	|	↘	|	↗
$f'(x)=2x(2x^2-1)$	neg	|	pos	|	neg	|	pos

(c) For large x, f behaves like its highest degree term, $y=x^4$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^4 = \infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^4 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 4x^3-2x \\ f''(x) &= 12x^2-2 = 2(6x^2-1)=0 \\ & \quad 6x^2-1=0 \\ & \quad x=\pm\frac{\sqrt{6}}{6} \end{aligned}$$

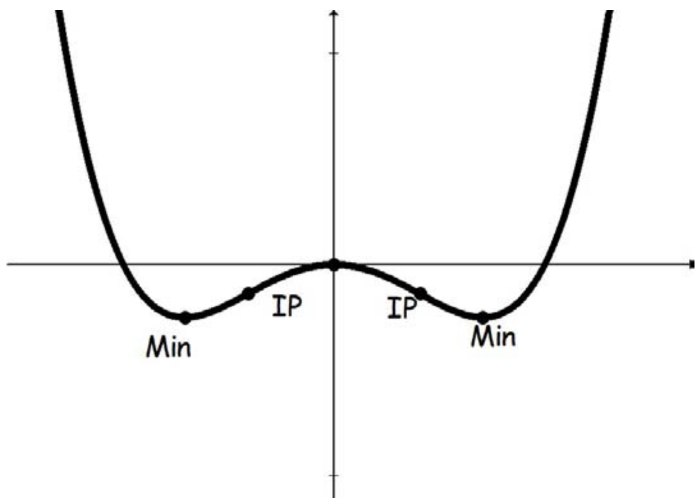
(e) Concavity Chart

		$x=-\sqrt{6}/6$ IP		$x=\sqrt{6}/6$ IP	
f	∪	|	∩	|	∪
$f''(x)=12x^2-2$	pos	|	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x)=x^4-x^2$.

	x	$f(x)$
min	$-\sqrt{2}/2$	$-1/4$
IP	$-\sqrt{6}/6$	$5/36$
max	0	
IP	$\sqrt{6}/6$	$-5/36$
min	$\sqrt{2}/2$	$-1/4$

(g) Sketch and label the graph of f.



9. $f(x) = x^3 + 6x^2 + 9x$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x^3 + 6x^2 + 9x \\ f'(x) &= 3x^2 + 12x + 9 = 3(x^2 + 4x + 3) = 3(x+1)(x+3) = 0 \\ & x = -3, \; x = -1 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = -3$ max		$x = -1$ min	
f	↗	|	↘	|	↗
$f'(x) = 3(x+1)(x+3)$	pos	|	neg	|	pos

(c) For large x, f behaves like its highest degree term, $y = x^3$

- $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} x^3 = \infty$
- $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} x^3 = -\infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 3x^2 + 12x + 9 \\ f''(x) &= 6x + 12 = 6(x+2) = 0 \\ & \quad x = -2 \end{aligned}$$

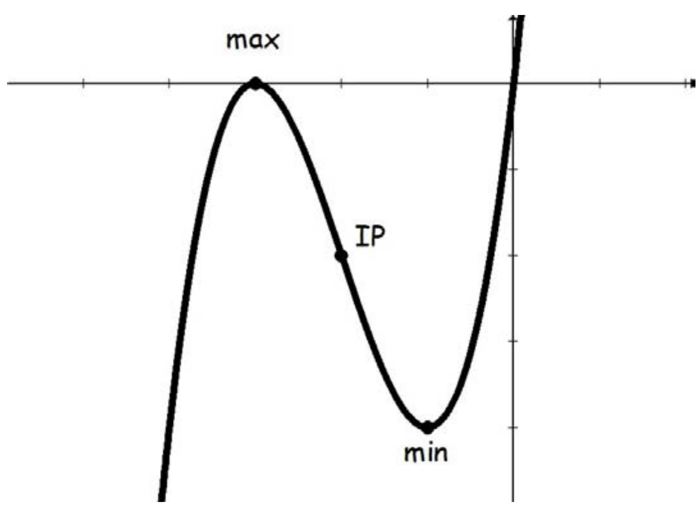
(e) Concavity Chart

		IP $x = -2$	
f		|	
$f''(x) = 6(x+2)$	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = x^3 + 6x^2 + 9x$.

	x	$f(x)$
max	-3	0
IP	-2	-2
min	-1	-4

(g) Sketch and label the graph of f.



10. $f(x) = 2 - 15x + 9x^2 - x^3 = -x^3 + 9x^2 - 15x + 2$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= -x^3 + 9x^2 - 15x + 2 \\ f'(x) &= -3x^2 + 18x - 15 = -3(x^2 - 6x + 5) = -3(x-1)(x-5) = 0 \\ & x = 1,\ x = 5 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = 1$ min		$x = 5$ max	
f	↘	|	↗	|	↘
$f'(x) = -3(x-1)(x-5)$	neg	|	pos	|	neg

(c) For large x, f behaves like its highest degree term, $y = -x^3$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} -x^3 = -\infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} -x^3 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= -3x^2 + 18x - 15 \\ f''(x) &= -6x + 18 = -6(x-3) = 0 \\ & x = 3 \end{aligned}$$

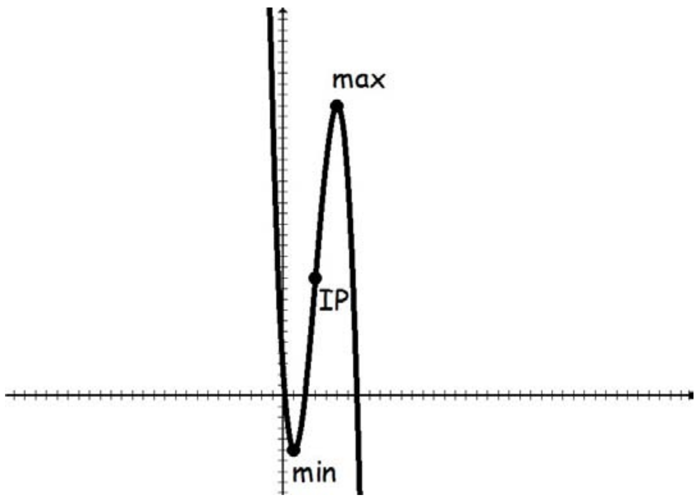
(e) Concavity Chart

		IP $x = 3$	
f	∪	|	∩
$f''(x) = -6(x-3)$	pos	|	neg

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = -x^3 + 9x^2 - 15x + 2$.

	x	$f(x)$
max	1	-5
IP	3	11
min	5	27

(g) Sketch and label the graph of f.



11. $f(x) = 8x^2 - x^4 = -x^4 + 8x^2$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= -x^4 + 8x^2 \\ f'(x) &= -4x^3 + 16x = -4x(x^2 - 4) = -4x(x - 2)(x + 2) = 0 \\ & \quad x = 0, \quad x = \pm 2 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x=-2$ max		$x=0$ min		$x=2$ max	
f	↗	|	↘	|	↗	|	↘
$f'(x) = -4x(x-2)(x+2)$	pos	|	neg	|	pos	|	neg

(c) For large x, f behaves like its highest degree term, $y = -x^4$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} -x^4 = -\infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} -x^4 = -\infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= -4x^3 + 16x \\ f''(x) &= -12x^2 + 16 = -12(x^2 - 4/3) = -12(x - 2\sqrt{3}/3)(x + 2\sqrt{3}/3) = 0 \\ & x = \pm\frac{2\sqrt{3}}{3} \end{aligned}$$

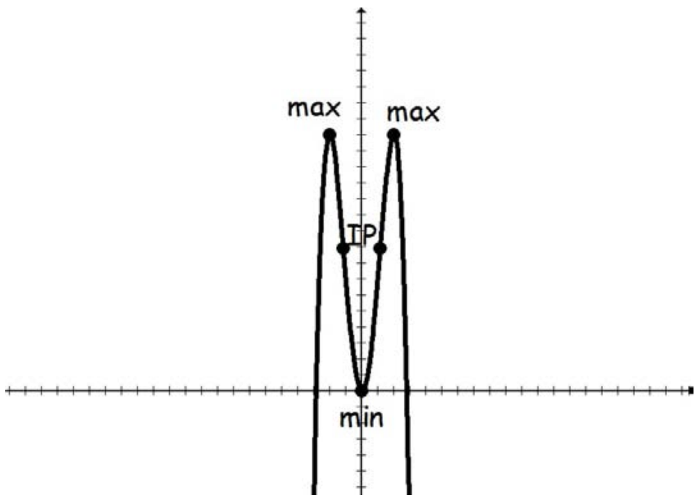
(e) Concavity Chart

		$x = -2\sqrt{3}/3$ IP		$x = 2\sqrt{3}/3$ IP	
f	∩	|	∪	|	∩
$f''(x) = -12(x - 2\sqrt{3}/3)(x + 2\sqrt{3}/3)$	neg	|	pos	|	neg

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = -x^4 + 8x^2$.

	x	$f(x)$
min	-2	16
IP	$-2\sqrt{3}/3$	80/9
max	0	0
IP	$2\sqrt{3}/3$	80/9
min	2	16

(g) Sketch and label the graph of f.



12. $f(x) = x^4 + 4x^3$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x^4 + 4x^3 \\ f'(x) &= 4x^3 + 12x^2 = 4x^2(x+3) = 0 \\ & x = -3; \quad x = 0 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x=-3$ min		$x=0$ neither	
f	↘	|	↗	|	↗
$f'(x) = 4x^2(x+3)$	neg	|	pos	|	pos

(c) For large x, f behaves like its highest degree term, $y = x^4$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} x^4 = \infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} x^4 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 4x^3 + 12x^2 \\ f''(x) &= 12x^2 + 24x = 12x(x+2) = 0 \\ & \quad x = 0 \;\; x = -2 \end{aligned}$$

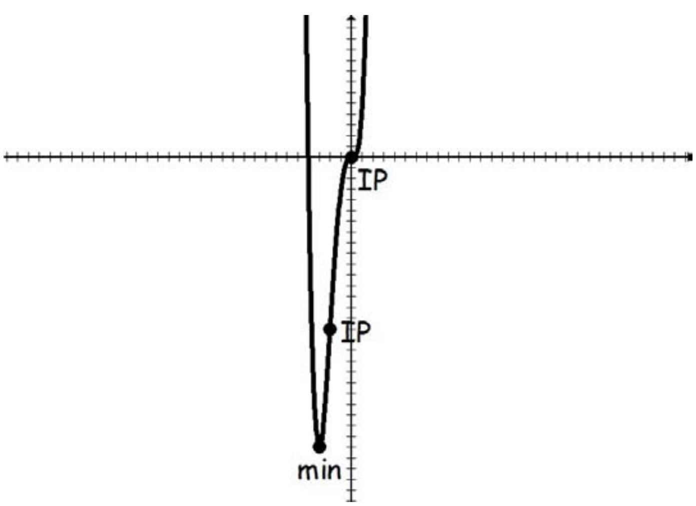
(e) Concavity Chart

		$x = 0$ IP		$x = 2$ IP	
f		|		|	
$f''(x) = 12x(x+2)$	pos	|	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = x^4 + 4x^3$.

	x	$f(x)$
min	-3	-27
IP	-2	-16
IP	0	0

(g) Sketch and label the graph of f.



13. $f(x) = x(x+2)^3$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= x(x+2)^3 \\ f'(x) &= (1)(x+2)^3 + x(3(x+2)^2) = (x+2)^2((x+2)+3x) \\ &= (x+2)^2(4x+2) = 4(x+2)^2(x+1/2) = 0 \\ & x=-2; \quad x=-1/2 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x=-2$ neither		$x=-1/2$ min	
f	↘	|	↘	|	↗
$f'(x) = 4(x+2)^2(x+1/2)$	neg	|	neg	|	pos

(c) For large x, f behaves like its highest degree term, $y = x^4$

- $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} x^4 = \infty$
- $\lim\limits_{x\to-\infty} f(x) = \lim\limits_{x\to-\infty} x^4 = \infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= (x+2)^2(4x+2) = (x^2+4x+4)(4x+2) \\ &= 4x^3+2x^2+16x^2+8x+16x+8 = 4x^3+18x^2+24x+8 \\ f''(x) &= 12x^2+36x+24 = 12(x^2+3x+2) = 12(x+2)(x+1) = 0 \\ & x=-2 \quad x=-1 \end{aligned}$$

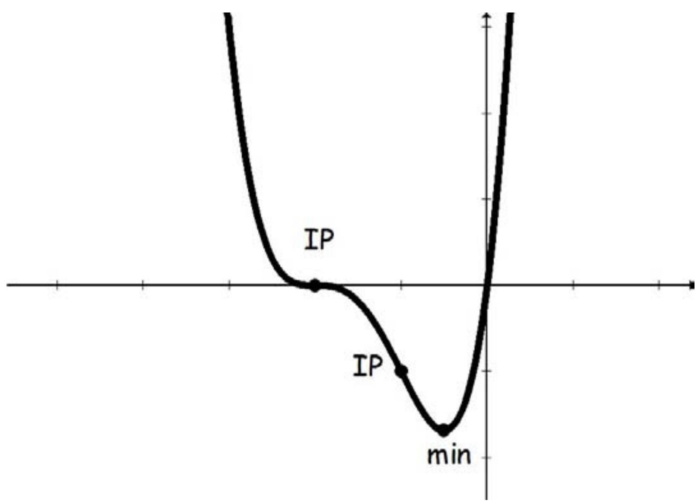
(e) Concavity Chart

		$x=-2$ IP		$x=-1$ IP	
f	∪	|	∩	|	∪
$f''(x) = 12(x+1)(x+2)$	pos	|	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = x(x+2)^3$.

	x	$f(x)$
IP	-2	0
IP	-1	-1
min	$-1/2$	$-27/16$

(g) Sketch and label the graph of f.



14. $f(x) = 2x^5 - 5x^2 + 1$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= 2x^5 - 5x^2 + 1 \\ f'(x) &= 10x^4 - 10x = 10x(x^3 - 1) = 10x(x-1)(x^2 + x + 1) = 0 \\ & x = 0,\ x = 1 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x=0$ max		$x=1$ min	
f	↗	|	↘	|	↗
$f'(x) = 10x(x-1)(x^2+x+1)$	pos	|	neg	|	pos

(c) For large x, f behaves like its highest degree term, $y = 2x^5$

- $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} 2x^5 = \infty$
- $\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty} 2x^5 = -\infty$

(d) Inflection Points

$$\begin{aligned} f'(x) &= 10x^4 - 10x \\ f''(x) &= 40x^3 - 10 = 40(x^3 - 1/4) = 0 \\ & \quad x^3 - 1/4 = 0; \quad x^3 = 1/4; \quad x = 1/\sqrt[3]{4} \end{aligned}$$

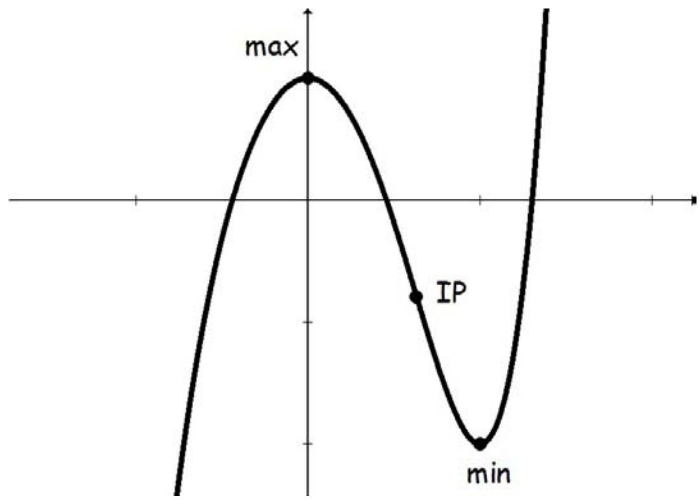
(e) Concavity Chart

		IP $x = 1/\sqrt[3]{4}$	
f	∩	|	∪
$f''(x) = -6(x-3)$	neg	|	pos

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = 2x^5 - 5x^2 + 1$.

	x	$f(x)$
max	0	1
IP	$1/\sqrt[3]{4}$	≈ -0.79
min	1	-2

(g) Sketch and label the graph of f.



15. $f(x) = 20x^3 - 3x^5 = -3x^5 + 20x^3$

(a) Critical Numbers.

$$\begin{aligned} f(x) &= -3x^5 + 20x^3 \\ f'(x) &= -15x^4 + 60x^2 = -15x^2(x^2-4) = -15x^2(x+2)(x-2) = 0 \\ & \quad x = -2,\ x = 0\ x = 2 \end{aligned}$$

(b) Increasing/Decreasing Chart

		$x = -1$ min		$x = 0$ neither		$x = 1$ max	
f	↘	|	↗	|	↗	|	↘
$f'(x) = -15x^2(x+2)(x-2)$	neg	|	pos	|	pos	|	neg

(c) For large x, f behaves like its highest degree term, $y = -3x^5$

- $\lim_{x \to \infty} f(x) = \lim_{x \to \infty} -3x^5 = -\infty$
- $\lim_{x \to -\infty} f(x) = \lim_{x \to -\infty} -3x^5 = \infty$

(d) Inflection Points

$$f'(x) = -15x^4 + 60x^2$$
$$f''(x) = -60x^3 + 120x = -60x(x^2 - 2) = -60x(x + \sqrt{2}(x - \sqrt{2}) = 0$$
$$x = 0 \quad x = \pm\sqrt{2}$$

(e) Concavity Chart

		$x = -\sqrt{2}$ IP		$x = 0$ IP		$x = \sqrt{2}$ IP	
f		|		|		|	
$f'(x) = -60x(x + \sqrt{2})(x - \sqrt{2})$	pos	|	neg	|	pos	|	neg

(f) Plot Points: Give the y-coordinates for the local max, local min, and inflection points for $f(x) = -3x^5 + 20x^3$.

	x	$f(x)$
min	-2	-64
IP	$-\sqrt{2}$	≈ -39.6
IP	0	
IP	$\sqrt{2}$	≈ 39.6
max	2	64

(g) Sketch and label the graph of f.