Practice Problems

§7.3 Let $\vec{r} = \langle 3, -2 \rangle$, $\vec{s} = \langle -1, 5 \rangle$, and $\vec{t} = \langle 4, -6 \rangle$. Perform the operations indicated.

① $-4\vec{s}$

$= -4\langle -1, 5 \rangle = \langle 4, -20 \rangle$

② $\vec{r} - \vec{t}$

$= \langle 3, -2 \rangle - \langle 4, -6 \rangle$

$= \langle -1, 8 \rangle$

③ $\dfrac{\vec{r} + \vec{s}}{2} = \dfrac{\langle 3, -2 \rangle + \langle -1, 5 \rangle}{2} = \dfrac{1}{2}\langle 2, 3 \rangle$

$= \langle 1, 3/2 \rangle$

④ $\vec{r} - \vec{s} - \vec{t}$

$= \langle 3, -2 \rangle - \langle -1, 5 \rangle - \langle 4, -6 \rangle$

$= \langle 3+1-4, -2-5+6 \rangle$

$= \langle 0, -1 \rangle$

⑤ $\vec{s} \cdot \vec{t}$

$= \langle -1, 5 \rangle \cdot \langle 4, -6 \rangle$

$= (-1)(4) + (5)(-6)$

$= -34$

§7.3

Find the angle to the nearest tenth of a degree between each given pair of vectors.

⑥ $\langle 2,3\rangle, \langle 1,5\rangle$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}$$

$$\cos\theta = \frac{17}{\sqrt{13}\sqrt{26}}$$

$$\theta = \cos^{-1}\left(\frac{17}{\sqrt{13\cdot 26}}\right)$$

$$\theta \approx 22.4^\circ$$

$\langle 2,3\rangle \cdot \langle 1,5\rangle$
$= 2\cdot 1 + 3\cdot 5$
$= 17$

$\|\langle 2,3\rangle\| = \sqrt{2^2+3^2}$
$= \sqrt{13}$

$\|\langle 1,5\rangle\| = \sqrt{1^2+5^2}$
$= \sqrt{26}$

⑦ $\langle -2,-5\rangle, \langle 1,-9\rangle$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|}$$

$$\cos\theta = \frac{43}{\sqrt{29}\sqrt{82}}$$

$$\theta = \cos^{-1}\left(\frac{43}{\sqrt{29}\sqrt{82}}\right)$$

$$\theta \approx 28.1^\circ$$

$\vec{a}\cdot\vec{b} = \langle -2,-5\rangle \cdot \langle 1,-9\rangle$
$= (-2)(1)+(-5)(-9)$
$= 43$

$\|\vec{a}\| = \sqrt{(-2)^2+(-5)^2}$
$= \sqrt{29}$

$\|\vec{b}\| = \sqrt{1^2+(-9)^2} = \sqrt{82}$

§7.3 Determine whether each pair of vectors is parallel, perpendicular, or neither.

⑧ $\langle 2,3\rangle, \langle 8,12\rangle$

$\langle 2,3\rangle \cdot \langle 8,12\rangle = 2\cdot 8 + 3\cdot 12$
$= 16 + 36 = 52$

$\|\langle 2,3\rangle\| = \sqrt{2^2+3^2} = \sqrt{13}$

$\|\langle 8,12\rangle\| = \sqrt{8^2+12^2} = \sqrt{208}$

$\cos\theta = \dfrac{\vec{a}\cdot\vec{b}}{\|\vec{a}\|\,\|\vec{b}\|} = \dfrac{52}{\sqrt{13}\sqrt{208}} = \dfrac{52}{\sqrt{13}\sqrt{16\cdot 13}}$

$= \dfrac{52}{52} = 1$

$\cos\theta = 1$
$\theta = 0$ parallel

208
104
52
13 4
2

⑨ $\langle 2,-4\rangle, \langle 2,1\rangle$

$\langle 2,-4\rangle \cdot \langle 2,1\rangle$

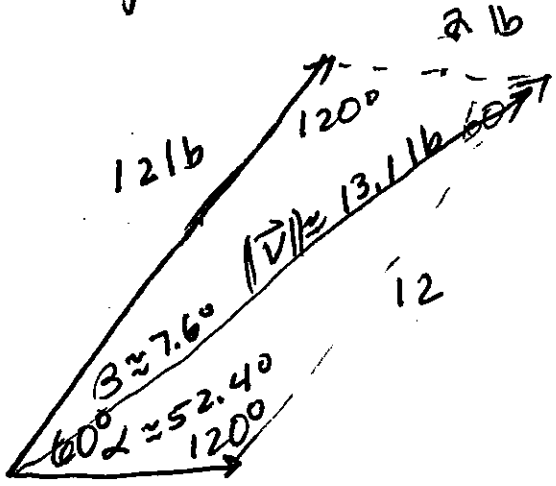
$= 4 - 4 = 0$

Write each vector as a linear combination of vectors $\vec{i}$ and $\vec{j}$.

(10) $\langle 1, 5 \rangle$
$= 1 \cdot \vec{i} + 5\vec{j}$
$= \vec{i} + 5\vec{j}$

(11) $\langle \sqrt{2}, -5 \rangle = \sqrt{2}\vec{i} - 5\vec{j}$

(12) Find the magnitude of the resultant force and the angle between the resultant and each force, when forces of 2lb and 12lb act at an angle of 60° to each other.



360
120
240

$\|\vec{v}\|^2 = 2^2 + 12^2 - 2 \cdot 2 \cdot 12 \cdot \cos 120°$

$\|\vec{v}\|^2 = 4 + 144 - 24(-\frac{1}{2})$

$\|\vec{v}\|^2 = 148 + 24 = 172$

$\|\vec{v}\| = \sqrt{172} \approx 13.1$ lb

$12^2 = 2^2 + (\sqrt{172})^2 - 2(2)(\sqrt{172}) \cos\alpha$

$\frac{144 - 4 - 172}{-4\sqrt{172}} = \cos\alpha$

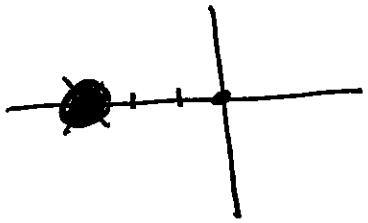
$\alpha = \cos^{-1}\left(\frac{-32}{-4\sqrt{172}}\right) \approx 52.4°$

$\beta = 60° - \alpha = 7.6°$

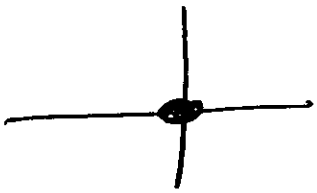
LAW OF COSINES
$c^2 = a^2 + b^2 - 2ab\cos C$

§7.6 Plot the points whose polar coordinates are given.

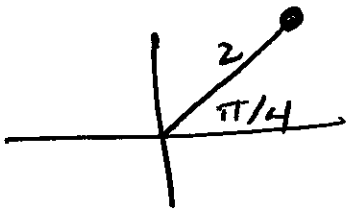
(14) $(-3, 0°)$



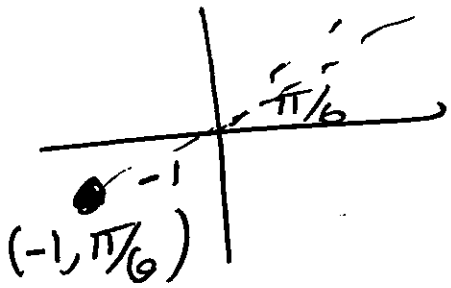
(15) $(0, 90°)$



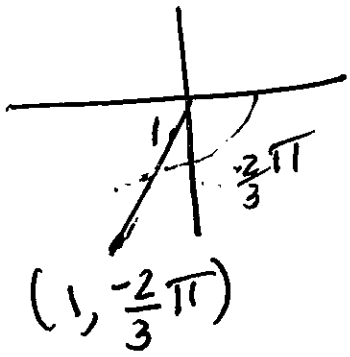
(16) $\left(2, \frac{\pi}{4}\right)$



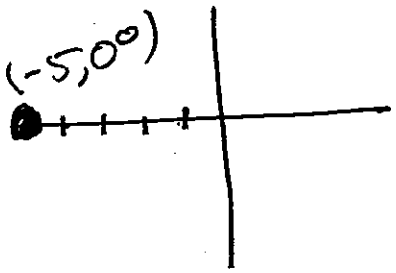
(17) $\left(-1, \frac{\pi}{6}\right)$



(18) $\left(1, -\frac{2\pi}{3}\right)$



(19) $(-5, 0°)$



Convert the polar coordinates to rectangular coordinates.

(20) $\left(1, \frac{\pi}{6}\right)$ $\quad r = 1, \ \theta = \pi/6$

$$x = r\cos\theta = 1 \cdot \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

$$y = r\sin\theta = 1 \sin\frac{\pi}{6} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

$$\left(\sqrt{3}/2, \frac{1}{2}\right)$$

(21) $(\sqrt{2}, 135^\circ)$ $\quad r = \sqrt{2}, \ \theta = 135^\circ$

$$x = r\cos\theta = \sqrt{2}\cos 135^\circ = \sqrt{2}\left(-\frac{\sqrt{2}}{2}\right) = -1$$

$$y = r\sin\theta = \sqrt{2}\sin 135^\circ = \sqrt{2}\left(\frac{\sqrt{2}}{2}\right) = 1$$

$$(-1, 1)$$

Convert the rectangular coordinates of each point to polar coordinates.

(22) $(-2, 2\sqrt{3})$ $\quad x = -2, \ y = 2\sqrt{3}$

$$x^2 + y^2 = r^2$$

$$(-2)^2 + (2\sqrt{3})^2 = r^2$$

$$4 + 4 \cdot 3 = r^2$$

$$16 = r^2, \quad \boxed{r = 4}$$

$$\sin\theta = \frac{y}{r} = \frac{2\sqrt{3}}{4} = \frac{\sqrt{3}}{2}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{4} = \frac{-1}{2}$$

$$\theta = \frac{2}{3}\pi$$

$$\left(4, \frac{2}{3}\pi\right)$$

(23) $(-2, -\sqrt{3})$ $x = -2,\ y = -\sqrt{3}$

$$r^2 = x^2 + y^2$$

$$r^2 = (-2)^2 + (-\sqrt{3})^2$$

$$= 4 + 3 = 7$$

$$r = \sqrt{7}$$

$$\cos\theta = \frac{x}{r} = \frac{-2}{\sqrt{7}}$$

$$\theta_R = \cos^{-1}\left(\frac{2}{\sqrt{7}}\right) \approx 40.9^\circ$$

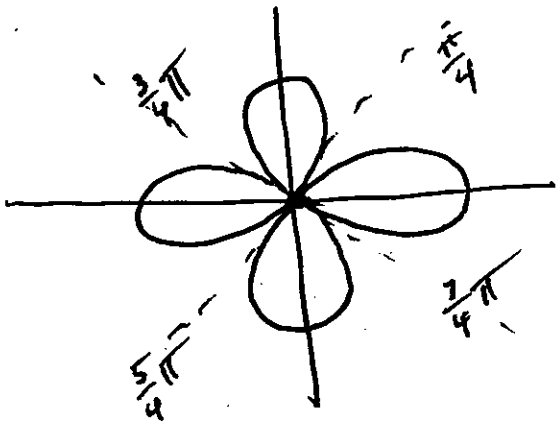
$$\theta = 180^\circ + 40.9^\circ = 220.9^\circ$$

$$(\sqrt{7}, 220.9^\circ)$$

§7.6

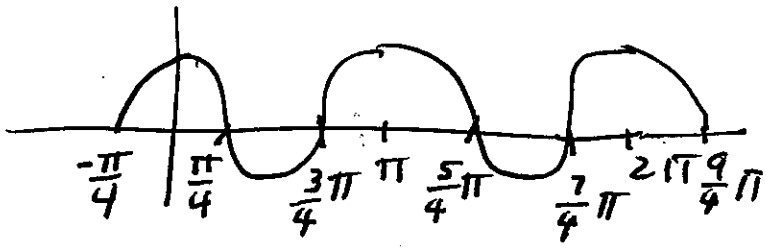
sketch the graph of the polar equation.

(24) $r = 4\cos 2\theta$

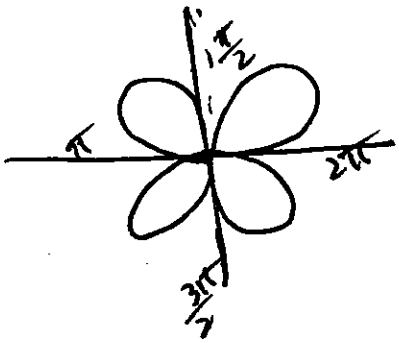


$y = 4\cos 2x$

$P = \frac{2\pi}{2} = \pi$

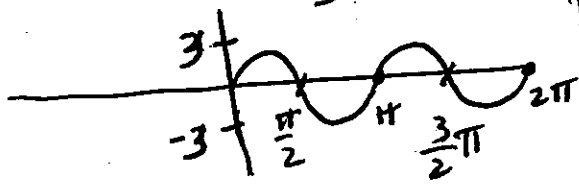


(25) $r = 3\sin 2\theta$

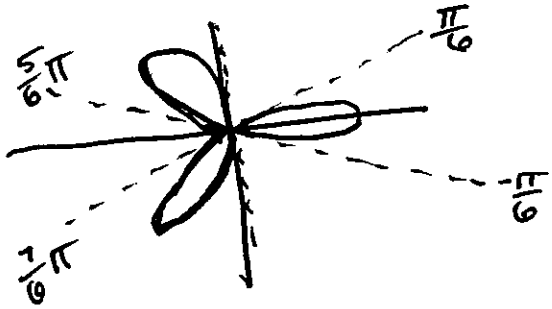


$y = 3\sin 2x$

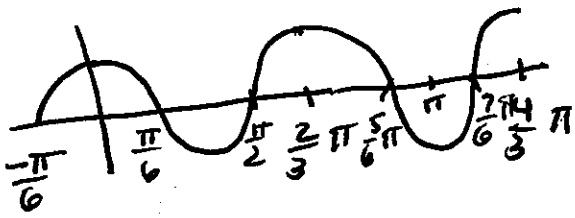
$P = \frac{2\pi}{2}$



(26) $r = 4\cos 3\theta$



$P = \frac{2\pi}{3}$

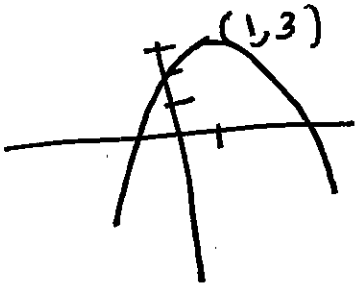


Conics

Sketch the graph of the parabola.

(27) $y = -(x-1)^2 + 3$

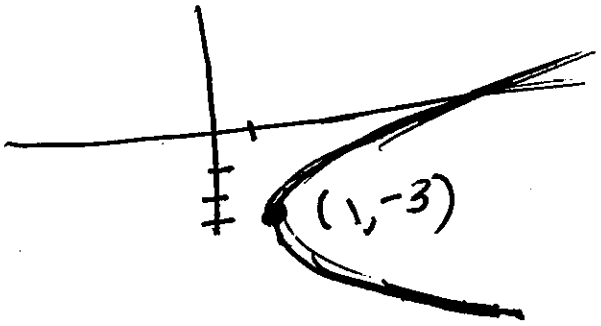
vertex $(1,3)$



(28) $x = (y+3)^2 - 1$

$(x-1) = (y+3)^2$

vertex $(1,-3)$

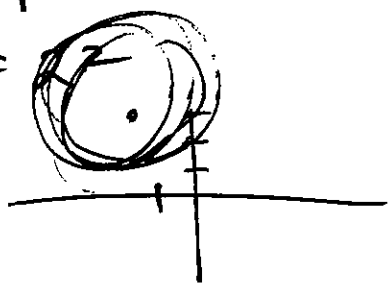


(29) Write the equation of the circle. Sketch the graph.

Center $(-1,3)$ and radius 2.

$(x-h)^2 + (y-k)^2 = r^2$

$(x+1)^2 + (y-3)^2 = 2^2$



(30) Write the circle in standard form $(x-h)^2+(y-k)^2=r^2$ state the center and radius, sketch the graph.

$$x^2-2x+y^2+4y=4$$

$$(x^2-2x+1)+(y^2+4y+4)=1+4+4$$

$$(x-1)^2+(y+2)^2=3^2$$

$(1,-2)$, $r=3$