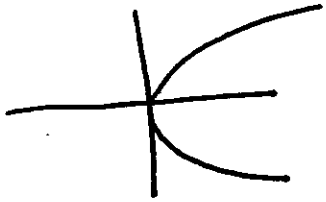
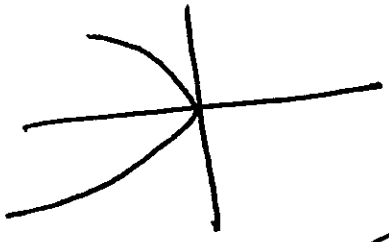
Conics Continued.

EXAMPLE: Sketch the parabola. Label the vertex.

① $x = y^2$



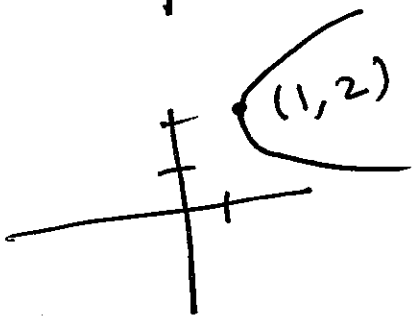
② $x = -y^2$



③ $x = (y-2)^2 + 1$

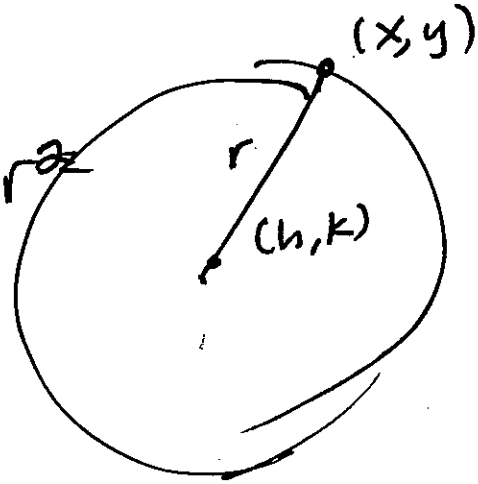
$(x-1) = (y-2)^2$

vertex $(1, 2)$



Circles

A circle is the set of all points (x, y) in the plane with constant distance r from a point (h, k), the center.



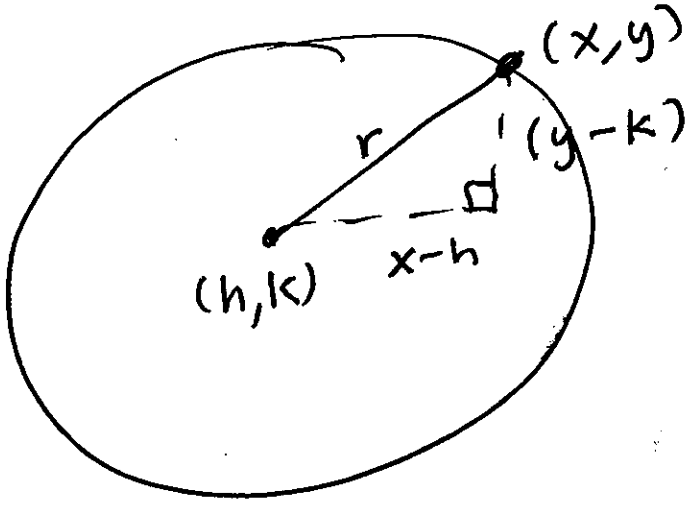
Equation for circle

$$(x-h)^2 + (y-k)^2 = r^2$$

(h, k) center

$$x^2 + y^2 = r^2$$

center $(0,0)$



Pyth Thm

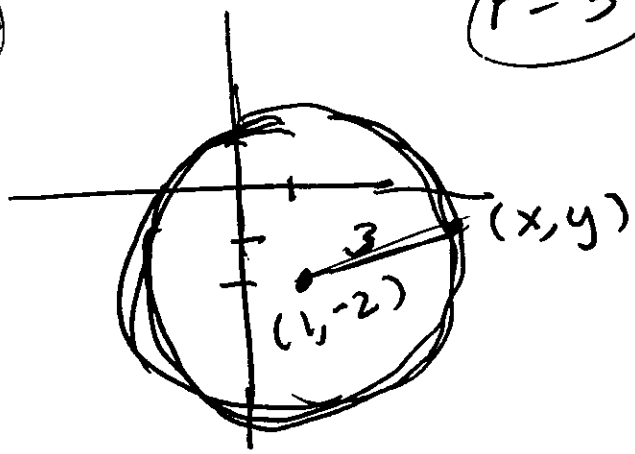
$$(x-h)^2 + (y-k)^2 = r^2$$

EXAMPLE Sketch the graph.

① $(x-1)^2 + (y+2)^2 = 9$

center $(1, -2)$

$r = 3$



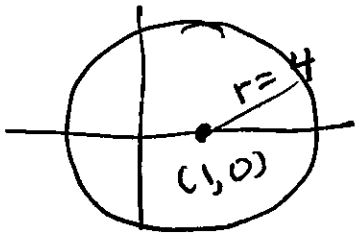
② $x^2 - 2x + y^2 = 15$

Complete the square.

$(x^2 - 2x + \underset{\searrow (-\frac{2}{2})^2}{1}) + y^2 = 15 + 1$

$(x-1)^2 + y^2 = 16$

Center $(1,0)$, $r = 4$



Ellipses

An ellipse is the set of all points (x, y) in a plane the sum of whose distances from two distances from two distinct points (foci) is constant.

Equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

EXAMPLE Sketch the graph.

① $\frac{x^2}{9} + y^2 = 1$

intercepts

$x = 0$

$\frac{0^2}{9} + y^2 = 1$

$y^2 = 1$

$y = \pm 1$

$(0, 1), (0, -1)$

$y = 0$

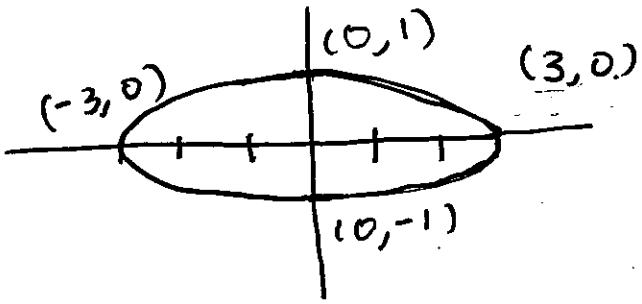
$\frac{x^2}{9} + 0^2 = 1$

$\frac{x^2}{9} = 1$

$x^2 = 9$

$x = \pm 3$

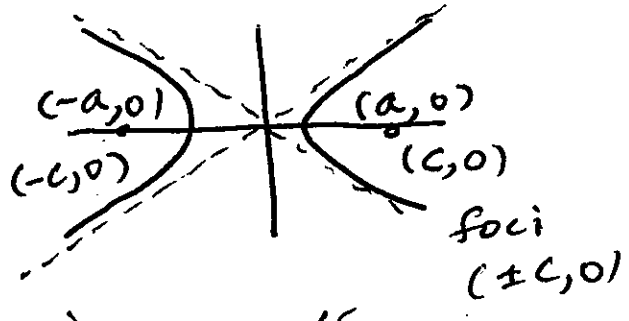
$(3, 0), (-3, 0)$



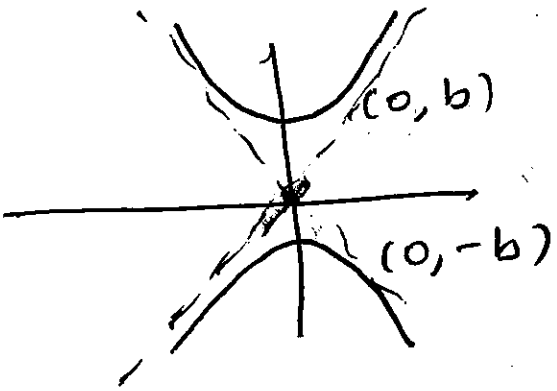
Hyperbolas

A hyperbola is the set of all points (x, y) in the plane the difference of whose distances from two distincts (foci) is a positive constant.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$



$$\frac{y^2}{b^2} - \frac{x^2}{a^2} = 1$$



Asymptote

$$y = \pm \frac{b}{a} x$$

EXAMPLE Sketch the graph.

① $\frac{x^2}{9} - y^2 = 1$ $\qquad x^2 - 9y^2 = 9$

Intercepts

- $y = 0$

$\frac{x^2}{9} = 1$

$x^2 = 9$

$x = \pm 3$

$(3, 0), (-3, 0)$

- $x = 0$

$\frac{0^2}{9} - y^2 = 1$

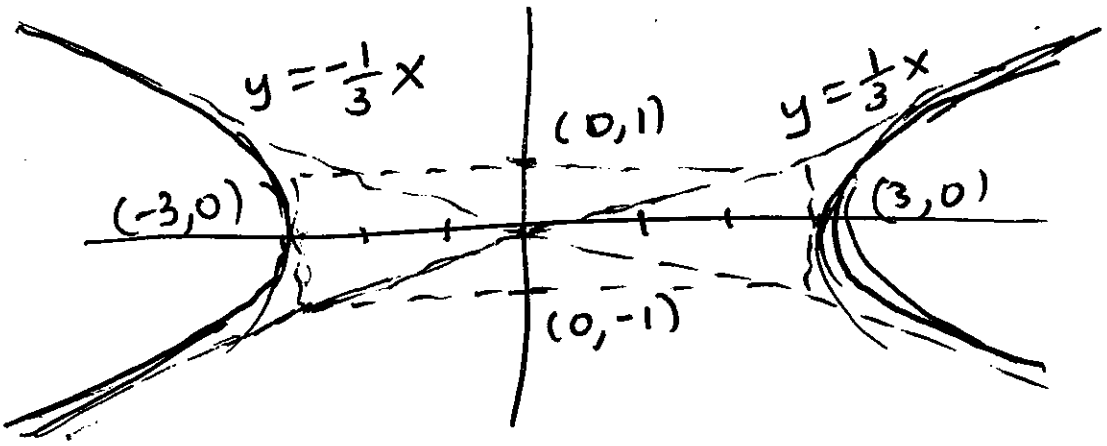
$-y^2 = 1$

$y^2 = -1$ no real sol.

No y-int.

$y^2 = -1$

$y = \pm 1 \cdot i$



② $y^2 - 16x^2 = 16$

$\frac{y^2}{16} - x^2 = 1$

$\frac{y^2}{(4)^2} - \frac{x^2}{(1)^2} = 1$

Intercepts

$x = 0$

$\frac{y^2}{4^2} = 1$

$y^2 = 4^2$

$y = \pm 4$

$(0, 4), (0, -4)$

$y = 0$

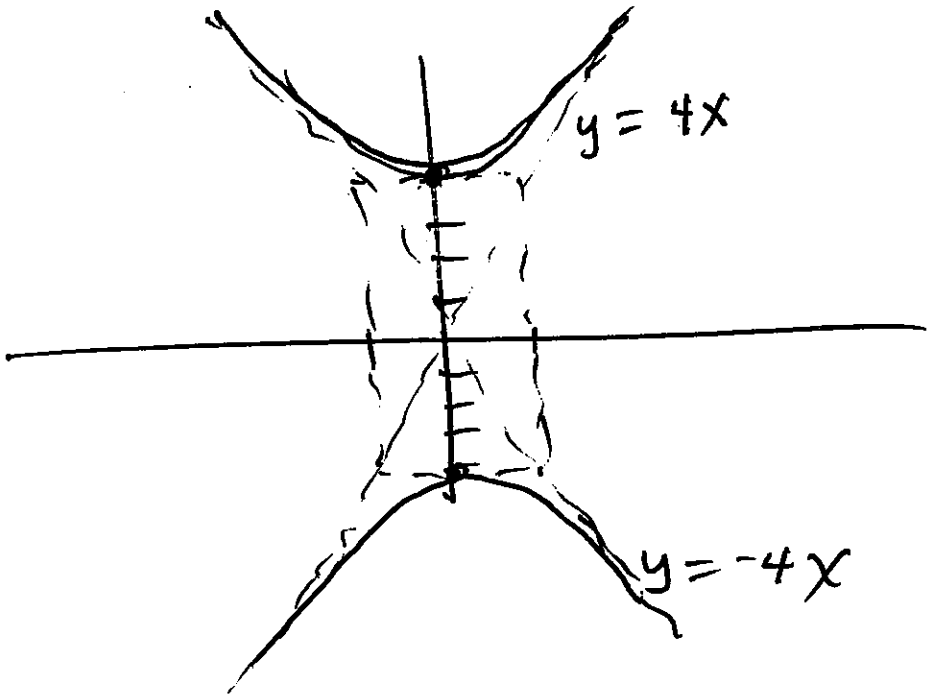
$\frac{0^2}{4^2} - \frac{x^2}{1^2} = 1$

$x^2 = -1$

no real sol

$x = \pm i$

↑ use this to find asymptotes.



Let's show that $y = \pm\frac{b}{a}x$ is the equation of the asymptote.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

$$\frac{x^2}{a^2} = \frac{y^2}{b^2} + 1$$

$$\frac{x^2}{a^2} - 1 = \frac{y^2}{b^2}$$

$$\frac{b^2}{a^2}x^2 - b^2 = y^2$$

$$y^2 = \frac{b^2}{a^2}x^2 - b^2$$

$$y = \pm\sqrt{\frac{b^2}{a^2}x^2 - b^2}$$

$$y = \pm\sqrt{\frac{b^2}{a^2}x^2\left(1 - \frac{a^2}{x^2}\right)}$$

$$y = \pm\frac{b}{a}x\sqrt{1 - \frac{a^2}{x^2}}$$

$\frac{a^2}{x^2} \to 0$ as $x \to \infty$

As $x \to \infty$ approaches infinity

$$\frac{a^2}{x^2} \longrightarrow 0$$

$$y \to \frac{\pm b}{a} x$$

Summations

homework online.

Let's look at the series

$$1 + 4 + 9 + 16 + 25 + 36$$

$$1^2 + 2^2 + 3^2 + 4^2 + 5^2 + 6^2$$

6 ← ends at 6.

$$= \sum_{i=1}^{6} i^2$$

↑ starts at 1

(Greek upper case sigma Σ)

EXAMPLE Write using summation notation.

① $2 + 4 + 6 + 8 + 10 + 12 + 14$

$= 2 \cdot 1 + 2 \cdot 2 + 2 \cdot 3 + 2 \cdot 4 + 2 \cdot 5 + 2 \cdot 6 + 2 \cdot 7$

$$= \sum_{k=1}^{7} 2k$$

② $1 + 2 + 4 + 8 + 16 + 32 = 2^0 + 2^1 + 2^2 + 2^3 + 2^4 + 2^5$

$$= \sum_{n=0}^{5} 2^n$$

(3) [illegible]

$1 - 3 + 9 - 27 + 81$

$= \sum_{K=0}^{4} (-3)^K$ Geometric Series.

(4) $2 + 5 + 8 + 11 + 14 + 17$

$+3 \quad +3 \quad +3 \quad +3$

arithmetic series

$\sum_{n=1}^{6} (3n - 1)$

choose this so that $3 \cdot 1 - 1 = 2$

$3 \cdot 2 - 1 = 5$

etc.

or

$\sum_{n=0}^{5} (3n + 2)$