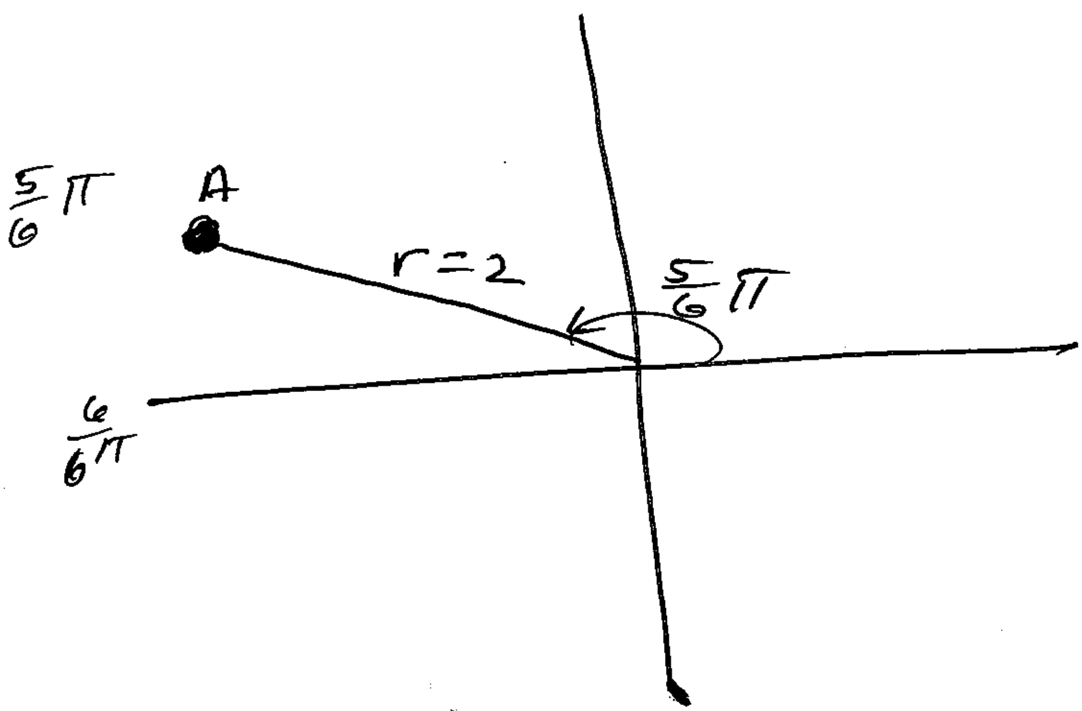
§7.6 Polar Equations

HW §7.6 ~~#1-73 odd~~
1-56, 65-68

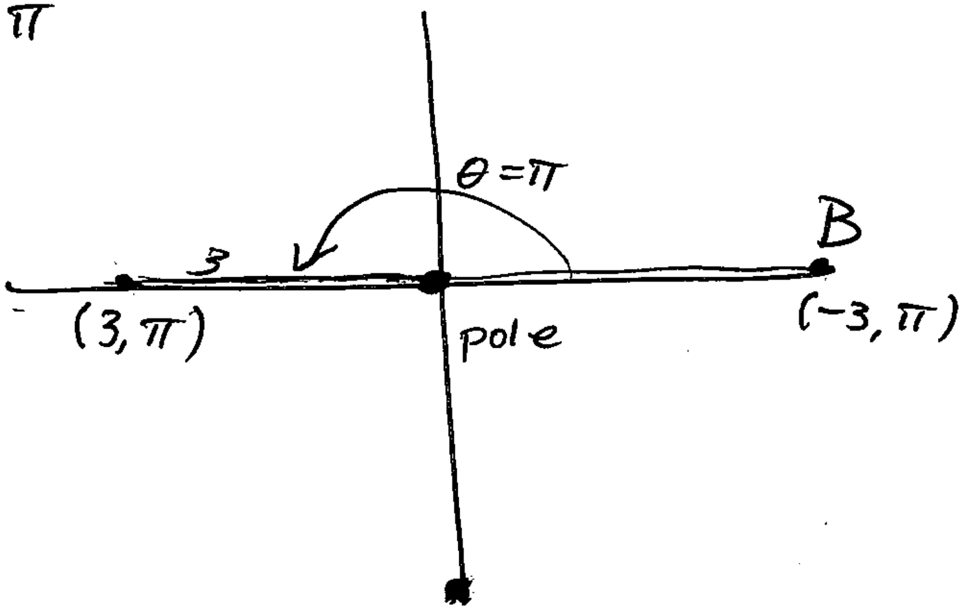
<u>EXAMPLE</u> Plot the polar coordinates $A(2, 5\pi/6)$, $B(-3, \pi)$, $C(1, -\pi/2)$, and $D(-1, 450°)$.

A point in polar coordinates is given by (r, θ), where r is radius, θ degrees or radians.

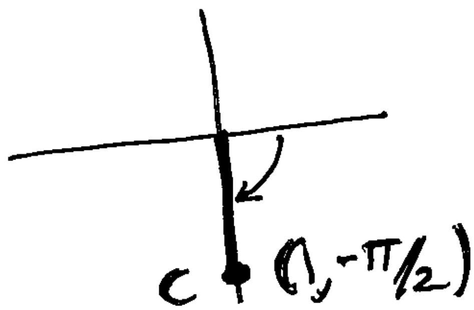


(B) $B(-3, \pi)$

$r=-3, \ \theta=\pi$

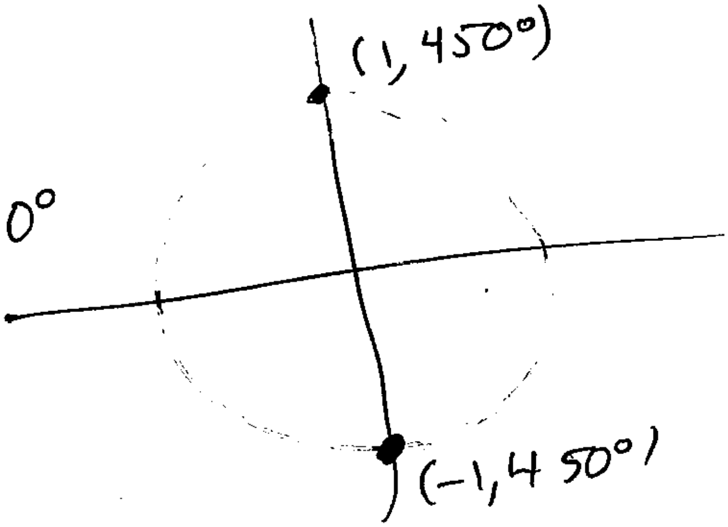


(C) $C(1, -\pi/2)$

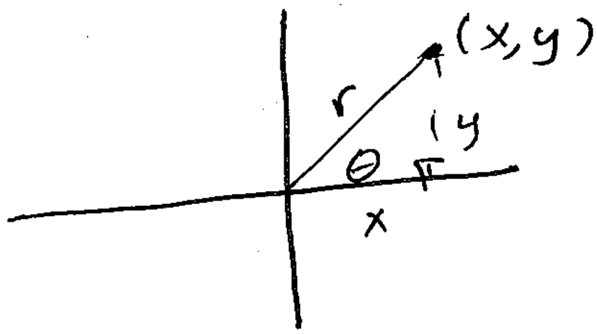


(D) $D(-1, 450^\circ)$

$\theta = 450^\circ$
$= 360^\circ + 90^\circ$



Polar - Rectangular Conversion Rules



$\cos\theta = \frac{x}{r}$

$\sin\theta = \frac{y}{r}$

$x = r\cos\theta, \; y = r\sin\theta$

$x^2 + y^2 = r^2$

<u>Example</u>: Convert from polar coordinates (r, θ), to rectangular coordinates, (x, y).

a) $(2, \pi/3)$

$r = 2, \; \theta = \frac{\pi}{3}$

$x = r\cos\theta$

$x = 2\cos\frac{\pi}{3} = 2(1/2) = 1$

$y = r\sin\theta$

$y = 2\sin\frac{\pi}{3} = 2\frac{\sqrt{3}}{2} = \sqrt{3}$

$\boxed{(1, \sqrt{3})}$

b) $(3, \frac{5}{4}\pi)$

$r = 3, \; \theta = \frac{5\pi}{4}$

$x = r\cos\theta = 3\cos\frac{5}{4}\pi = 3\left(-\frac{\sqrt{2}}{2}\right) = \frac{-3\sqrt{2}}{2}$

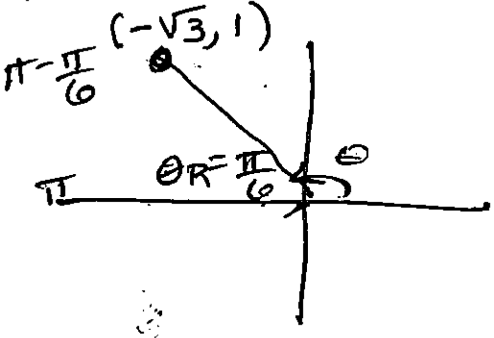
$y = r\sin\theta = 3\sin\frac{5}{4}\pi = 3\left(-\frac{\sqrt{2}}{2}\right) = -\frac{3}{2}\sqrt{2}$

$\boxed{\left(-\frac{3}{2}\sqrt{2}, \frac{-3\sqrt{2}}{2}\right)}$

EXAMPLE Convert from rectangular coordinates, (x, y), to ~~fo~~ polar coordinates, (r, θ).

a) $(-\sqrt{3}, 1)$

$x = -\sqrt{3},\ y = 1$



Find r

$\cancel{x^2+y^2}$

$x^2 + y^2 = r^2$

$(-\sqrt{3})^2 + (1)^2 = r^2$

$3 + 1 = r^2$

$r^2 = 4$

$\boxed{r = 2}$

Find θ

$x = r\cos\theta$

$-\sqrt{3} = 2\cos\theta$

$\cos\theta = -\frac{\sqrt{3}}{2}$

$\theta = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right) = \frac{5}{6}\pi$

$\theta = \frac{5}{6}\pi$

$\boxed{\left(2, \frac{5}{6}\pi\right)}$

$\theta_R = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{6}$

b) $(-3, 3)$

b) $(-3, -3)$

$x = -3, \quad y = -3$

Find r

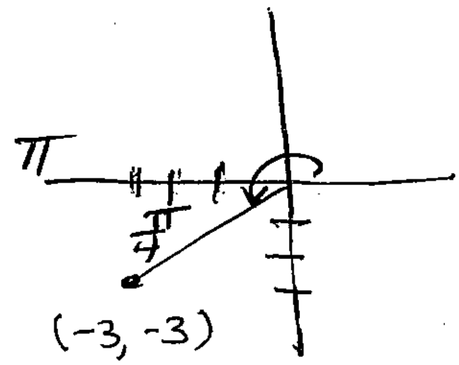
$x^2 + y^2 = r^2$

$(-3)^2 + (-3)^2 = r^2$

$9 + 9 = r^2$

$r^2 = 18$

$r = \sqrt{18} = \sqrt{9 \cdot 2} = 3\sqrt{2}$ $\quad \boxed{r = 3\sqrt{2}}$



Find θ

$\frac{y}{x} = \tan\theta$

$\tan\theta = \frac{y}{x}$

$\tan\theta = \frac{-3}{-3} = 1$

$\theta_R = \tan^{-1}(1) = \frac{\pi}{4}$

$\boxed{\theta = \frac{5\pi}{4}}$

$\boxed{\left(3\sqrt{2}, \frac{5}{4}\pi\right)}$

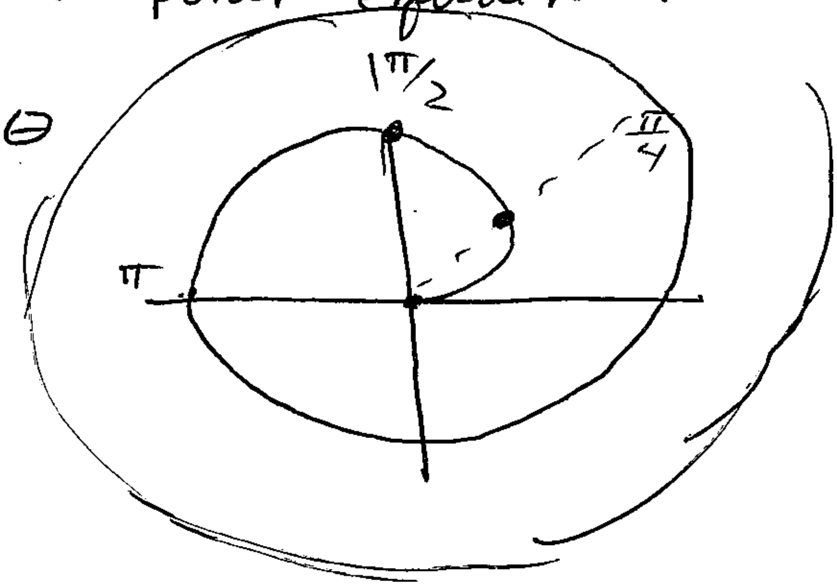
Polar Equations

EXAMPLE: Sketch the graph of the polar equation.

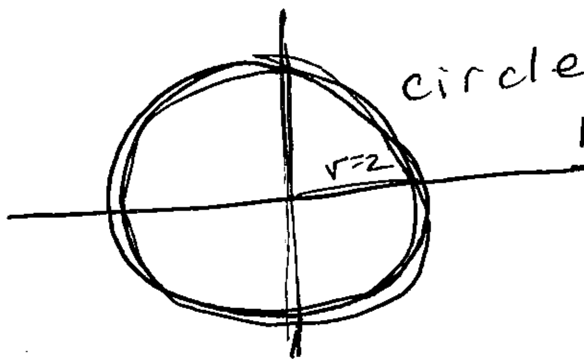
① $r = \theta$

$\theta \geq 0$

θ	r
0	0
$\frac{\pi}{4}$	$\frac{\pi}{4}$
$\frac{\pi}{2}$	$\frac{\pi}{2}$

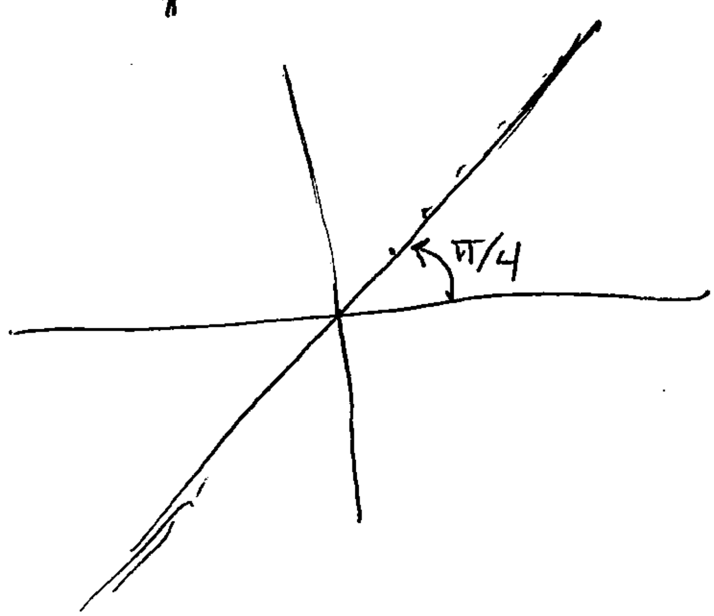


② $r = 2$



circle with radius $r=2$

③ $\theta = \frac{\pi}{4}$



④ $r = 2\cos\theta$

Trick multiply both sides by r.

$$r^2 = \underbrace{2r\cos\theta}_{x}$$

$$x^2+y^2 = 2x$$

$$x^2 - 2x + y^2 = 0$$

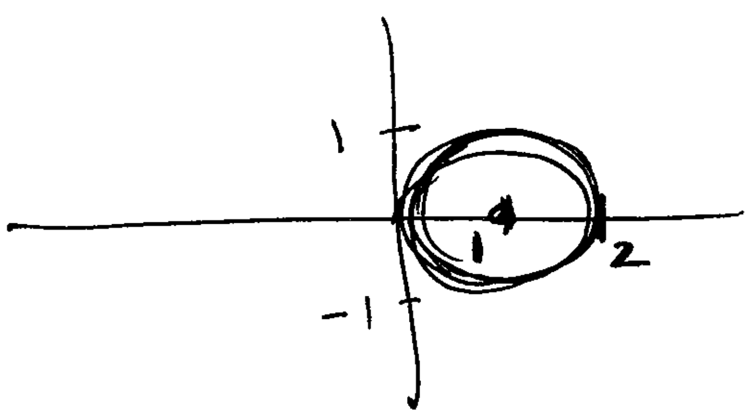
$$(x^2 - 2x + 1) + y^2 = 1$$

$$\left(\frac{-2}{2}\right)^2$$

$$(x-1)^2 + y^2 = 1$$

Circle
center $(1,0)$
$r = 1$

Circle: $(x-h)^2+(y-k)^2=r^2$
center (h,k)
radius r



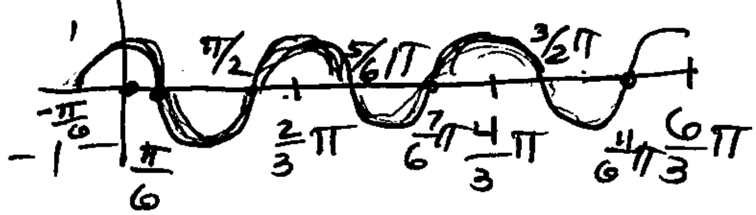
5 $r = \cos 3\theta$

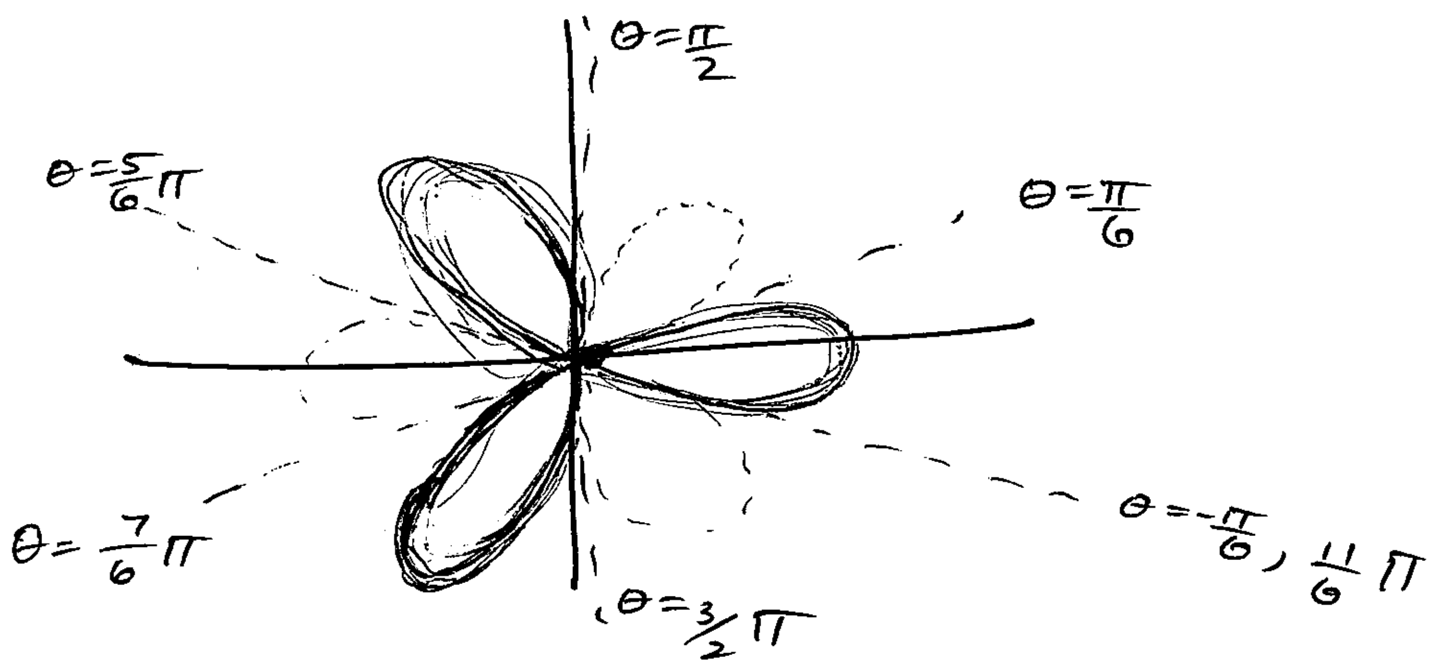
θ	r
$-\frac{\pi}{6}$	0
0	1
$\frac{\pi}{6}$	0

sketch $y = \cos 3x$

$P = \frac{2\pi}{3}$ Qtr Per $\frac{2}{3}\pi \cdot \frac{1}{4} = \frac{\pi}{6}$

Optional





~~3 leaf rose~~

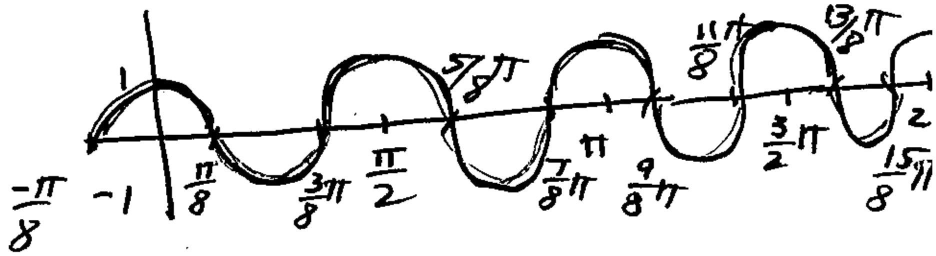
3 pedal rose

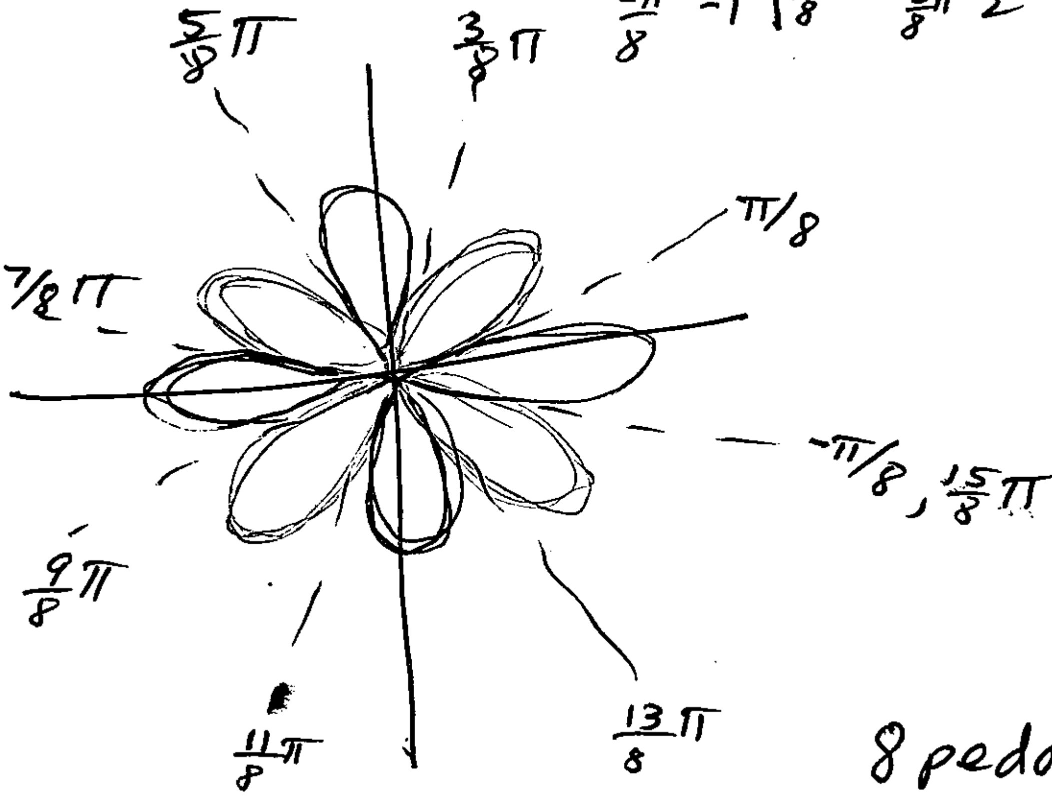
⑥ $r = \cos 4\theta$

$y = \cos 4x$

$p = \frac{2\pi}{4} = \frac{\pi}{2}$

Qtr per: $\frac{\pi}{2} \cdot \frac{1}{4} = \pi/8$





Zeros

$\cos 4x = 0$

[illegible]

~~$x = \frac{k\pi}{2}$~~

$4x = \frac{\pi}{2} + k\pi$

$x = \frac{\pi}{8} + \frac{k\pi}{4}$

8 pedal rose

For $r = \cos n\theta$
or $r = \sin n\theta$

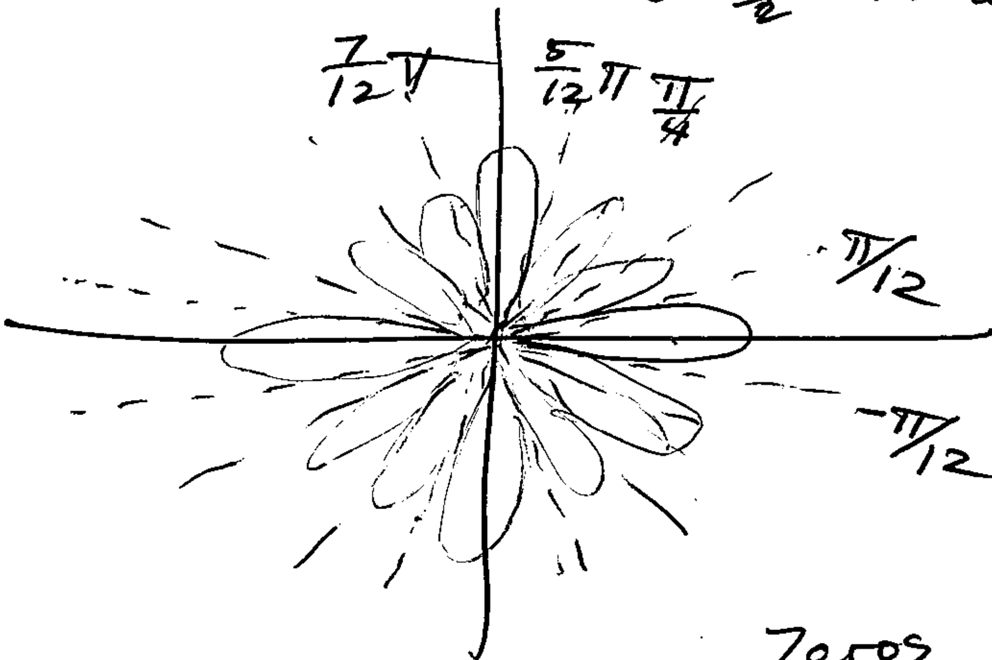
- if n is odd, we get an nth pedal rose (n pedals)
- if n is even, we get a 2nth pedal rose. (2n pedals)

(7) $r = \cos 6\theta$

even, 12 pedals

$\theta = 0, \quad r = \cos 6 \cdot 0 = 1$

$\theta = \frac{\pi}{2} \quad r = \cos 6\frac{\pi}{2} = \cos \frac{3}{\cancel{4}}\pi = -$



<u>Zeros</u>

$\cos 6\theta = 0$

$6\theta = \frac{\pi}{2} + k\pi$

$\theta = \frac{\pi}{12} + k\frac{\pi}{6}$