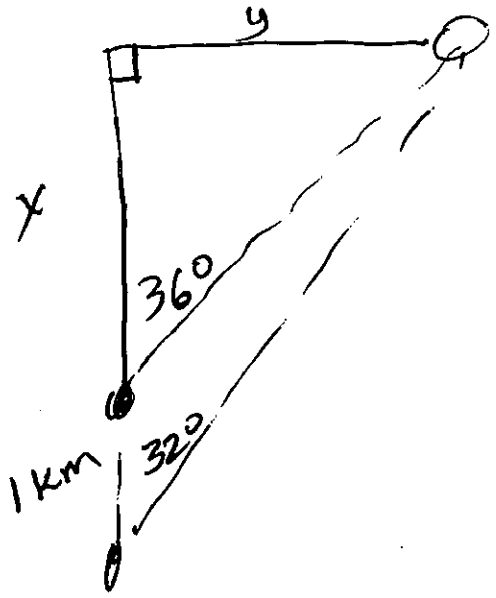
Practice

§5.6 ① Passing in the Night: A boat sailing north sights a lighthouse to the east at an angle of 32° from the north. After the boat travels 1 km, the angle of the lighthouse from the north is 36°. If the boat continues to sail north, then how close will it come to the lighthouse?




Find y

Sol'n $\tan 36° = \frac{y}{x}$

$\tan 32° = \frac{y}{x+1}$

$x \tan 36° = y$

$x = \frac{y}{\tan 36°}$

$\tan 32° = \frac{y}{x+1} = \frac{y}{\left(\frac{y}{\tan 36°}\right)+1}$

$$\tan 32^\circ = \frac{y}{\frac{y}{\tan 36^\circ} + 1}$$

$$\tan 32^\circ \left(\frac{y}{\tan 36^\circ} + 1\right) = y$$

$$\tan 32^\circ = \frac{y \tan 36^\circ}{y + \tan 36^\circ}$$

$$\tan 32^\circ (y + \tan 36^\circ) = y \tan 36^\circ$$

$$y \tan 32^\circ + \tan 32^\circ \tan 36^\circ = y \tan 36^\circ$$

$$y \tan 32^\circ - y \tan 36^\circ = -\tan 32^\circ \tan 36^\circ$$

$$y(\tan 32^\circ - \tan 36^\circ) = -\tan 32^\circ \tan 3$$

$$y = -\frac{\tan 32^\circ \tan 36^\circ}{\tan 32^\circ - \tan 36^\circ}$$

$$y = \frac{\tan 32^\circ \tan 36^\circ}{\tan 36^\circ - \tan 32^\circ} \approx 4.5 \text{ km}$$

(2) $\sin 2x = \cos x$

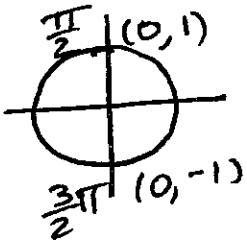
$2\sin x \cos x = \cos x$

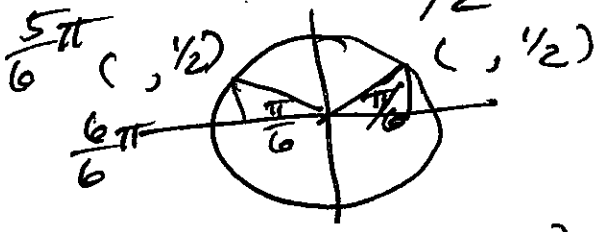
$2\sin x \cos x - \cos x = 0$

$\cos x(2\sin x - 1) = 0$

$\cos x = 0 \qquad 2\sin x - 1 = 0$

$\sin x = 1/2$




$\left\{ \frac{\pi}{2} + k\pi,\ \frac{\pi}{6} + 2k\pi,\ \frac{5}{6}\pi + 2k\pi \right\}$

(3) $\sin 2x = \sin x$

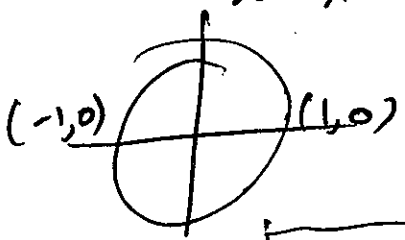
$2\sin x \cos x = \sin x$

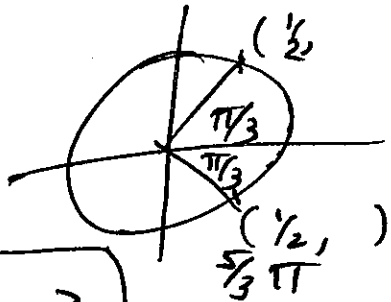
$2\sin x \cos x - \sin x = 0$

$\sin x(2\cos x - 1) = 0$

$\sin x = 0, \qquad 2\cos x - 1 = 0$

$\cos x = 1/2$




$\left\{ k\pi,\ \frac{\pi}{3} + 2k\pi,\ \frac{5}{3}\pi + 2k\pi \right\}$

§ 6.3 Find the exact value.

④ $\cos\left(\frac{5}{12}\pi\right) = \cos\left(\frac{3}{12}\pi + \frac{2}{12}\pi\right)$

$= \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right)$

$= \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6}$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

⑤ $\cos\left(\frac{7}{12}\pi\right) = \cos\left(\frac{4}{12}\pi + \frac{3}{12}\pi\right)$

$= \cos\left(\frac{\pi}{3} + \frac{\pi}{4}\right)$

$= \cos\frac{\pi}{3}\cos\frac{\pi}{4} - \sin\frac{\pi}{3}\sin\frac{\pi}{4}$

$= \frac{1}{2}\frac{\sqrt{2}}{2} - \frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

⑥ $\sin 15^\circ = \sin(45^\circ - 30^\circ)$

$= \sin 45^\circ \cos 30^\circ - \cos 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2}$

$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

⑦ $\cos 105^\circ = \cos(45^\circ + 60^\circ)$

$= \cos 45^\circ \cos 60^\circ - \sin 45^\circ \sin 60^\circ$

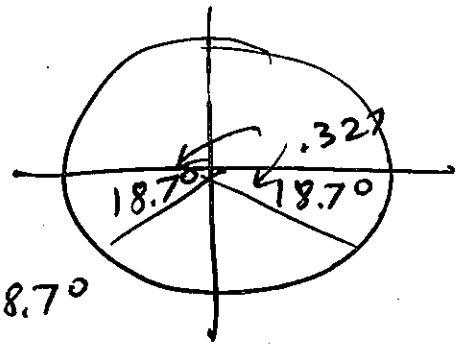
$= \frac{\sqrt{2}}{2}\frac{1}{2} - \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}$

$= \frac{\sqrt{2} - \sqrt{6}}{4}$

§6.6 Find all real solutions.

(8) $\sin\alpha = -0.321$

$\alpha_R = \sin^{-1}(.321) \approx 18.7° = .327$




$180 + 18.7°$

$= 198.7°$

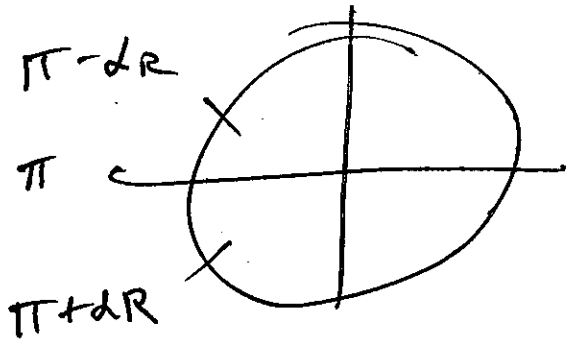
$\pi + .327 \approx 3.47$

$\{198.7° + 360°k,$
$-18.7° + 360°\}$

$\{-.327 + 2k\pi, 3.47 + 2k\pi\}$

(9) $\cos\alpha = -0.75$

$\alpha_R = \cos^{-1}(.75) \approx 0.72$

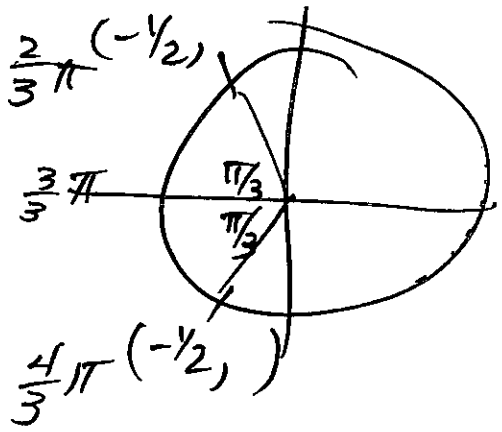



$\{2.42 + 2k\pi, 3.86 + 2k\pi\}$

(10) $\cos 2\alpha = -\frac{1}{2}$

$2\alpha = \frac{2}{3}\pi + 2k\pi$

$\boxed{\alpha = \frac{\pi}{3} + k\pi}$




$2\alpha = \frac{4}{3}\pi + 2k\pi$

$\boxed{\alpha = \frac{2}{3}\pi + k\pi}$

§6.3 (11) Find the exact value of $\sin(\alpha - \beta)$, given that $\sin\alpha = -\frac{4}{5}$ and $\cos\beta = \frac{12}{13}$, with $\alpha$ in quadrant III, $\beta$ in quadrant IV.

SOLUTION $\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$= \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{-5}{13}\right)$

$= \frac{-48 + 15}{65} = \frac{-63}{65}$

Find $\cos\alpha$

$\cos\alpha = \pm\sqrt{1-\sin^2\alpha}$

$= \pm\sqrt{1-(-4/5)^2} = \pm\sqrt{9/25}$

$= \pm\frac{3}{5}$

$\cos\alpha = -\frac{3}{5}$ quad 3

Find $\sin\beta$

$\sin\beta = \pm\sqrt{1-\cos^2\beta}$

$= \pm\sqrt{1-(12/13)^2}$

$= \pm\sqrt{25/169} = \pm 5/13$

$\sin\beta = -\frac{5}{13}$

↑ quad 4