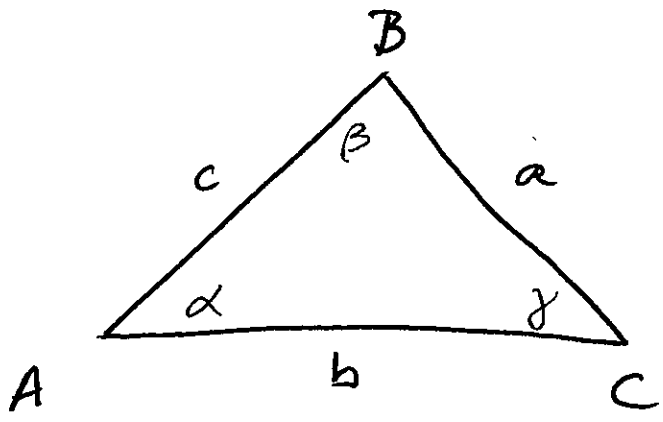
§7.1 The Law of Sines #1-8 odd



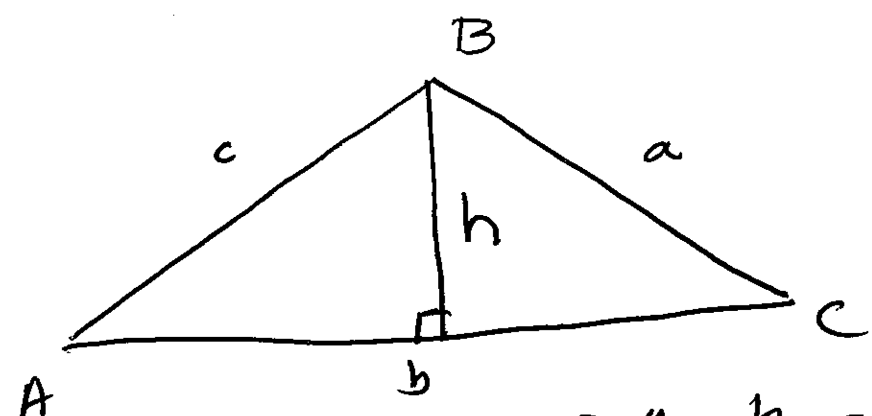
The Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

or write

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

<u>Proof</u>



we have $\sin A = \frac{h}{c} = \frac{opp}{hyp}$

$h = c \sin A$

and $\sin C = \frac{h}{a}$ so ~~h = sin C~~

$h = a \sin C$

So $c \sin A = a \sin C$

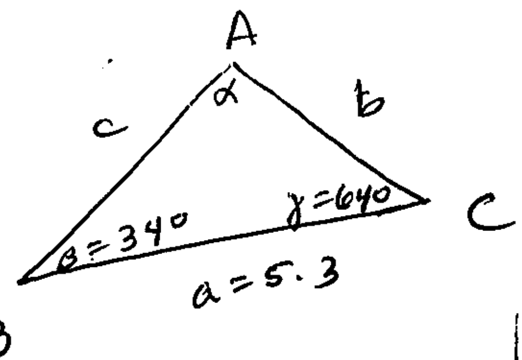
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

We can do the same for the other angles.

EXAMPLE Give two angles and an included side (ASA).

Given $\beta = 34°$, $\gamma = 64°$, and $a = 5.3$, solve the triangle.

SOLUTION:



$\alpha = 82°$

$c \approx 4.8$

$b \approx 3.0$

* Find α

$$\alpha = 180° - 34° - 64°$$

$$\alpha = 82°$$

* Find c

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 64°}{c} = \frac{\sin 82°}{5.3}$$

$$5.3 \sin 64° = c \sin 82°$$

$$c = \frac{5.3 \sin 64°}{\sin 82°} \approx 4.8$$

* Find b

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin 34^\circ}{b} = \frac{\sin 82^\circ}{5.3}$$

$$5.3 \sin 34^\circ = b \sin 82^\circ$$

$$b = \frac{5.3 \sin 34^\circ}{\sin 82^\circ}$$

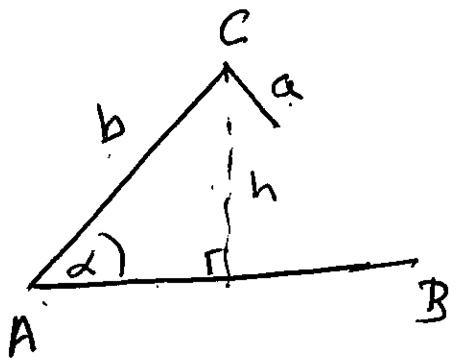
$$b \approx 3.0$$

The Ambiguous (ASS) case

Not on Test or Homework

Four Possibilities

Ⓘ



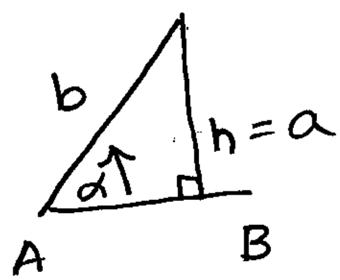
α, a, b are given

$\sin\alpha = \frac{h}{b}$

$h = b\sin\alpha$

if $a < h$, there's no triangle

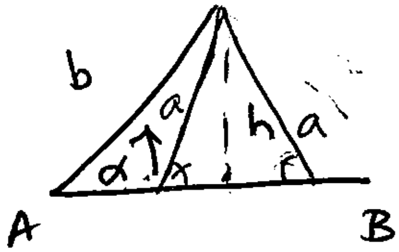
Ⓘⓘ



$h = a$

then there is exactly one triangle

Ⅲ



$h < a < b$

Two possible triangles.

Ⅳ

$a \geq b$ one triangle

§7.2 The Law of Cosines

HW §7.2 # 1–14 odd

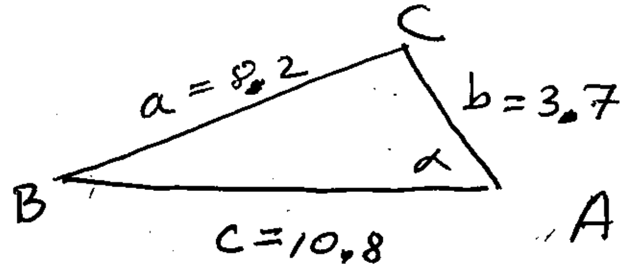
If triangle ABC is an oblique triangle with sides a, b, and c and angles α, β, and γ, then

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$
$$b^2 = a^2 + c^2 - 2ac\cos\beta$$
$$c^2 = a^2 + b^2 - 2ab\cos\gamma$$

<u>EXAMPLE</u> Given three sides of a triangle (SSS)

Given $a = 8.2$, $b = 3.7$, and $c = 10.8$, solve the triangle.

<u>SOLUTION</u>



Find α

$$a^2 = b^2 + c^2 - 2bc\cos\alpha$$

$$(8.2)^2 = (3.7)^2 + (10.8)^2 - 2(3.7)(10.8)\cos\alpha$$

$$(8.2)^2 - (3.7)^2 - (10.8)^2 = -2(3.7)(10.8)\cos\alpha$$

$$\cos\alpha = \frac{(8.2)^2 - (3.7)^2 - (10.8)^2}{-2(3.7)(10.8)}$$

$$\alpha = \cos^{-1}\left(\frac{(8.2)^2 - (3.7)^2 - (10.8)^2}{-2(3.7)(10.8)}\right)$$

$$\approx 37.9^\circ$$

$$\boxed{\alpha = 37.9^\circ}$$

Find β

$$\frac{\sin\beta}{b} = \frac{\sin\alpha}{a}$$

$$\frac{\sin\beta}{3.7} = \frac{\sin 37.9^\circ}{8.2}$$

$$\sin\beta = \frac{3.7\sin 37.9^\circ}{8.2}$$

$$\beta = \sin^{-1}\left(\frac{3.7\sin 37.9^\circ}{8.2}\right) \approx 16.1^\circ$$

$$\boxed{\beta = 16.1^\circ}$$

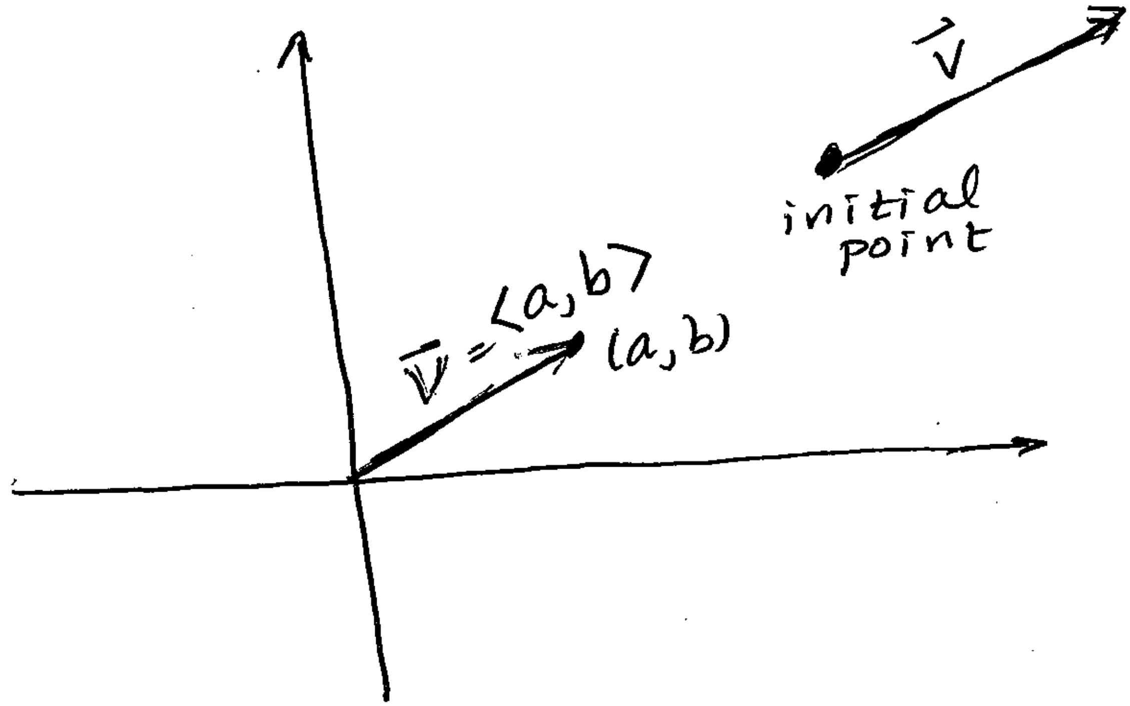
Find γ, $\gamma = 180^\circ - \alpha - \beta$

$$\gamma = 180 - 37.9 - 16.1$$

$$\boxed{\gamma = 126^\circ}$$

§7.3 Vectors

HW §7.3 #1-87 odd



- A vector is a directed line segment.
  - A vector is defined by its direction and its length.
- If the initial point is the origin, then the vector can be described by the coordinates of its endpoint

Scalar Multiplication.

If $\vec{v} = \langle a, b \rangle$ and c is a scalar (a real number in our case)

then $\quad c\vec{v} = c\langle a, b\rangle = \langle ca, cb\rangle$

<u>Example</u>: Perform the given operation.

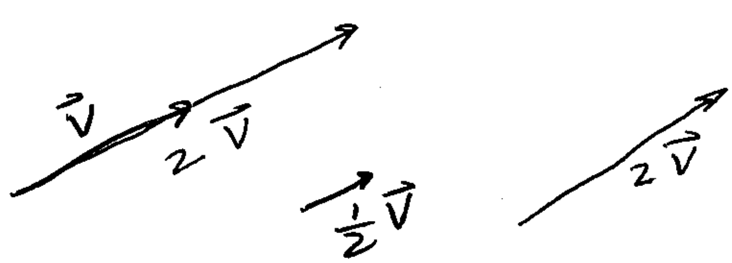
a) $5\langle 2, -3\rangle$

$= \langle 10, -15\rangle$

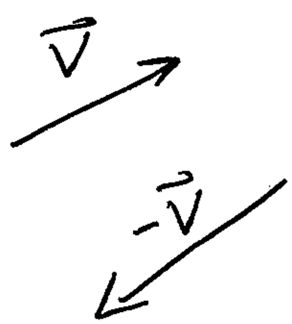
b) $-2\langle 7, 5\rangle$

$= \langle -14, -10\rangle$

Multiplying a vector $\vec{v}$ by a positive scalar c will change the length by a factor of c. The direction stays the same.



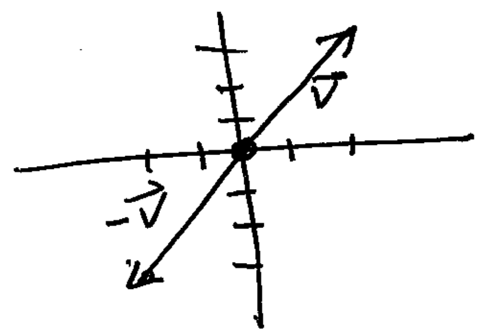
Multiplying a vector $\vec{v}$ by a negative scalar changes the direction by 180°.



Example: $\vec{v} = \langle 2,3 \rangle$

$-1 \cdot \vec{v} = -1 \cdot \langle 2,3 \rangle$

$= \langle -2,-3 \rangle$



Addition of Vectors

Def: $\langle a,b \rangle + \langle c,d \rangle$

$= \langle a+c,\ b+d \rangle$

we add componentwise.

$\langle a,b \rangle - \langle c,d \rangle = \langle a-c,\ b-d \rangle$

Example: Perform the operation.

① $2\langle 3,5 \rangle - 3\langle 1,4 \rangle$

$= \langle 6,10 \rangle + \langle -3,-12 \rangle$

$= \langle 6-3,\ 10-12 \rangle = \langle 3,-2 \rangle$

② $\langle 7,5 \rangle + \langle 2,1 \rangle = \langle 9,6 \rangle$