Practice.

§6.2 Prove that each equation is an identity.

① $\cos 2x \tan^2 x = \sin^2 x$

LHS

$\cos^2 x \tan^2 x$

$= \cos^2 x \dfrac{\sin^2 x}{\cos^2 x}$

$= \sin^2 x$ equals RHS

② $1 - \csc x \sin^3 x = \cos^2 x$

LHS

$1 - \csc x \sin^3 x$

$= 1 - \dfrac{1}{\sin x} \cdot \sin^3 x$

$= 1 - \sin^2 x$

$= \cos^2 x$ equals RHS

(3) $1 + \csc^2 x \cos^2 x = \csc^2 x$

LHS

$1 + \csc^2 x \cos^2 x$

$= 1 + \frac{1}{\sin^2 x} \cos^2 x$

$= 1 + \cot^2 x$

$= \csc^2 x$ equals RHS

(4) $\dfrac{\cos x \sin 2x + \cos^3 x}{\sin x} = \cot x$

LHS

$\dfrac{\cos x \sin^2 x + \cos^3 x}{\sin x}$

$= \dfrac{\cos x (\sin^2 x + \cos^2 x)}{\sin x}$

$= \dfrac{\cos(x) \cdot 1}{\sin x}$

$= \cot x$ equals RHS

⑤ $\sin^3 x \csc x + \cos^3 x \sec x = 1$

LHS

$\sin^3 x \csc x + \cos^3 x \sec x$

$= \sin^3 x \frac{1}{\sin x} + \cos^3 x \frac{1}{\cos x}$

$= \sin^2 x + \cos^2 x$

$= 1$

⑥ $\frac{-1}{\tan\theta - \sec\theta} = \frac{1+\sin\theta}{\cos\theta}$

LHS

$\frac{-1}{\tan\theta - \sec\theta}$

$= \left(\frac{-1}{\tan\theta - \sec\theta}\right)\left(\frac{\tan\theta + \sec\theta}{\tan\theta + \sec\theta}\right)$

$= -\frac{\tan\theta + \sec\theta}{\tan^2\theta - \sec^2\theta}$ → Aside: $1 + \tan^2\theta = \sec^2\theta$, $\tan^2\theta - \sec^2\theta = -1$

$= -\frac{\tan\theta + \sec\theta}{-1}$

$= \tan\theta + \sec\theta$

$= \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}$

$= \frac{1+\sin\theta}{\cos\theta}$ equals RHS

(7) $\dfrac{\sec x - \cos x}{\cos x} = \tan^2 x$

LHS

$$\frac{\sec x - \cos x}{\cos x}$$

$$= \left(\frac{\frac{1}{\cos x} - \cos x}{\cos x}\right)\left(\frac{\cos x}{\cos x}\right)$$

$$= \frac{1 - \cos^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \tan^2 x$$ equals RHS

(8) $\tan^2 x = \dfrac{1 - \sin^2 x \csc^2 x + \sin^2 x}{\cos^2 x}$

RHS $\dfrac{1 - \sin^2 x \csc^2 x + \sin^2 x}{\cos^2 x}$

$$= \frac{1 - \sin^2 x \frac{1}{\sin^2 x} + \sin^2 x}{\cos^2 x}$$

$$= \frac{1 - 1 + \sin^2 x}{\cos^2 x}$$

$$= \frac{\sin^2 x}{\cos^2 x}$$

$$= \tan^2 x$$

(9) $\tan^4 z = \dfrac{\sec^2 z - \csc^2 z + \csc^2 z \cos^2 z}{\cot^2 z}$

RHS $\dfrac{\sec^2 z - \csc^2 z + \csc^2 z \cos^2 z}{\cot^2 z}$

$= \dfrac{\sec^2 z - \csc^2 z (1 - \cos^2 z)}{\cot^2 z}$

$= \dfrac{\sec^2 z - \csc^2 z \sin^2 z}{\cot^2 z}$

$= \dfrac{\sec^2 z - \csc^2 z \frac{1}{\csc^2 z} \sin^2 z}{\cot^2 z}$

$= \dfrac{\sec^2 z - 1}{\cot^2 z}$

$= \dfrac{\tan^2 z}{\cot^2 z}$

$= \tan^2 z \tan^2 z$

$= \tan^4 z$

(10) $\tan^2(-x) - \dfrac{\sin(-x)}{\sin x} = \sec^2 x$

LHS

$$\tan^2(-x) - \frac{\sin(-x)}{\sin x}$$

$$= (\tan(-x))^2 - \frac{(-\sin x)}{\sin x}$$

$$= (-\tan x)^2 + 1$$

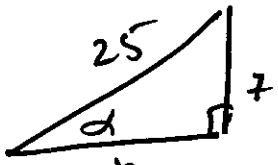
$$= \tan^2 x + 1$$

$$= \sec^2 x \text{ equals RHS}$$

§6.3 #11 Find the exact value of $\sin(\alpha+\beta)$ if $\sin\alpha = \frac{7}{25}$ and $\sin\beta = \frac{-8}{17}$, with α in quadrant II and β in quadrant III.

Find ~~sin~~ cos β

$\sin\alpha = \frac{7}{25} = \frac{opp}{hyp}$



$7^2 + b^2 = 25^2$

$b^2 = 625 - 49$

$b^2 = 576$

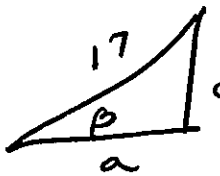
$b = 24$

$\cos\alpha = \frac{adj}{hyp} = \frac{-24}{25}$

neg because quad II

Find cos β

$\sin\beta = \frac{-8}{17} = \frac{opp}{hyp}$



$a^2 + 8^2 = 17^2$

$a^2 = 17^2 - 8^2$

$a^2 = 225$

$a = 15$

$\cos\beta = \frac{adj}{hyp} = \frac{-15}{17}$

neg Quad III

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(\frac{7}{25}\right)\left(\frac{-15}{17}\right) + \left(\frac{-24}{25}\right)\left(\frac{-8}{17}\right)$

$= \frac{87}{425}$

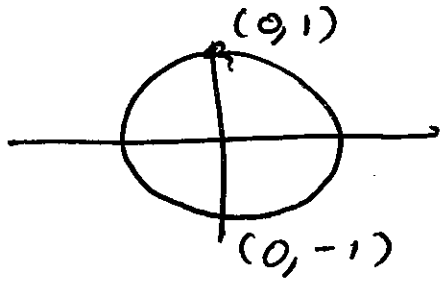
§6.6 Find all real solutions

(12) $\cos x = 0$

$x = \frac{\pi}{2} + 2k\pi$

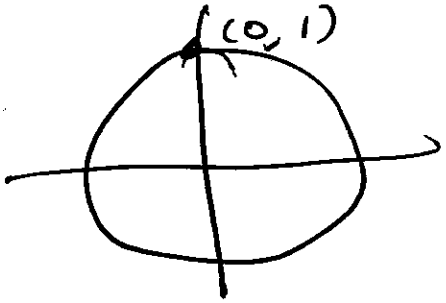
$x = \frac{3}{2}\pi + 2k\pi$

$\{\pi/2 + k\pi\}$



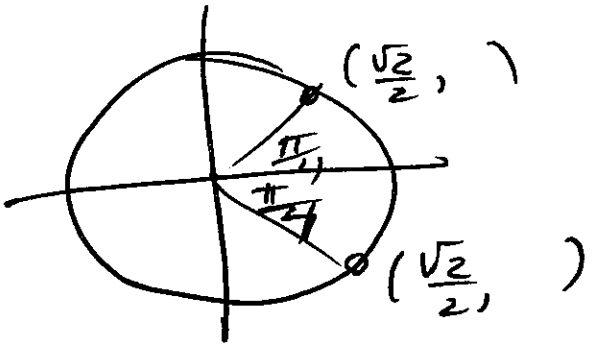
(13) $\sin x = 1$

$x = \frac{\pi}{2} + 2k\pi$



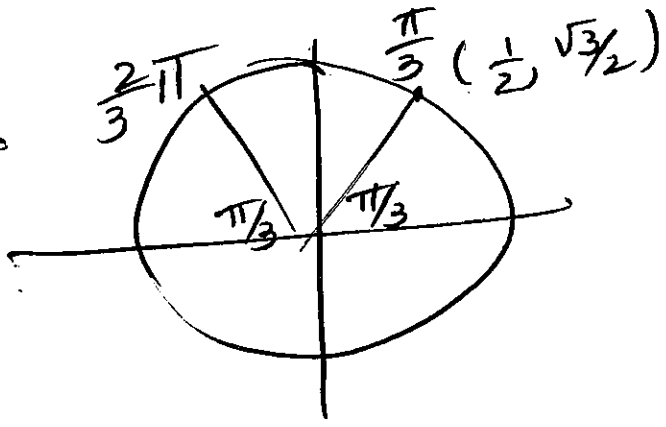
(14) $\cos x = \frac{\sqrt{2}}{2}$

$\{\frac{\pi}{4} + 2k\pi, \frac{7}{4}\pi + 2k\pi\}$

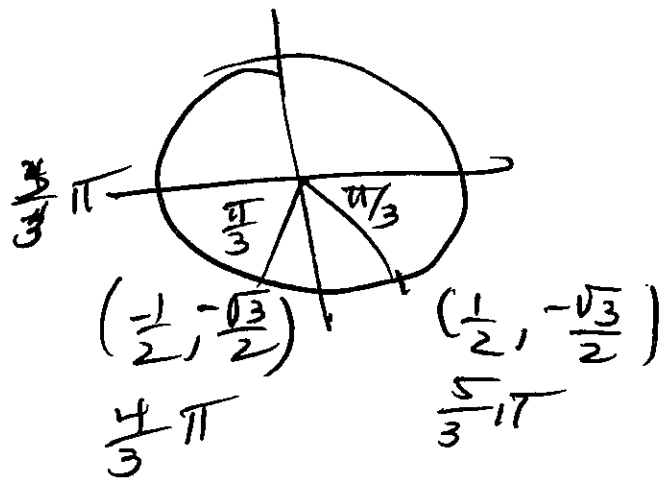


(15) $\sin x = \frac{\sqrt{3}}{2}$

$\{\frac{\pi}{3} + 2k\pi, \frac{2}{3}\pi + 2k\pi\}$



(16) $\sin x = -\frac{\sqrt{3}}{2}$



$\left\{\frac{4}{3}\pi + 2k\pi, \frac{5}{3}\pi + 2k\pi\right\}$