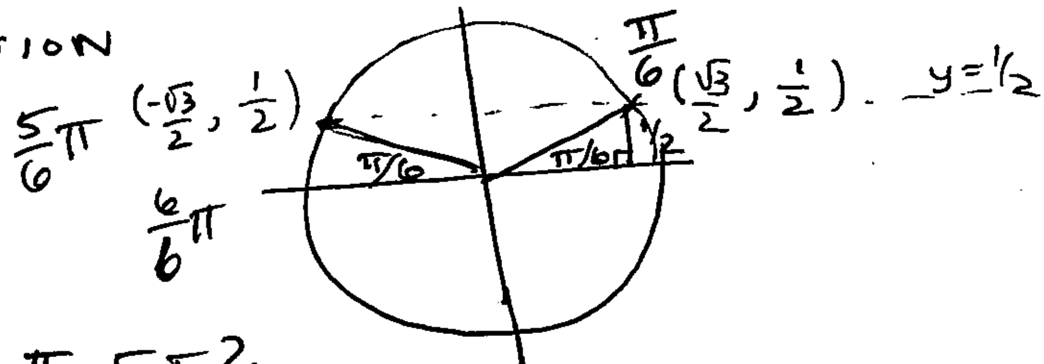
§6.6 Conditional Trigonometric Equations.

HW §6.6 #1–69 odd

EXAMPLE Find all real solutions in the interval $[0, 2\pi)$.

(i) $\sin\theta = \frac{1}{2}$

SOLUTION

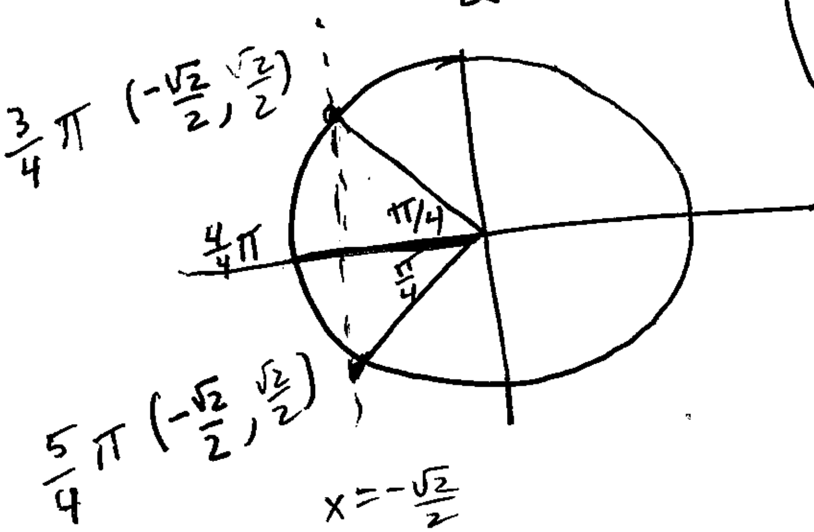


$\{\frac{\pi}{6}, \frac{5}{6}\pi\}$

$x = \frac{\pi}{6}$, $x = \frac{5}{6}\pi$

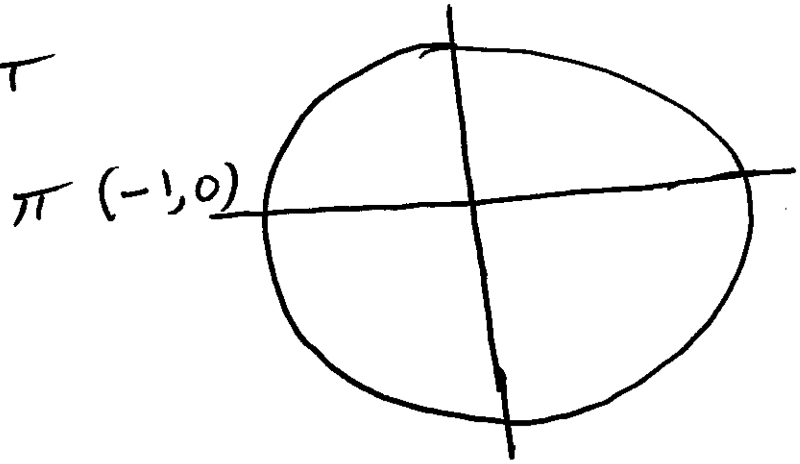
(ii) $\cos\theta = -\frac{\sqrt{2}}{2}$

$\boxed{x = \frac{3}{4}\pi,\ x = \frac{5}{4}\pi}$



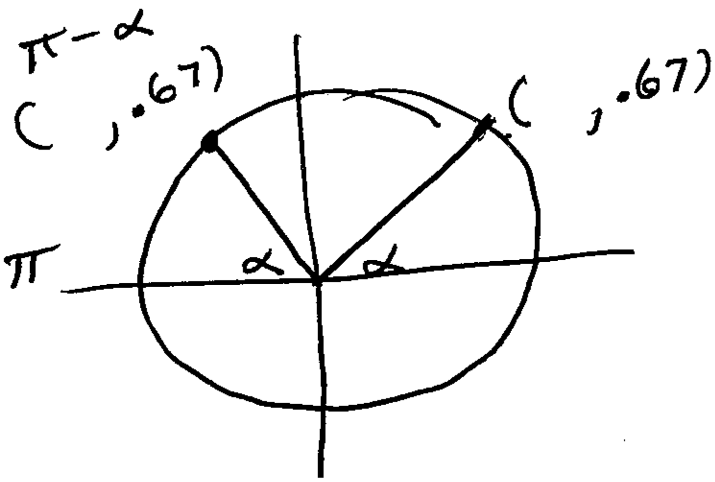
(iii) $\cos\theta = -1$

$\theta = \pi$



(iv) $\sin\theta = 0.67$

Use the calculator to find an approximation rounded to 2 decimal places



$\sin\alpha = .67$, α acute.

$\alpha = \sin^{-1}(.67)$

α is a reference angle.

~~find the other solution~~

The other angle is

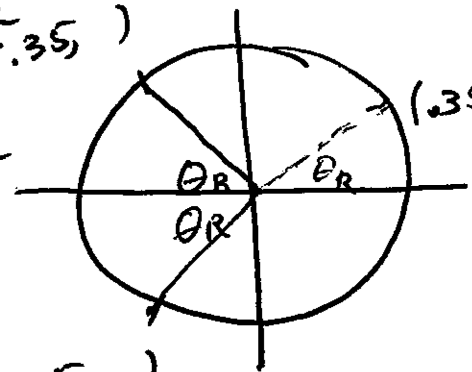
$\pi - \alpha$

$\theta = \sin^{-1}(.67)$, $\theta = \pi - \sin^{-1}(.67)$

$\{0.73, 2.41\}$

(V) $\cos\theta = -.35$

$\theta_1 = \pi - \theta_R$ $(-.35, \ )$



θ_2 $(-.35, \ )$
$= \pi + \theta_R$

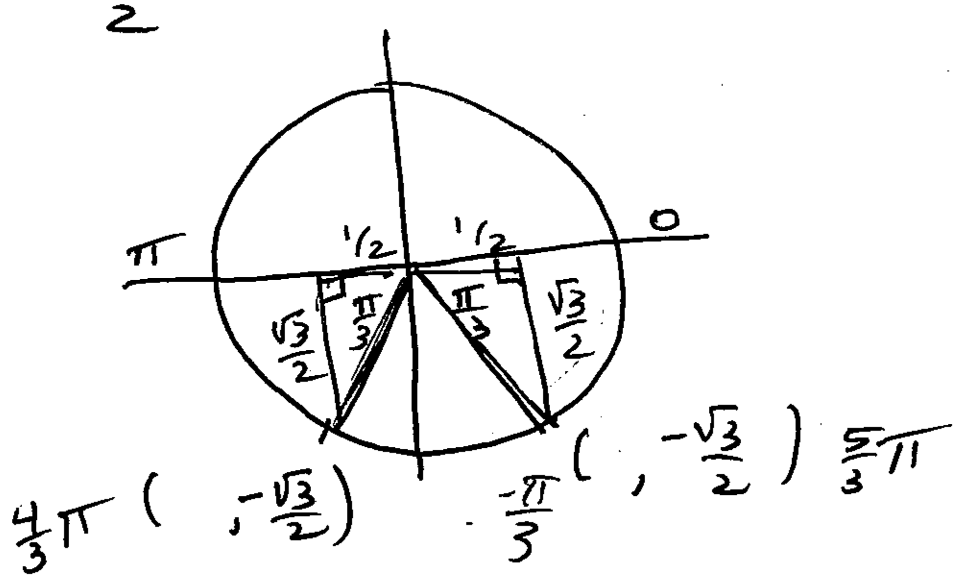
Step 1 Find the reference angle θ_R by throwing away the negative on $-.35$ and applying cosine inverse.

$\theta_R = \cos^{-1}(.35)$

Answer: $\theta_1 = \pi - \theta_R = \pi - \cos^{-1}(.35) \approx 1.93$
$\theta_2 = \pi + \theta_R = \pi + \cos^{-1}(.35) \approx 4.35$

EXAMPLE Find all real solutions.

(a) $\sin x = -\frac{\sqrt{3}}{2}$



$x = \frac{4}{3}\pi + 2k\pi$

$x = \frac{5}{3}\pi + 2k\pi$

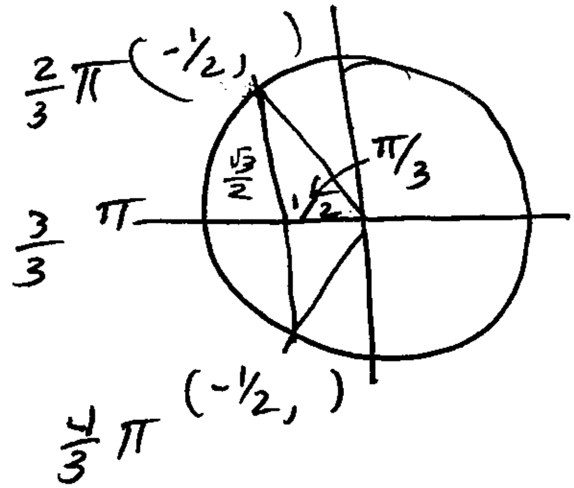
k is an integer.

This means:

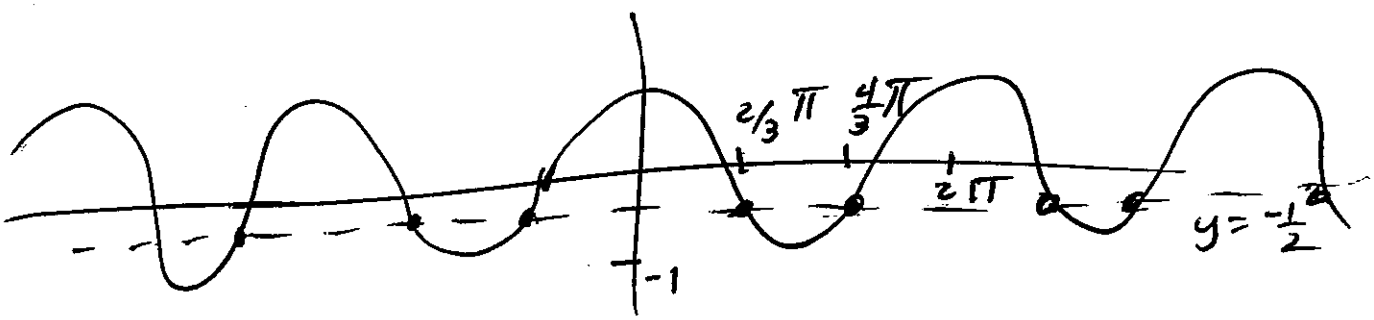
$\left\{ \ldots \frac{4}{3}\pi - 4\pi, \frac{4}{3}\pi - 2\pi, \frac{4}{3}\pi, \frac{4}{3}\pi + 2\pi, \frac{4}{3}\pi + 4\pi \ldots \right\}$

$\cup \left\{ \ldots \frac{5}{3}\pi - 4\pi, \frac{5}{3}\pi - 2\pi, \frac{5}{3}\pi, \frac{5}{3}\pi + 2\pi, \ldots \right\}$

(b) $\cos\theta = -\frac{1}{2}$ Find all real solutions.



$$\left\{ \frac{2}{3}\pi + 2k\pi,\ \frac{4}{3}\pi + 2k\pi \right\}$$

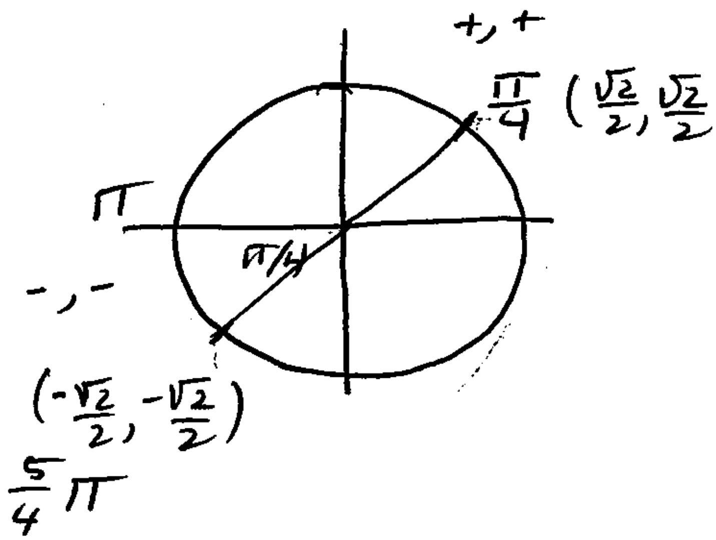


Find all real sol'n.

c) $\tan\theta = 1$

$$\frac{\sin\theta}{\cos\theta} = 1$$

$$\sin\theta = \cos\theta$$



$$\frac{\pi}{4} = \tan^{-1}(1)$$

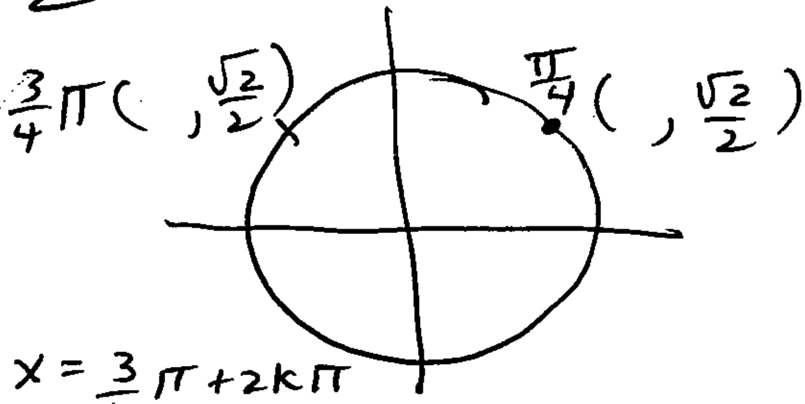
$$\left\{\frac{\pi}{4} + 2k\pi,\ \frac{5}{4}\pi + 2k\pi\right\}$$

or write

$$\left\{\frac{\pi}{4} + k\pi\right\}$$

Find all real sol'n.

d) $\sin(2x) = \frac{\sqrt{2}}{2}$



$2x = \frac{\pi}{4} + 2k\pi$, $\quad 2x = \frac{3}{4}\pi + 2k\pi$

solve for x by diving by 2.

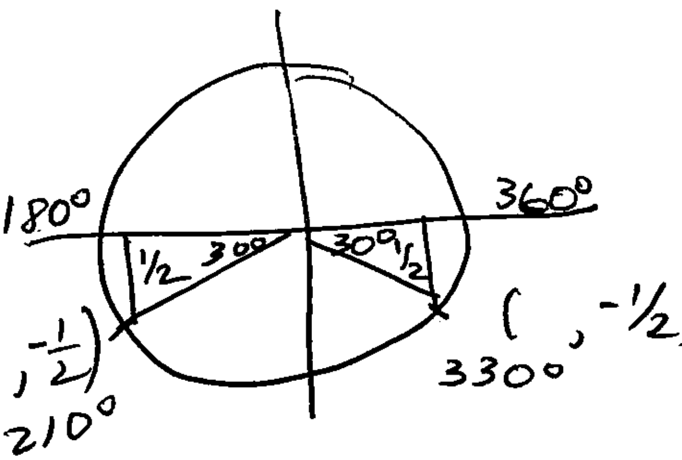
$x = \frac{1}{2}\left(\frac{\pi}{4} + 2k\pi\right)$, $\quad x = \frac{1}{2}\left(\frac{3}{4}\pi + 2k\pi\right)$

$$\boxed{x = \frac{\pi}{8} + k\pi , \quad x = \frac{3}{8}\pi + k\pi}$$

EXAMPLE Find all solutions in the interval $[0, 360°)$

(a) $\sin(2x) = -\frac{1}{2}$

$2x = 210° + 360°k$, $\quad 2x = 330° + 360°k$,



$x = \frac{1}{2}(210° + 360°k)$, $\quad x = \frac{1}{2}(330° + 360°k)$

$x = 105° + 180°k$, $\quad x = 165° + 180°k$

We put in values for k and find the angles between 0° and 360°.

k = -1 too small

k = 0 $\quad x = 105°$, $\quad x = 165°$

k = 1 $\quad x = 105 + 180 = 285°$, $\quad x = 165 + 180 = 345°$

k = 2 too big

$\{105°, 165°, 285°, 345°\}$

Example (A Quadratic Type Equation)

Find all solutions in $[0, 2\pi)$

$$2\cos^2 x + 3\cos x = -1$$

SOLUTION

$$2\cos^2 x + 3\cos x + 1 = 0$$

Let $u = \cos x$

$$2(\cos x)^2 + 3(\cos x) + 1 = 0$$ Move everything to LHS.

$$2u^2 + 3u + 1 = 0$$

solve for u

$$(2u + 1)(u + 1) = 0$$

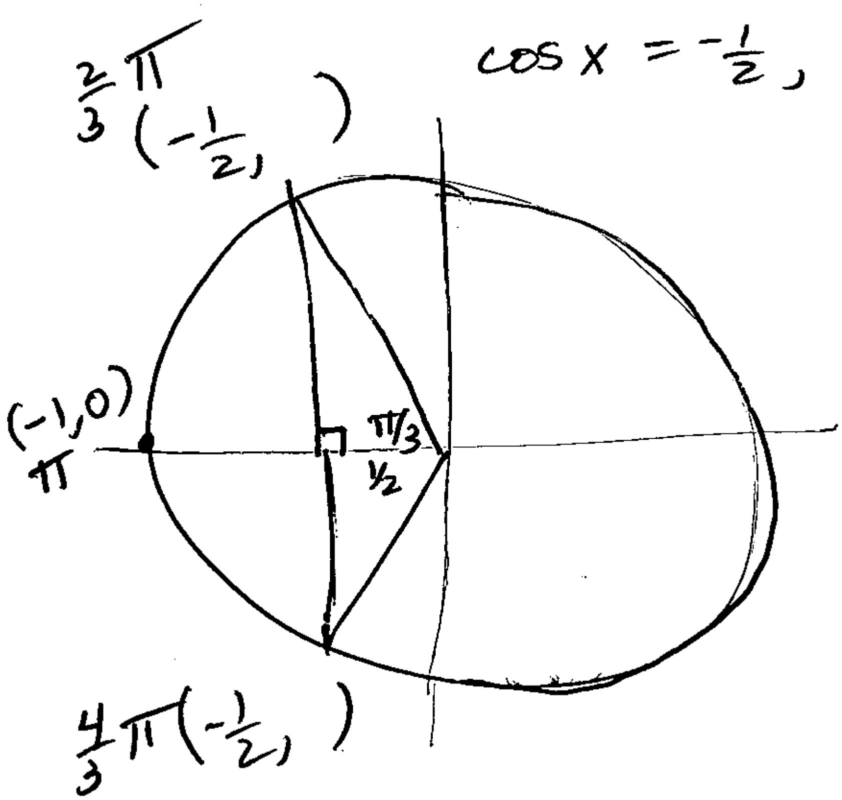
$$2u + 1 = 0 \quad , \quad u = -1$$

$$2u = -1$$

$$u = -\tfrac{1}{2}$$

• substitute back $u = \cos x$

$$\cos x = -\tfrac{1}{2}, \quad \cos x = -1$$



$$\left\{ \tfrac{2}{3}\pi, \pi, \tfrac{4}{3}\pi \right\}$$