③ $\tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}$

<u>EXAMPLE</u> If $\sin\alpha = \frac{4}{5}$, α in Quadrant II, find the exact values of $\sin 2\alpha$ and $\cos 2\alpha$.

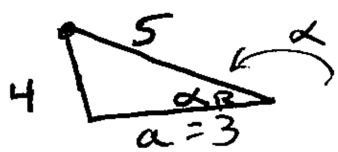
<u>SOLUTION</u>

- $\sin 2\alpha = 2\sin\alpha\cos\alpha$

$$= 2\left(\frac{4}{5}\right)\left(-\frac{3}{5}\right) = \frac{-24}{25}$$

?

<u>Find $\cos\alpha$</u>



$\sin\alpha = \frac{4}{5}$ $\frac{opp}{hyp}$

$a^2 + 4^2 = 5^2$

$a = 3$

$\cos\alpha = \frac{adj}{hyp} = -\frac{3}{5}$

cosine neg in Quad II

- $\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$$= (-3/5)^2 - (4/5)^2$$

$$= 9/25 - \frac{16}{25} = \frac{-7}{25}$$

In which quadrant does 2α lie?

Answer: $\sin 2\alpha = \frac{-24}{25}$, $\cos 2\alpha = \frac{-7}{25}$

So 2α lies in Quad III.
because sine & cosine are both negative.

Fermat's Last Theorem.

There are no nonzero pos. integer solutions to

$a^n + b^n = c^n$ if n is an integer greater or equal to 3

e.g. $a^3 + b^3 = c^3$
has no nonzero positive integer solutions for a, b, & c.

Half Angle Identies

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2} = \pm\sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \pm\sqrt{\frac{1-\cos x}{1+\cos x}} \qquad \tan\frac{x}{2} = \frac{\sin x}{1+\cos x}$$

$$\tan\frac{x}{2} = \frac{1-\cos x}{\sin x}$$

<u>Pf</u> Let's show that

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

We have

$$\cos 2\theta = 1 - 2\sin^2\theta$$

Let $\theta = \frac{x}{2}$

$$\cos 2\left(\frac{x}{2}\right) = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

$$\cos x = 1 - 2\sin^2\left(\frac{x}{2}\right)$$

Now solve for $\sin\left(\frac{x}{2}\right)$

$$-1 + \cos x = -2\sin^2\left(\frac{x}{2}\right)$$

$$\frac{-1+\cos x}{-2} = \sin^2\left(\frac{x}{2}\right)$$

$$\frac{-(1-\cos x)}{-(2)} = \sin^2\left(\frac{x}{2}\right)$$

$$\sin\left(\frac{x}{2}\right) = \pm\sqrt{\frac{1-\cos x}{2}}$$

We also have:

(i) $\cos^2\theta = \dfrac{1+\cos 2\theta}{2}$

(ii) $\sin^2\theta = \dfrac{1-\cos 2\theta}{2}$

<u>Pf</u> Let's prove (i)

We have: $\cos 2\theta = 2\cos^2\theta - 1$

solve for $\cos^2\theta$

$$1+\cos 2\theta = 2\cos^2\theta$$

$$\frac{1+\cos 2\theta}{2} = \cos^2\theta \checkmark$$

§6.5 Product and Sum Identities

$$\sin A \cos B = \frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right]$$

__Pf__ we have

$$\sin(A+B) = \sin A \cos B + \cos A \sin B$$
$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

__add__ $\sin(A+B) + \sin(A-B) = 2\sin A \cos B$

Divide by 2 $\frac{1}{2}\left[\sin(A+B) + \sin(A-B)\right] = \sin A \cos B$

Hidden Fortress

There's more in the text book.

p547 Dugapolski

__Example:__

$$\sin \overbrace{2X}^{A} \cos \overbrace{3X}^{B} = \frac{1}{2}\left[\sin(2X+3X) + \sin(2X-3X)\right]$$
$$= \frac{1}{2}\sin 5X - \frac{1}{2}\sin X$$

$$\sin A \sin B = \frac{1}{2}\left[\cos(A-B) - \cos(A+B)\right]$$

$$\cos A \sin B = \frac{1}{2}\left[\sin(A+B) - \sin(A-B)\right]$$

$$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$$