EXAMPLE Verify.

$$\frac{\csc x - \sin x}{\sin x} = \cot^2 x$$

Aside

Simplify

$$\frac{\left(\frac{1}{2} - \frac{1}{x}\right)}{\left(\frac{1}{x}\right)} \cdot \frac{2x}{2x}$$

$$= \frac{x - 2}{2}$$

LHS	RHS
$\dfrac{\csc x - \sin x}{\sin x}$	$\cot^2 x$ ✓
$\left(\dfrac{\left(\frac{1}{\sin x}\right) - \sin x}{\sin x}\right) \cdot \dfrac{\sin x}{\sin x}$	
$\dfrac{1 - \sin^2 x}{\sin^2 x}$	
$\dfrac{\cos^2 x}{\sin^2 x}$	
$\cot^2 x$ ✓	

EXAMPLE Verify.

$$2\tan^2 x = \frac{1}{\csc x - 1} - \frac{1}{\csc x + 1}$$

LHS

$2\tan^2 x$ ✓

RHS

$$\frac{1}{\csc x - 1} - \frac{1}{\csc x + 1}$$

when you have two fractions, consider adding them.

$$\left(\frac{1}{\csc x - 1}\right)\left(\frac{\csc x + 1}{\csc x + 1}\right) - \left(\frac{1}{\csc x + 1}\right)\left(\frac{\csc x - 1}{\csc x - 1}\right)$$

$$\frac{(\csc x + 1) - (\csc x - 1)}{\csc^2 x - 1^2}$$

$$\frac{\cancel{\csc x} + 1 - \cancel{\csc x} + 1}{\csc^2 x - 1}$$

$$\frac{2}{\csc^2 x - 1}$$

$$\frac{2}{\cot^2 x} \qquad \frac{2}{\left(\frac{1}{\tan^2 x}\right)}$$

$2\tan^2 x$ ✓

Aside

$\cot^2 x + 1 = \csc^2 x$

$\cot^2 x = \csc^2 x - 1$

EXAMPLE

$$\frac{1-\sin^2 t}{1-\csc(-t)} = \frac{1+\sin(-t)}{\csc t}$$

LHS

$$\frac{1-\sin^2 t}{1-\csc(-t)}$$

$$\frac{1-\sin^2 t}{1+ +\csc t}$$

$$\frac{\cos^2 t}{1+\csc t}$$

$$\left(\frac{\cos^2 t}{1+\left(\frac{1}{\sin t}\right)}\right)\frac{\sin t}{\sin t}$$

$$\frac{\cos^2 t\sin t}{\sin t+1} = \left(\frac{\cos^2 t\sin t}{1+\sin t}\right)\left(\frac{1-\sin t}{1-\sin t}\right)$$

$$\frac{\cos^2 t\sin t(1-\sin t)}{1-\sin^2 t}$$

$$\frac{\cancel{\cos^2 t}\sin t(1-\sin t)}{\cancel{\cos^2 t}}$$

$$\sin t(1-\sin t)$$ ✓

RHS

$$\frac{1+\sin(-t)}{\csc t}$$

$$\frac{1-\sin t}{\csc t}$$

$$\frac{(1-\sin t)\sin t}{\left(\frac{1}{\sin t}\right)\sin t}$$

$$\sin t(1-\sin t)$$ ✓

§ 6.3 Sum and Difference Identities

HW
#1-70 odd

__Identity__

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

__EXAMPLE__ The exact value of a cosine.

Find the exact value of $\cos(75^\circ)$.

__SOLUTION__

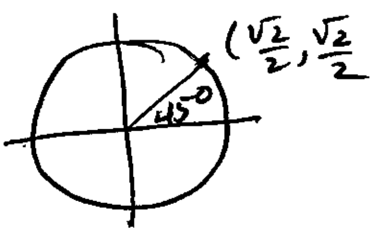
$\cos(45^\circ + 30^\circ)$

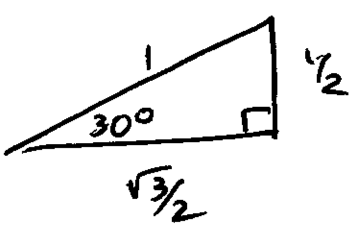
$= \cos 45^\circ \cos 30^\circ - \sin 45^\circ \sin 30^\circ$

$= \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\,\frac{1}{2}$

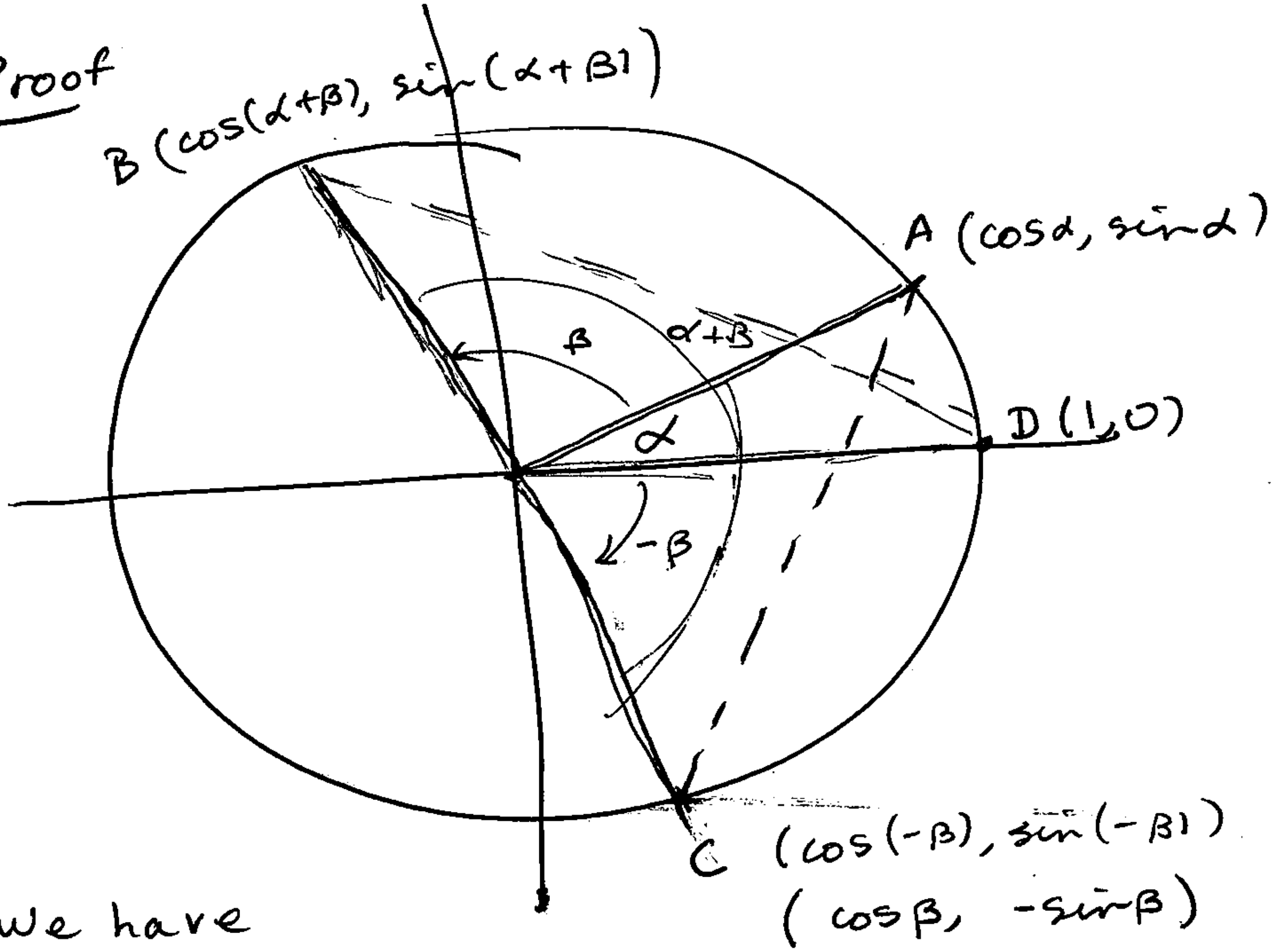
$= \frac{\sqrt{6}}{4} - \frac{\sqrt{2}}{4}$

$= \frac{\sqrt{6} - \sqrt{2}}{4}$





Proof



We have

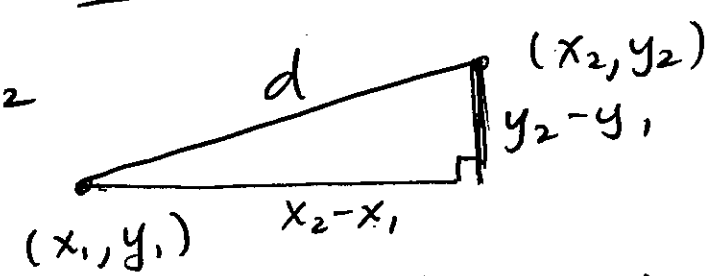
$\overline{BD} = \overline{AC}$

$(\overline{BD})^2 = (\overline{AC})^2$

$(\cos(\alpha+\beta) - 1)^2 + (\sin(\alpha+\beta) - 0)^2$

$= (\cos\alpha - \cos\beta)^2 + (\sin\alpha - -\sin\beta)^2$

Distance Formula



$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$\left(\cos^2(\alpha+\beta) - 2\cos(\alpha+\beta) + 1^2\right) + \sin^2(\alpha+\beta)$$

$$= \left(\cos^2\alpha - 2\cos\alpha\cos\beta + \cos^2\beta\right) + \left(\sin^2\alpha + 2\sin\alpha\sin\beta + \sin^2\beta\right)$$

$$\underbrace{\left(\cos^2(\alpha+\beta) + \sin^2(\alpha+\beta)\right)}_{1} - 2\cos(\alpha+\beta) + 1$$

$$= \underbrace{\left(\cos^2\alpha + \sin^2\alpha\right)}_{1} - 2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta$$

$$\underbrace{(\cos^2\beta + \sin^2\beta)}_{1}$$

$$-2\cos(\alpha+\beta) \cancel{+2} = -2\cos\alpha\cos\beta + 2\sin\alpha\sin\beta \cancel{+2}$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

QED.