§5.5 Continued.

Inverses of Cotangent, secant, and cosecant.

EXAMPLE: Evaluate.

a) $\sec^{-1}(-2) = \theta$

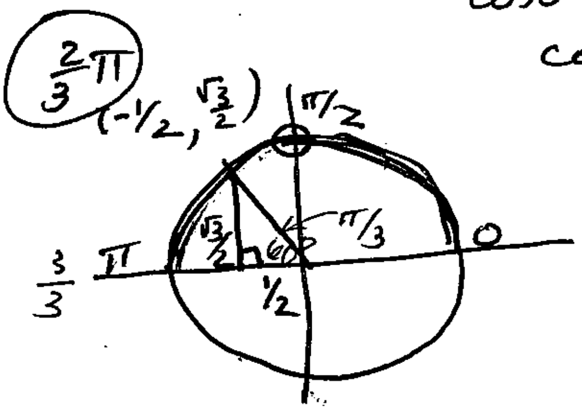
$\sec\theta = -2$

$\frac{1}{\cos\theta} = -2$

$\cos\theta = -\frac{1}{2} = x$

$0 < \theta < \pi/2$ or $\pi/2 < \theta \le \pi$

~~We use~~ The range of $y = \sec^{-1} x$ is the same as the range of $y = \cos^{-1} x$



Answer: $\frac{2}{3}\pi$

Some books have the range of $y = \sec^{-1} x$ as $[0, \pi/2) \cup [\pi, \frac{3}{2}\pi)$ because we sometimes use the identity

$\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta = \sec^2\theta - 1$

$\tan\theta = \pm\sqrt{\sec^2\theta - 1}$

if θ lies ~~Quad~~ QI, QIII then $\tan\theta$ is positive, so we don't need $\pm$.

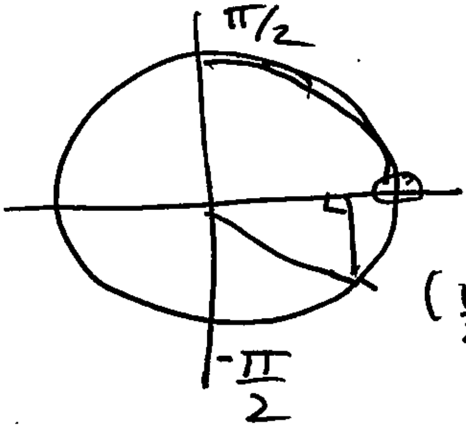
b) $\csc^{-1}(-\sqrt{2}) = \theta$

$$\csc\theta = -\sqrt{2}$$

$$\frac{1}{\sin\theta} = -\sqrt{2}$$

$$\sin\theta = \frac{-1}{\sqrt{2}} = \frac{-\sqrt{2}}{2}$$

θ in $(0, \frac{\pi}{2}] \cup [\frac{-\pi}{2}, 0)$

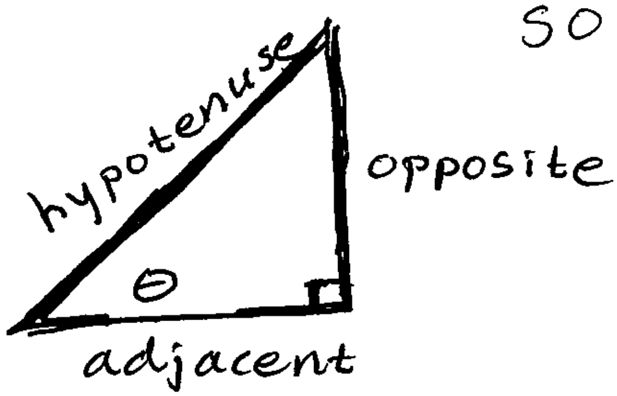


$\left(\frac{\sqrt{2}}{2}, \frac{-\sqrt{2}}{2}\right)$ $-\frac{\pi}{4}$

§5.6 Right Triangle Trigonometry

HW #1–53 odd

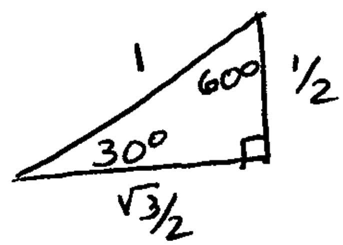
SOH – CAH – TOA



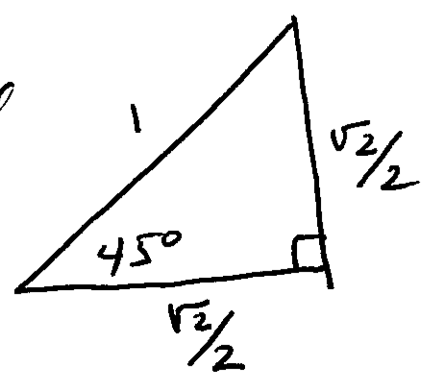
Sine
Opposite
Hypotenuse
|
Cosine
Adjacent
Hypotenuse
|
Tangent
Opposite
Adjacent

<u>EXAMPLE</u>: Evaluate.

① $\sin 30^\circ = \dfrac{opp}{hyp} = \dfrac{1/2}{1} = \dfrac{1}{2}$



② $\sin 45^\circ = \dfrac{opp}{hyp} = \dfrac{\sqrt{2}/2}{1} = \dfrac{\sqrt{2}}{2}$

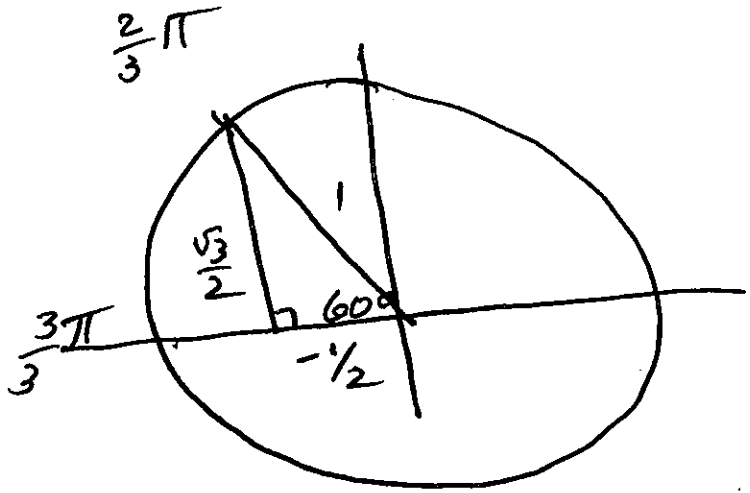


- An angle between 0 and 90° is called <u>acute</u>.
- An angle between 90 and 180° is called <u>obtuse</u>.
- A 90° angle is called a <u>right</u> angle.

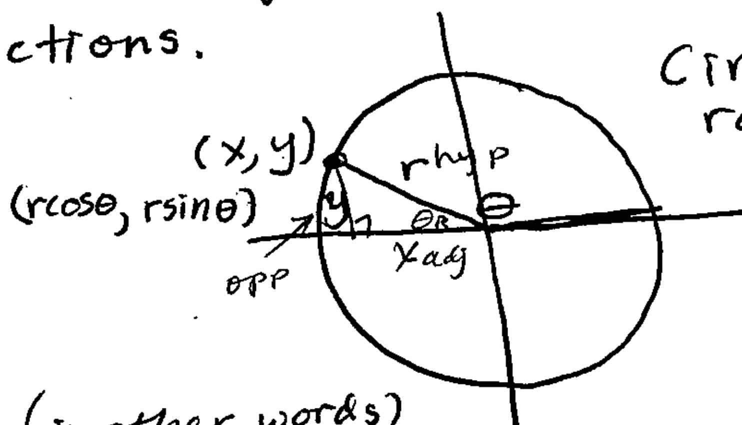
③ $\tan\left(\frac{2}{3}\pi\right)$

$= \frac{opp}{adj} = \frac{\frac{\sqrt{3}}{2}}{-\frac{1}{2}}$

$= \frac{\sqrt{3}}{2} \cdot \frac{-2}{1} = -\sqrt{3}$



Yet Another Way to Define the Six Trig Functions.



Circle of radius r.

$x = r\cos\theta$

$y = r\sin\theta$

$x^2 + y^2 = r^2$

Eqn of circle

i.e. (in other words)

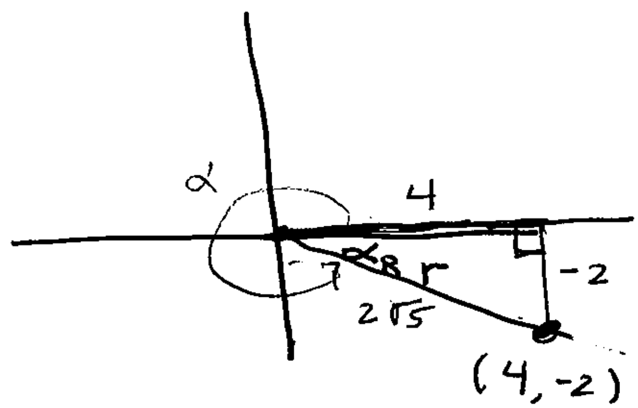
$\cos\theta = \frac{x}{r} = \frac{adj}{hyp}$ $\quad \sec\theta = \frac{r}{x}$

$\sin\theta = y/r = \frac{opp}{hyp}$ $\quad \csc\theta = \frac{r}{y}$

$\tan\theta = y/x = \frac{opp}{adj}$ $\quad \cot\theta = \frac{x}{y}$

EXAMPLE Find the values of the six trigonometric functions of the angle α in standard position whose terminal side passes through $(4, -2)$.

SOLUTION



Find r

$$r^2 = x^2 + y^2$$
$$r^2 = (4)^2 + (-2)^2$$
$$r^2 = 16 + 4$$
$$r^2 = 20$$
$$r = \sqrt{20} = \sqrt{4 \cdot 5} = 2\sqrt{5}$$

↑ r is always positive

$$\cos\alpha = \frac{x}{r} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}} = \frac{2\sqrt{5}}{5}$$

$$\sin\alpha = \frac{y}{r} = \frac{opp}{hyp} = \frac{-2}{2\sqrt{5}} = \frac{-1}{\sqrt{5}} = \frac{-\sqrt{5}}{5}$$

$$\tan\alpha = \frac{y}{x} = \frac{opp}{adj} = \frac{-2}{4} = \frac{-1}{2}$$

$$\sec\alpha = \frac{\sqrt{5}}{2} = \frac{1}{\cos\alpha}$$

$$\csc\alpha = \frac{1}{\sin\alpha} = -\sqrt{5}$$

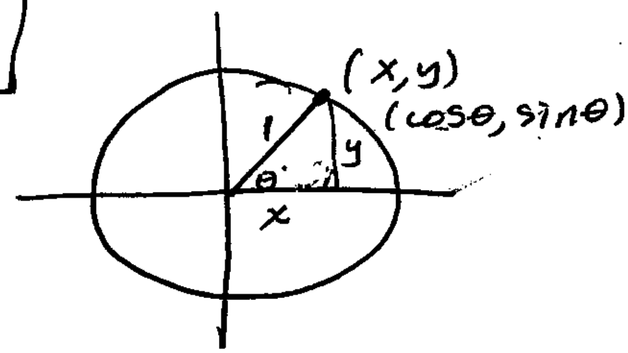
$$\cot\alpha = \frac{1}{\tan\alpha} = -2$$

§6.1 Basic Identities

Pythagorean Identities

$$\boxed{\sin^2\theta + \cos^2\theta = 1}$$

$x^2+y^2=1$



Divide through by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$$\boxed{\tan^2\theta + 1 = \sec^2\theta}$$

Divide through by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$\boxed{1 + \cot^2\theta = \csc^2\theta}$$

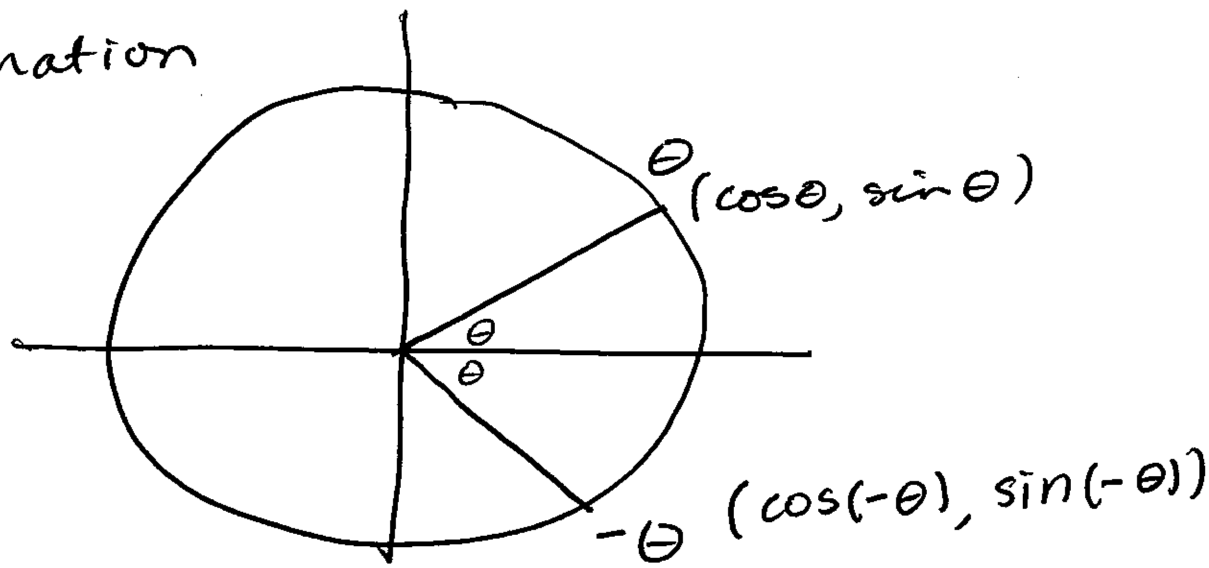
Even and Odd Identies

Odd: $\sin(-x) = -\sin x$ $\qquad \csc(-x) = -\csc x$

$\tan(-x) = -\tan x$ $\qquad \cot(-x) = -\cot x$

Even: $\cos(-x) = \cos x$ $\qquad \sec(-x) = \cos x$

Explanation



- The x-coord of the two points are the same, so

$$\cos(-\theta) = \cos\theta$$

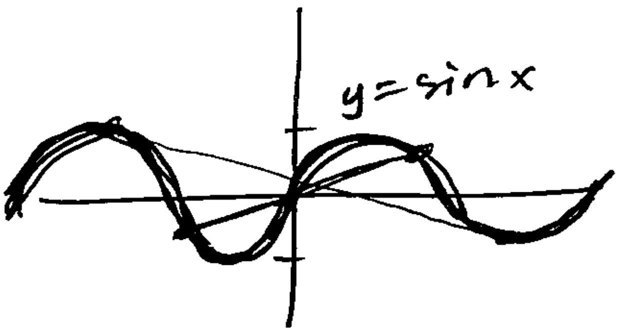
- The y-coords of the two points are opposite of each other.

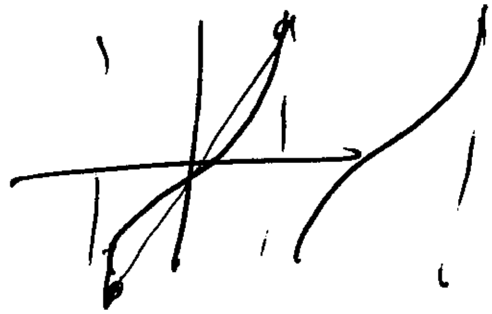
So $\quad \sin(-\theta) = -\sin\theta$

- We have $\tan(-\theta) = \dfrac{\sin(-\theta)}{\cos(-\theta)} = \dfrac{-\sin\theta}{\cos\theta}$

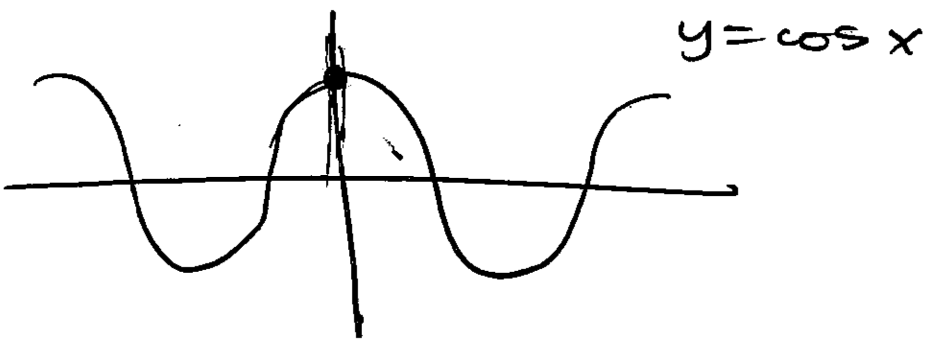
$= -\tan\theta$

So $y=\sin x$, $y=\tan x$ are odd functions (symmetric about the origin).





And $y=\cos x$ is an even function (symmetric about the y-axis).



§6.2 Verifying Identities

HW # 55 – 79

EXAMPLE Verify the Identity.

$$1 + \sec x \sin x \tan x = \sec^2 x$$

LHS (left hand side)

$1 + \sec x \sin x \tan x$

$1 + \frac{1}{\cos x} \cdot \sin x \ \frac{\sin x}{\cos x}$

$1 + \frac{\sin^2 x}{\cos^2 x}$

$1 + \tan^2 x$

$\sec^2 x$ ✓

LHS = RHS

RHS (left hand side)

$\sec^2 x$ ✓

□

Q.E.D.

EXAMPLE Verify.

$$\frac{\cos\alpha}{1-\sin\alpha} = \frac{1+\sin\alpha}{\cos\alpha}$$

Aside $(a-b)(a+b)$
$= a^2 - b^2$

Aside:
$\cos^2\alpha + \sin^2\alpha =$
$\cos^2\alpha = 1 - \sin$

LHS

$$\frac{\cos\alpha}{1-\sin\alpha}$$

$$\frac{\cos\alpha}{1-\sin\alpha} \cdot \frac{1+\sin\alpha}{1+\sin\alpha}$$

$$\frac{\cos\alpha(1+\sin\alpha)}{1-\sin^2\alpha}$$

$$\frac{\cos\alpha(1+\sin\alpha)}{\cos^2\alpha}$$

$$\frac{1+\sin\alpha}{\cos\alpha} \checkmark$$

equals RHS

RHS

$$\frac{1+\sin\alpha}{\cos\alpha} \checkmark$$