§5.5 The Inverse Trigonometric Functions

HW # 1 – 63 odd

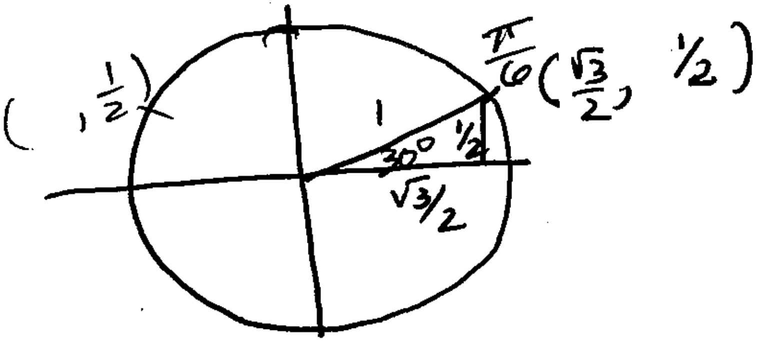
Jeapordy:

The answer is 1776.
What is the question?
When was the Declaration of Independence written?

EXAMPLE Find θ if θ is in quadrant I. $\left(0 \le \theta \le \frac{\pi}{2}\right)$

a) $\sin\theta = \frac{1}{2}$ ← y-coord

$\theta = \frac{\pi}{6}$ ✓



$\sin^{-1}\left(\frac{1}{2}\right) = \theta$

means $\sin\theta = \frac{1}{2}$

and [illegible]

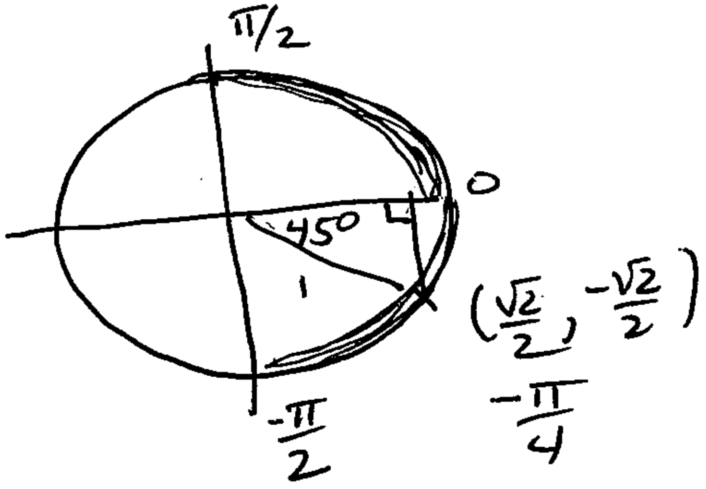
$0 \le \theta \le \frac{\pi}{2}$

We say "sine inverse of ½".

Example : Find θ if
$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

a) $\sin\theta = -\frac{\sqrt{2}}{2}$

$\theta = \frac{-\pi}{4}$



Definition : The inverse sine function is written $y = \sin^{-1} x$ or $y = \arcsin x$.

We have that $y = \sin^{-1} x$ means $x = \sin y$ where $\frac{-\pi}{2} \le y \le \frac{\pi}{2}$

$\underset{\text{angle}}{y} = \sin^{-1} \underset{\text{coord on unit circle}}{x}$

- We have $\sin^{-1}\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

this is equivalent to part(a) above.

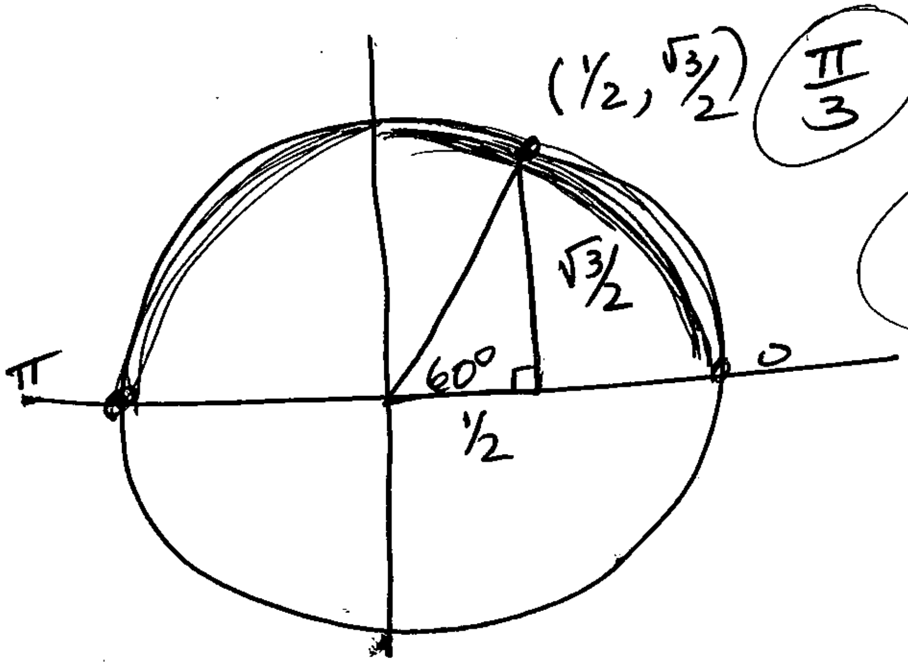
Inverse Cosine.

EXAMPLE Evaluate.

(i) $\cos^{-1}\left(\frac{1}{2}\right)$

SOLUTION $\cos^{-1}(1/2) = \theta$

means $\cos\theta = \frac{1}{2}$ $\quad 0 \le \theta \le \pi$

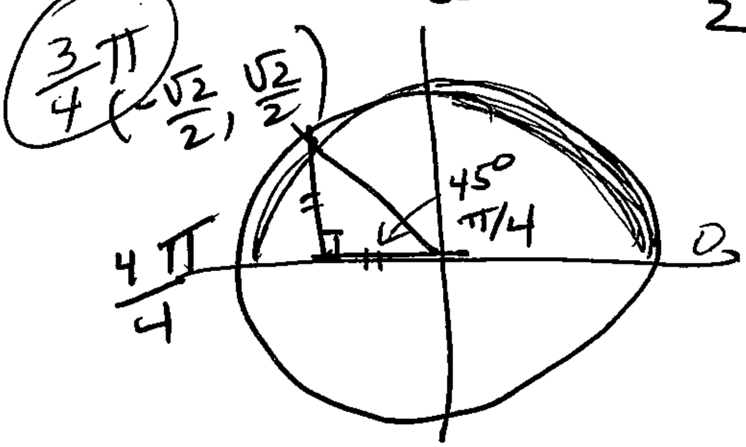


Answer: $\frac{\pi}{3}$

(ii) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

SOL $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right) = \theta$

$\cos\theta = -\frac{\sqrt{2}}{2}$ $\quad 0 \le \theta \le \pi$



Definition of Inverse Cosine.

$y = \cos^{-1} x$ means

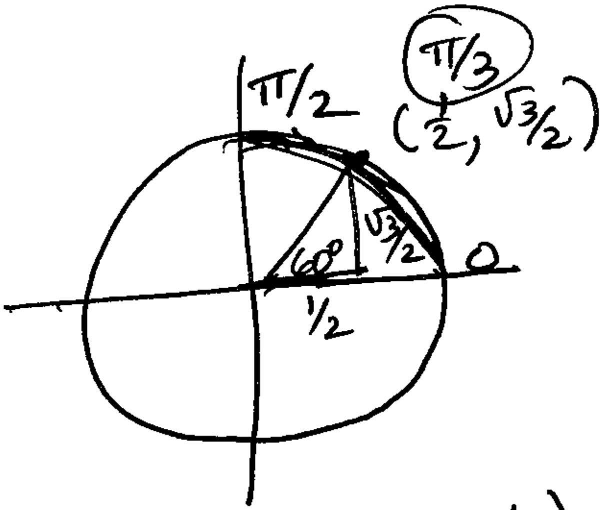
$\cos y = x, \; 0 \le y \le \pi$.

EXAMPLE Evaluate.

(i) $\sin^{-1} \frac{\sqrt{3}}{2}$

SOLUTION $\sin^{-1} \frac{\sqrt{3}}{2} = \theta$

$\sin\theta = \frac{\sqrt{3}}{2} \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

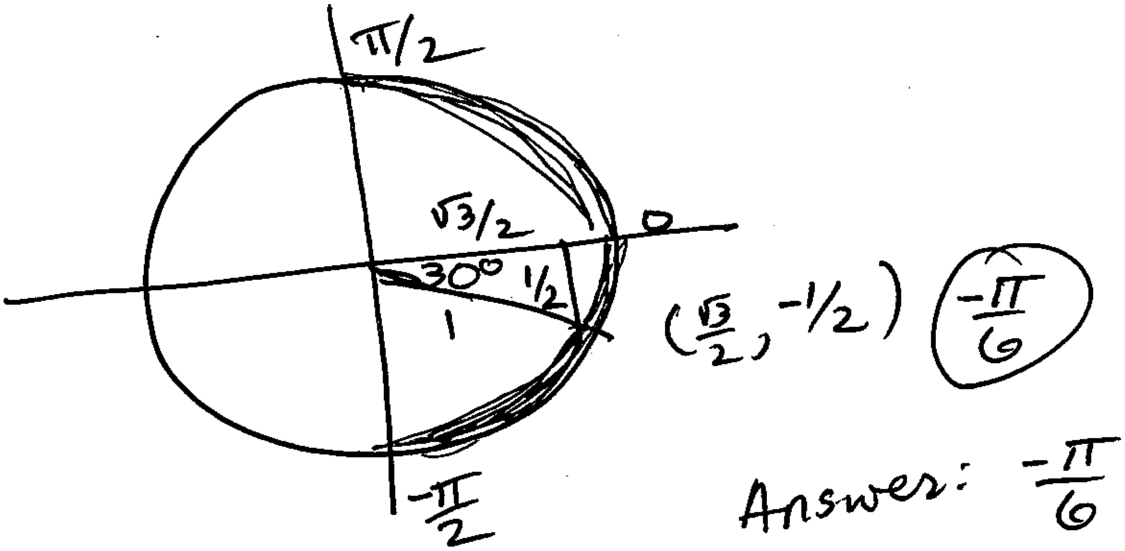


Answer $\boxed{\frac{\pi}{3}}$

(ii) $\sin^{-1}\left(-\frac{1}{2}\right)$

SOLUTION $\sin^{-1}(-1/2) = \theta$

$\sin\theta = -1/2, \qquad -\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$



Answer: $-\frac{\pi}{6}$

Inverse Tangent

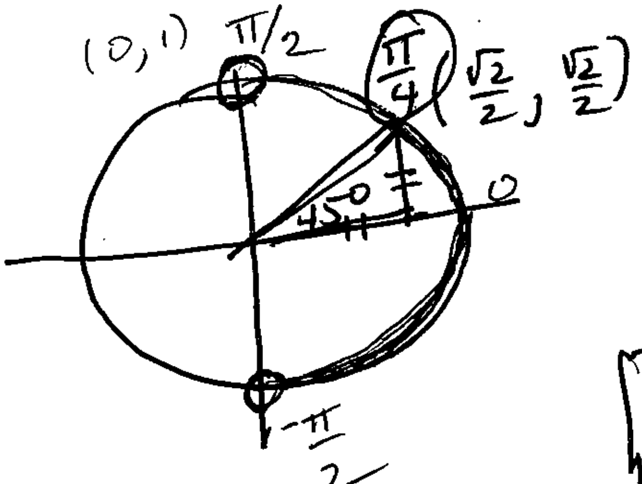
EXAMPLE Evaluate.

a) $\tan^{-1}(1)$

SOLUTION $\tan^{-1}(1)=\theta$

means $\tan\theta = 1$, $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$\tan\theta=\frac{y}{x}=\frac{\sin\theta}{\cos\theta}$



$\frac{\sin\theta}{\cos\theta}=1$

$\sin\theta=\cos\theta$

$y=x$

$$\boxed{\text{Answer: } \frac{\pi}{4}}$$

b) $\tan^{-1}(-1)$

SOL $\tan^{-1}(-1)=\theta$

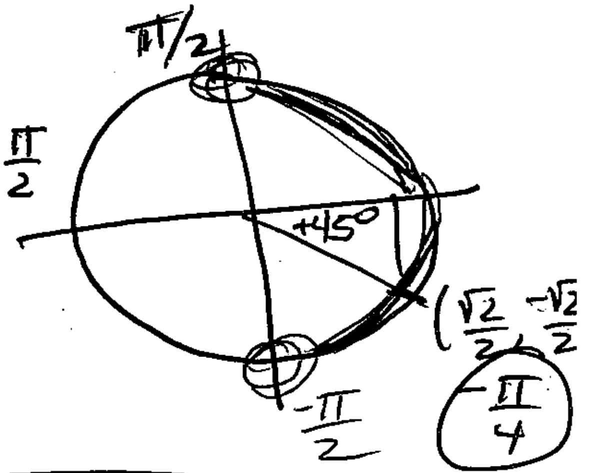
$\tan\theta=-1$, $-\frac{\pi}{2}<\theta<\frac{\pi}{2}$

$\frac{y}{x}=-1$

$y=-x$

$\frac{\sin\theta}{\cos\theta}=-1$

$\sin\theta=-\cos\theta$

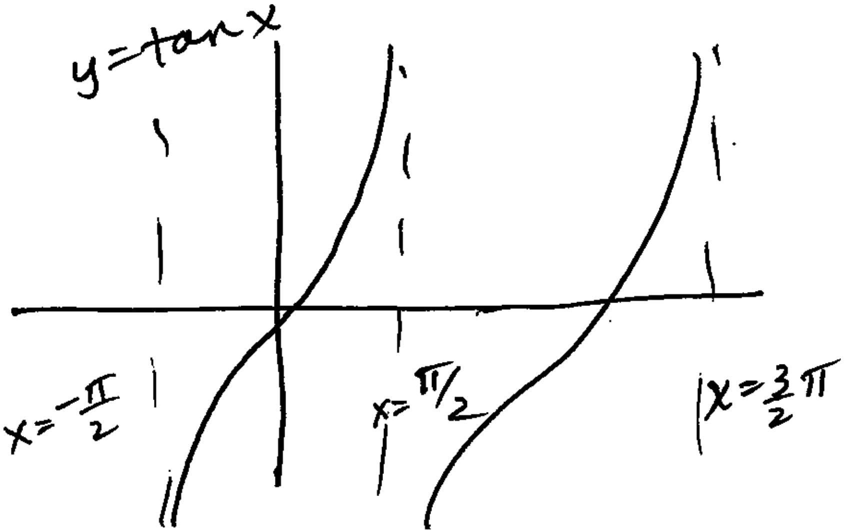


Definition of Tangent Inverse:

$y=\tan^{-1}x$ means $\tan y = x$

$-\frac{\pi}{2}<y<\frac{\pi}{2}$

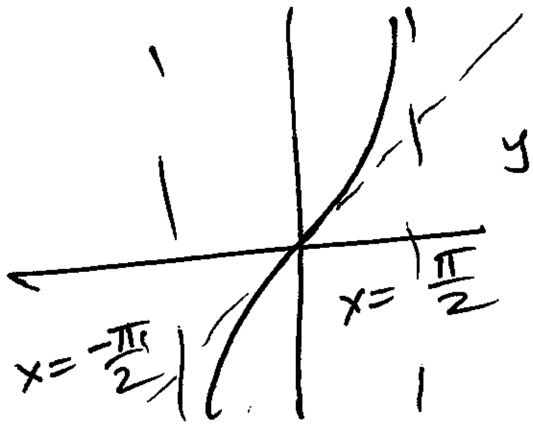
The Graph of Inverse Tangent.



$y = \tan x$ is/ [✓ is not] one-to-one

It fails the horizontal line tes

We restrict the domain of $y = \tan x$ to get one-to-one function.



$y = \tan x$

$-\frac{\pi}{2} < x < \frac{\pi}{2}$

This function is one-to-one.

To find the graph of $y = \tan^{-1} x$, we reflect the restricted graph of $y = \tan x$ about the line $y = x$.

Domain: $\mathbb{R}$

Range: $(-\frac{\pi}{2}, \frac{\pi}{2})$