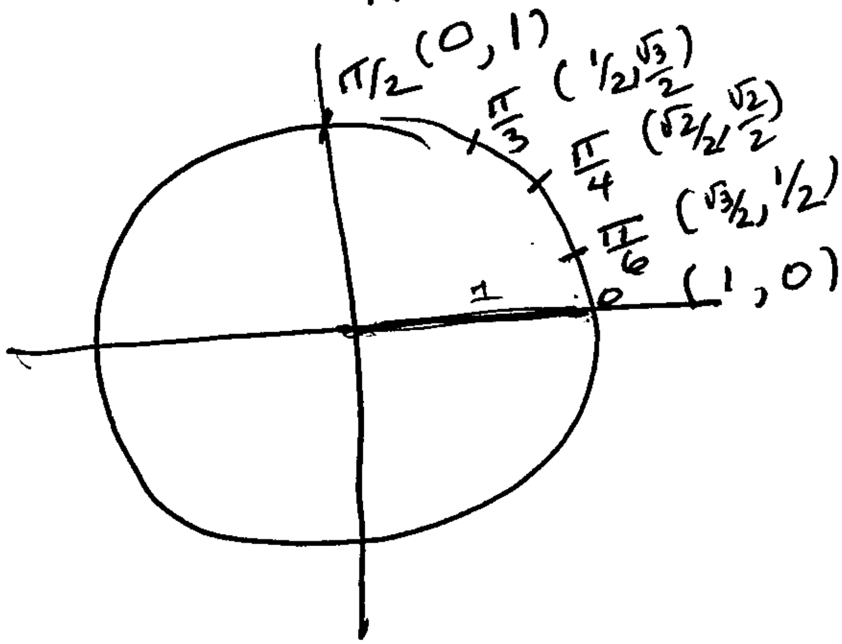
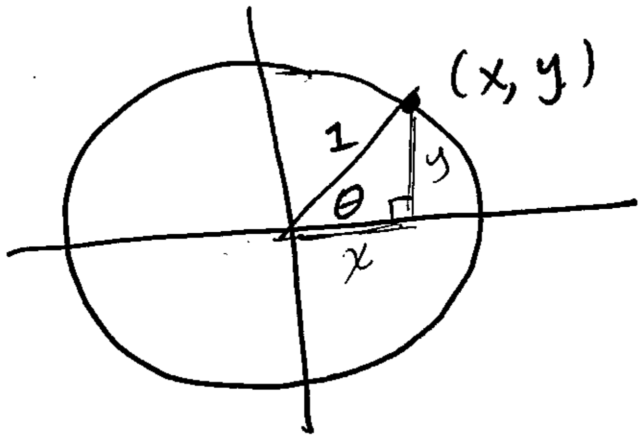
§ 5.4 Other Trig Functions and their Graphs

HW # 1-81 odd

A note regarding the unit circle.



	0	$\frac{\pi}{6}$	$\pi/4$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
cosine	$\frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{0}}{2}$
sine	$\frac{\sqrt{0}}{2}$	$\frac{\sqrt{1}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{4}}{2}$



hyp = 1
opp = y
adj = x
SOH - CAH - TOA

Define

$\cos\theta = x$ $\quad\quad$ $\sec\theta = \frac{1}{x}$

$\sin\theta = y$ $\quad\quad$ $\csc\theta = 1/y$

$\tan\theta = y/x$ $\quad\quad$ $\cot\theta = \frac{x}{y}$

We have the following identies

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

The Cos <u>don't</u> go

$$\sec\theta = \frac{1}{\cos\theta}$$

$$\csc\theta = \frac{1}{\sin\theta}$$

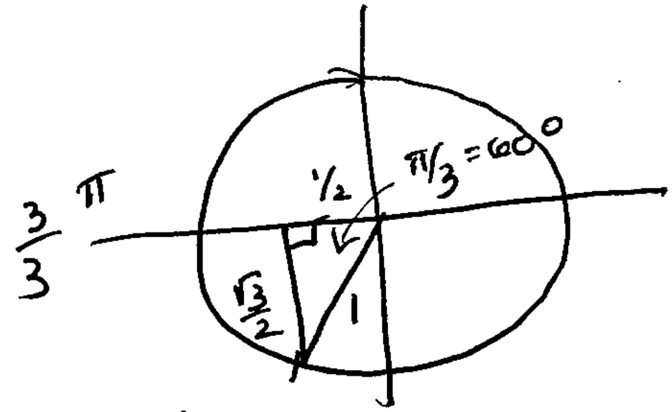
$$\cot\theta = \frac{1}{\tan\theta}$$

EXAMPLE Evaluate.

a) $\tan \frac{4}{3}\pi$

$= \frac{y}{x} = \frac{-\sqrt{3}/2}{-\frac{1}{2}}$

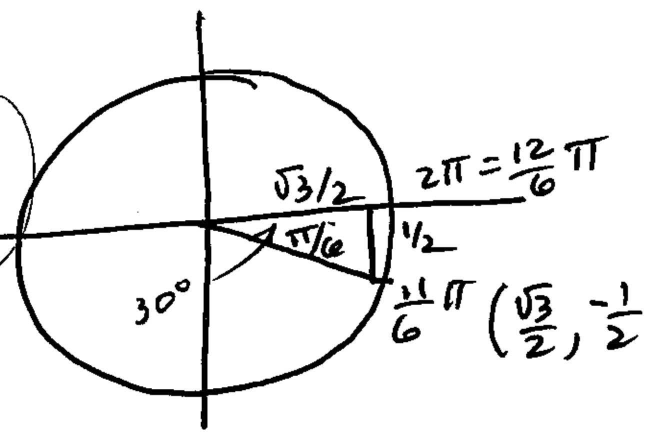
$= \frac{\sqrt{3}}{2} \cdot \frac{2}{1} = \sqrt{3}$

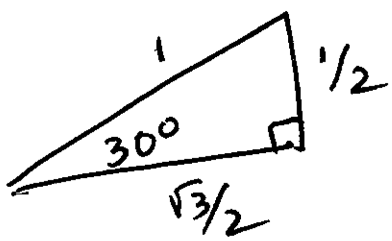


$\left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right) \frac{4}{3}\pi$

b) $\sec \frac{11}{6}\pi = \frac{1}{x}$

$= \frac{1}{\cos \frac{11}{6}\pi} = \frac{1}{\frac{\sqrt{3}}{2}} = \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$



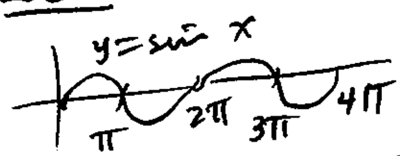


The Graph of $y = \tan x$

$$= \frac{\sin x}{\cos x}$$

<u>zero</u> when $\sin x = 0$

when

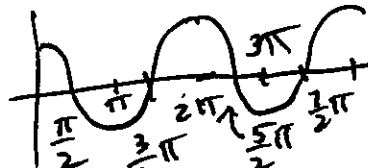


$\{\ldots -2\pi, -\pi, 0, \pi, 2\pi, 3\pi, 4\pi \ldots\}$

$x = k\pi$

Undef when $\cos x = 0$

when x is



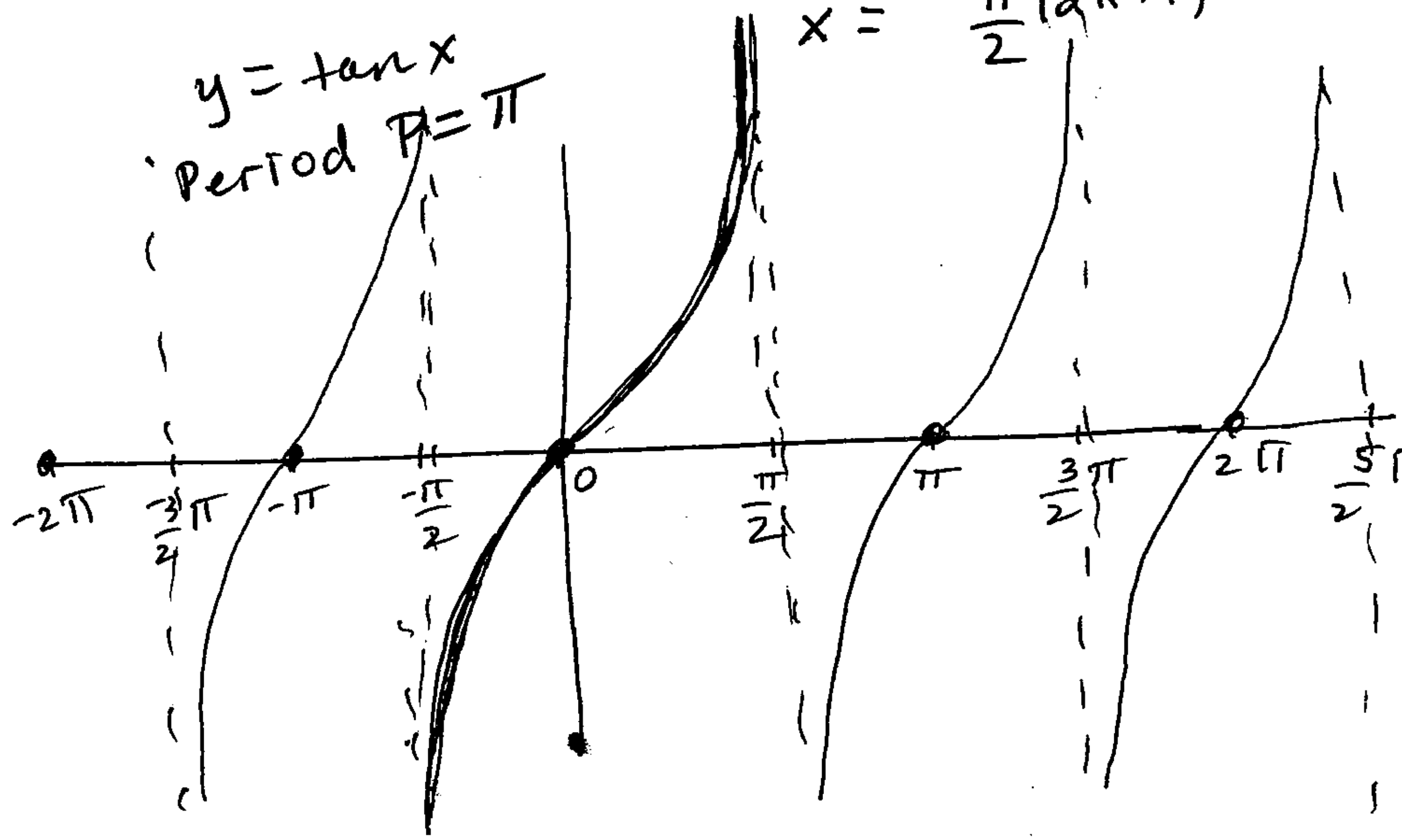
$\{\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi, \frac{7}{2}\pi \ldots\}$

(vertical asymptote

$x = \frac{\pi}{2}(2k+1)$

$y = \tan x$

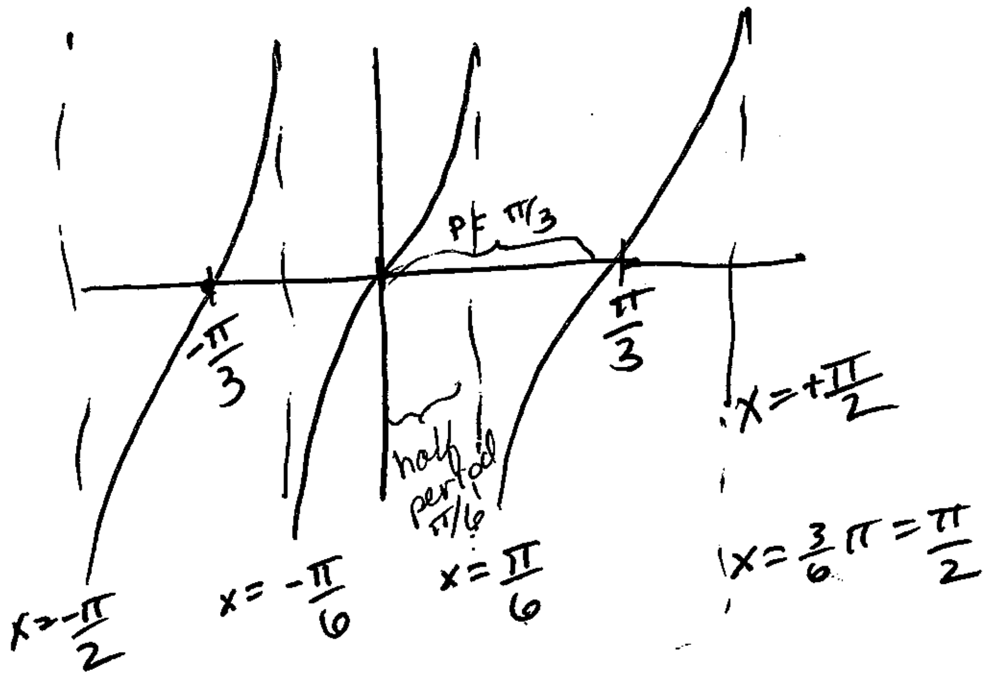
Period $P = \pi$



EXAMPLE Sketch the graph.

a) $y = \tan(3x)$

Period $P = \frac{\pi}{3} = \frac{2}{6}\pi$



• To find the zeros. Solve

zeros of $y = \tan$ are $x = k\pi$

∘ $3x = 0$
$x = 0$

∘ $3x = \pi$
$x = \frac{\pi}{3}$

∘ $3x = 2\pi$
$x = \frac{2}{3}\pi$

∘ $3x = k\pi$
$x = \frac{k\pi}{3}$

• To find the vertical asymptotes, ~~solve~~ solve

V.A. for $y = \tan x$ are $\ldots -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5}{2}\pi$

∘ $3x = \frac{\pi}{2}$
$x = \frac{1}{3}\frac{\pi}{2} = \frac{\pi}{6}$

∘ $3x = \frac{3}{2}\pi$
$x = \frac{1}{3}\frac{3}{2}\pi = \frac{\pi}{2}$

∘ $3x = (2k+1)\frac{\pi}{2}$
$x = (2k+1)\frac{\pi}{6}$ an odd number times $\frac{\pi}{6}$.

(b) Graph $y = \tan 2(x - \frac{\pi}{6})$

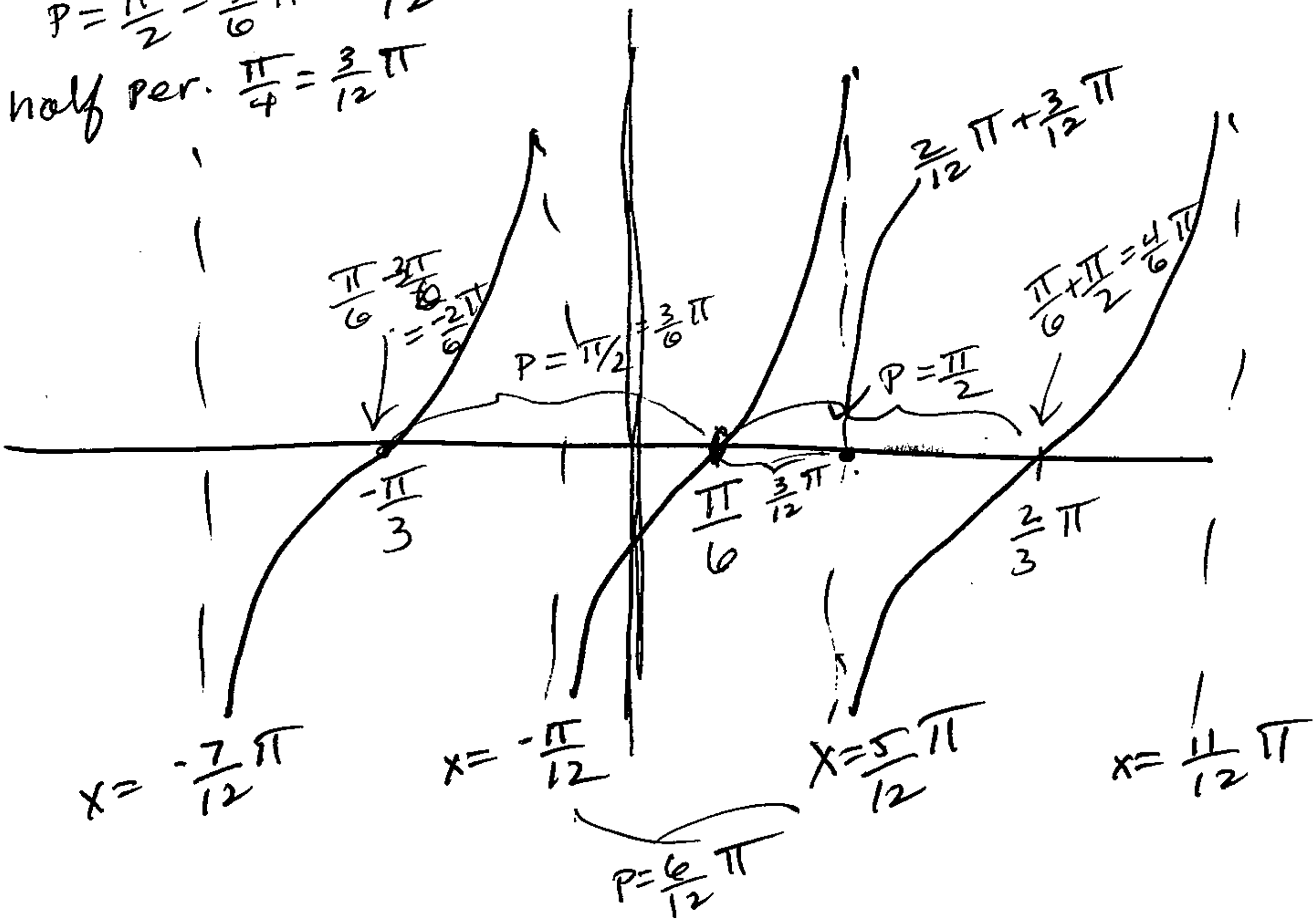
$P = \frac{\pi}{2}$

Phase shift : $\frac{\pi}{6}$ right

- Find the zeros
  - $2(x - \pi/6) = 0$
    $x - \pi/6 = 0$
    $x = \frac{\pi}{6}$
  - $2(x - \frac{\pi}{6}) = \pi$
    $x - \pi/6 = \frac{\pi}{2}$
    $x = \frac{\pi}{2} + \frac{\pi}{6} = \frac{3}{6} + \frac{\pi}{6} = \frac{4}{6}\pi = \frac{2\pi}{3}$
  - $2(x - \frac{\pi}{6}) = -\pi$
    $x - \pi/6 = -\pi/2$
    $x = -\pi/2 + \pi/6 = -\frac{3}{6}\pi + \frac{\pi}{6} = \frac{-2\pi}{6}$
    $= -\frac{\pi}{3}$

$P = \frac{\pi}{2} = \frac{3}{6}\pi = \frac{6}{12}\pi$

half per. $\frac{\pi}{4} = \frac{3}{12}\pi$



Find the vertical asymptotes

- $2(x - \frac{\pi}{6}) = \frac{\pi}{2}$ ← V.A. of $y = \tan x$

  $x - \frac{\pi}{6} = \frac{\pi}{4}$

  $x = \frac{\pi}{4} + \frac{\pi}{6} = \frac{3}{12}\pi + \frac{2}{12}\pi = \frac{5}{12}\pi$

- $2(x - \frac{\pi}{6}) = -\frac{\pi}{2}$

  $x - \frac{\pi}{6} = -\frac{\pi}{4}$

  $x = -\frac{\pi}{4} + \frac{\pi}{6} = \frac{-3}{12}\pi + \frac{2\pi}{12} = \frac{-\pi}{2}$

$y = \cot x$

$= \dfrac{\cos x}{\sin x}$

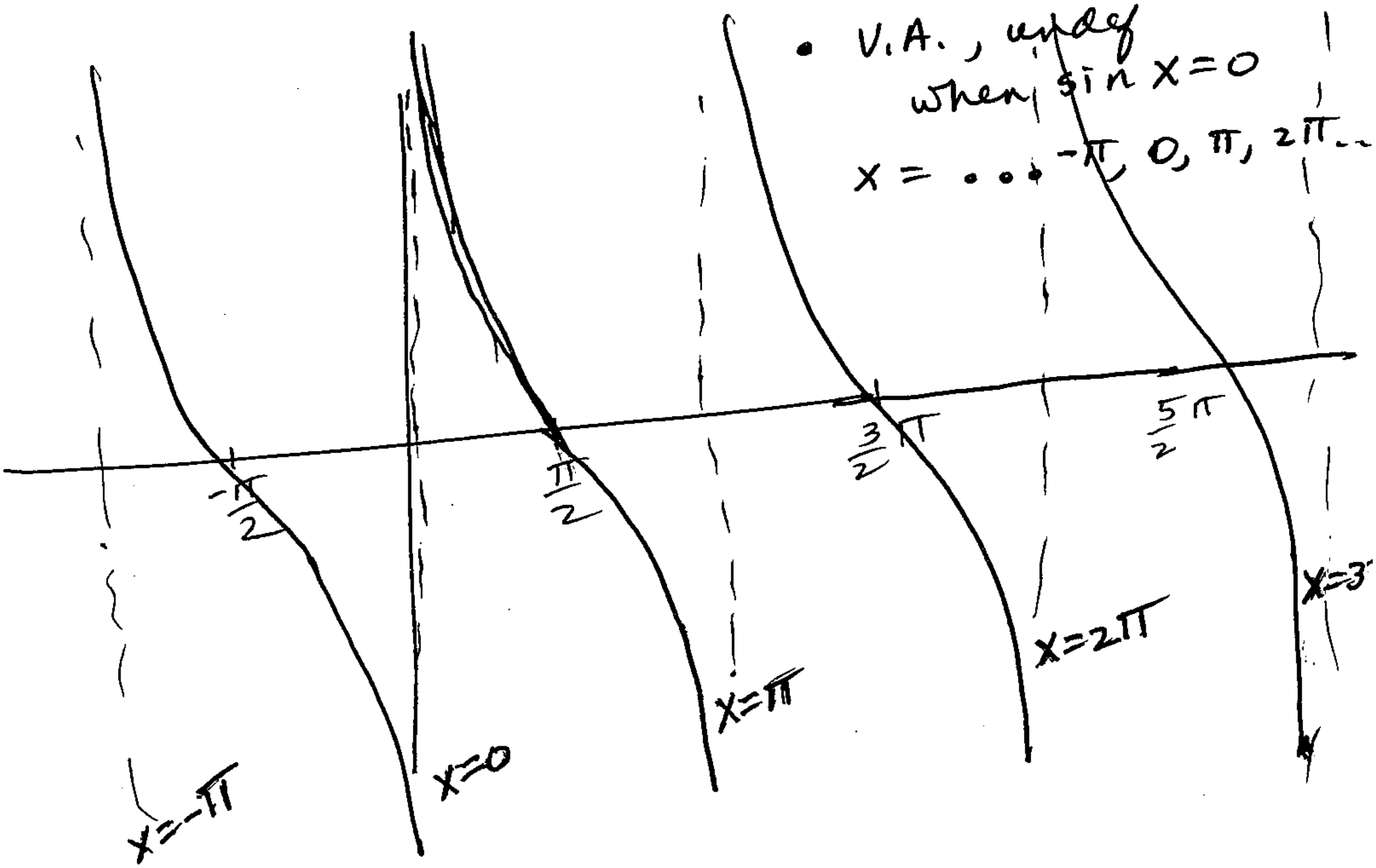
$p = \pi$

- Zero when $\cos x = 0$

  $x = \ldots -\frac{3}{2}\pi, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3}{2}\pi$

- V.A., undef when $\sin x = 0$

  $x = \ldots -\pi, 0, \pi, 2\pi \ldots$



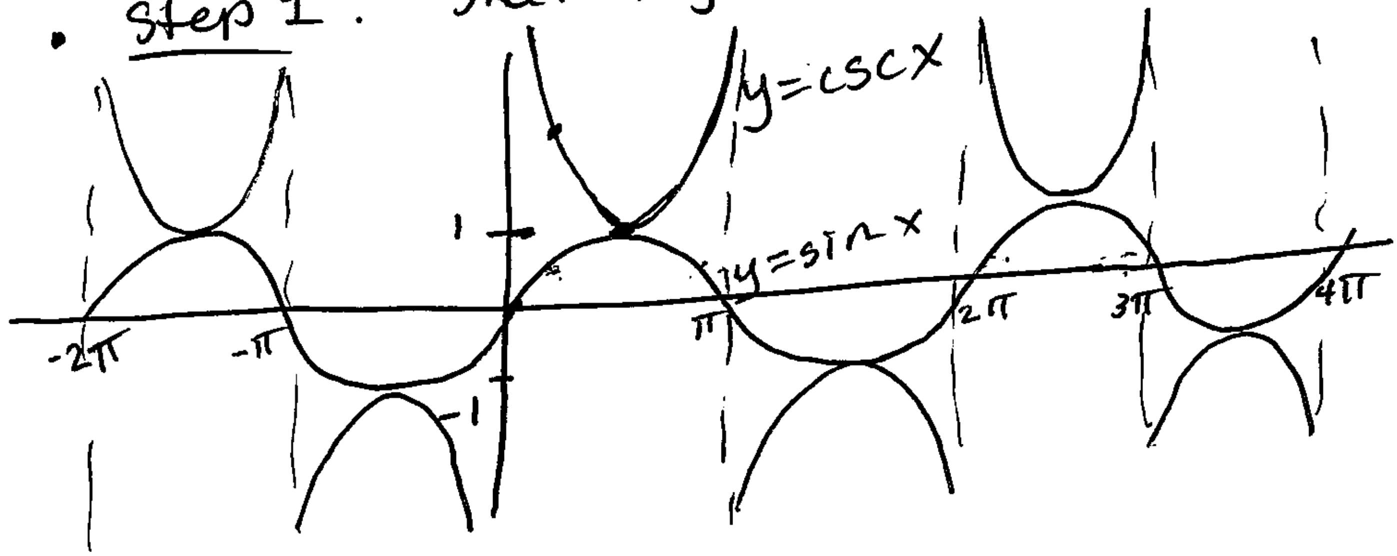
- The zeros of $y = \tan x$ are the vertical asymptotes of $y = \cot x$
- The V.A. of $y = \tan x$ are the zeros of $y = \cot x$.

The Graph of $y = \csc x$.

$y = \csc x$

$= \frac{1}{\sin x}$

- Step 1: Sketch $y = \sin x$



- Step 2: Draw loops

The Graph of

$y = \sec x$

$= \frac{1}{\cos x}$

Step 1 Sketch $y = \cos x$