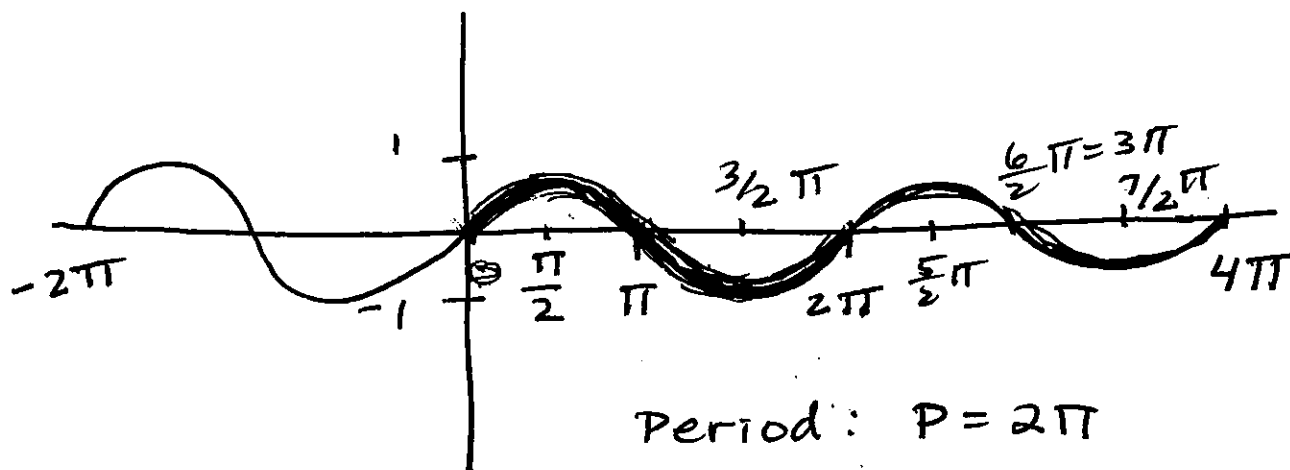


# § 5.3 The Graphs of the Sine and Cosine Functions

HW § 5.3 #1-75 odd

The graph of  $y = \sin x$



Period:  $P = 2\pi$

Amplitude:  $A = 1$

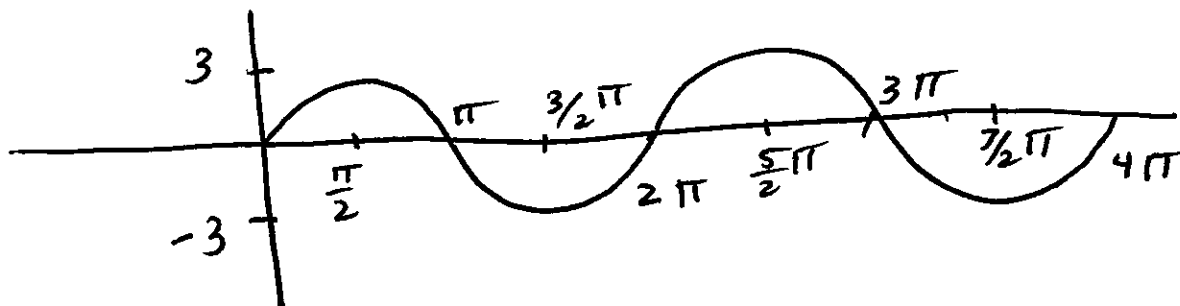
~~Qtr~~ Quarter Period:  $\frac{P}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$

EXAMPLE Sketch the graph.

①  $y = 3 \sin x$

$P = 2\pi$

$A = 3 \leftarrow$  amplitude

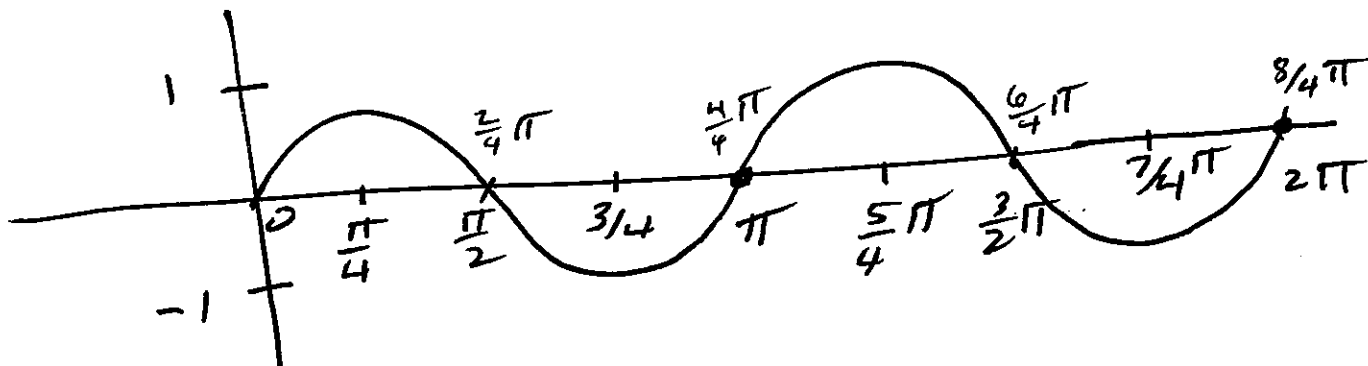


EXAMPLE Change in Period. sketch the graph.

①  $y = \sin(2)x$

Period  $P = \frac{2\pi}{2} = \pi$

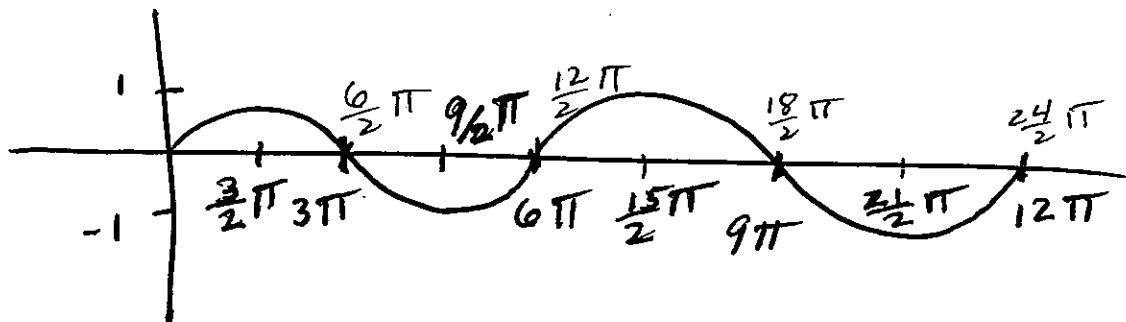
Qtr Per:  $\pi/4$



$$\textcircled{2} \quad y = \sin \frac{x}{3} = \sin \left( \frac{1}{3}x \right)$$

$$P = \frac{2\pi}{\frac{1}{3}} = 6\pi, \quad \text{Qtr Per: } \frac{6\pi}{4} = \frac{3}{2}\pi$$

$$A = 1 \quad \frac{1}{3}$$



## Phase Shifts

EXAMPLE: Sketch the graph.

$$\textcircled{1} \quad y = \sin \left( x - \frac{\pi}{4} \right)$$

← shift right by  $\frac{\pi}{4}$ .

•  $P = 2\pi$ , • Qtr Per.  $2\pi/4 = \pi/2$

•  $A = 1$

• One Cycle:

ends of cycle

Solve:  $x - \frac{\pi}{4} = 0$   
 $x = \frac{\pi}{4}$

$x - \frac{\pi}{4} = 2\pi$   
 $x = \frac{9\pi}{4}$

$$0 \leq x - \frac{\pi}{4} \leq 2\pi$$

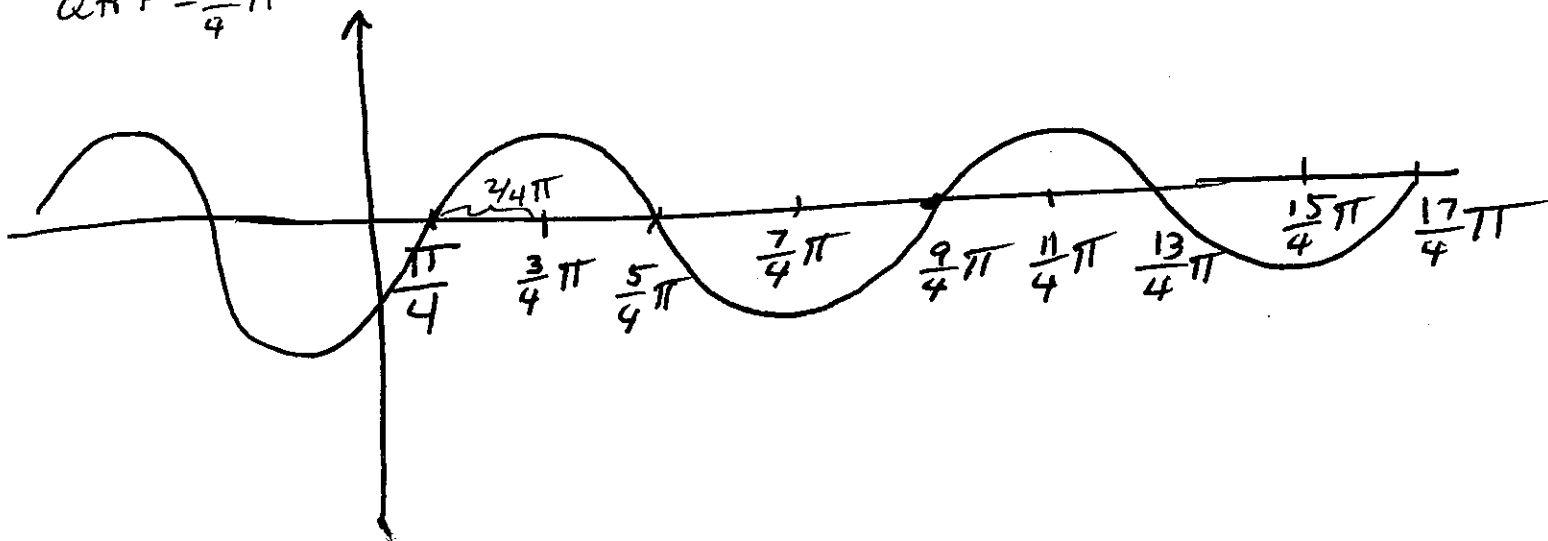
$$+ \frac{\pi}{4} \quad + \frac{\pi}{4} \quad + \frac{\pi}{4}$$

$$\frac{\pi}{4} \leq x \leq 2\pi + \frac{\pi}{4}$$

$$\frac{\pi}{4} \leq x \leq \frac{9\pi}{4}$$

$$P = \frac{8\pi}{4}$$

$$Q + P = \frac{2\pi}{4}$$

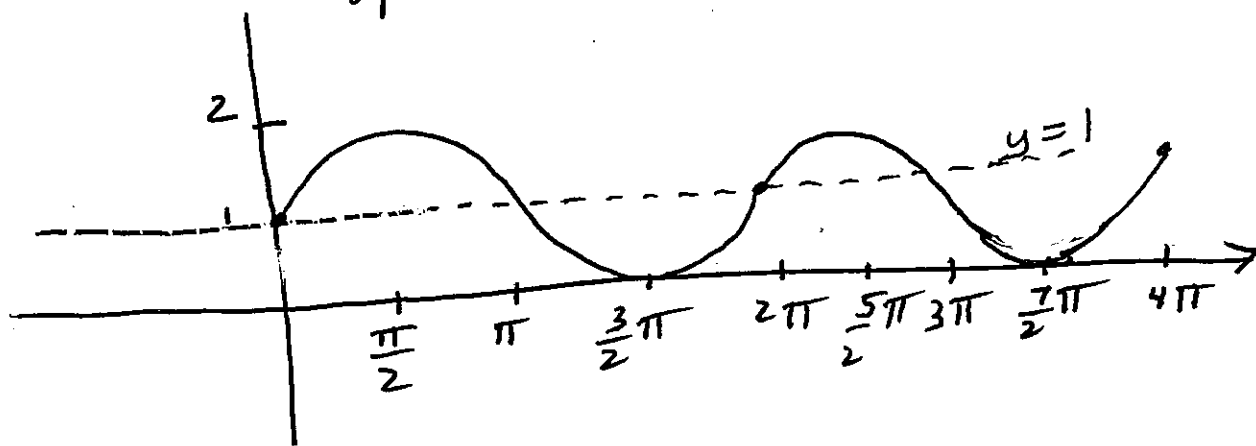


EXAMPLE: (Vertical shift)

$$y = 1 + \sin x$$

↑  
shift  
up

$$P = 2\pi$$



EXAMPLE Sketch

$$y = 2 \sin(3x + \pi) + 1$$

$$y = 2 \sin\left[3\left(x + \frac{\pi}{3}\right)\right] + 1$$

- $A = 2$
- $P = \frac{2\pi}{3}$
- Qtr Per:  $\frac{2\pi}{3 \cdot 4} = \frac{\pi}{6}$
- shift up: 1
- shift left:  $\frac{\pi}{3}$

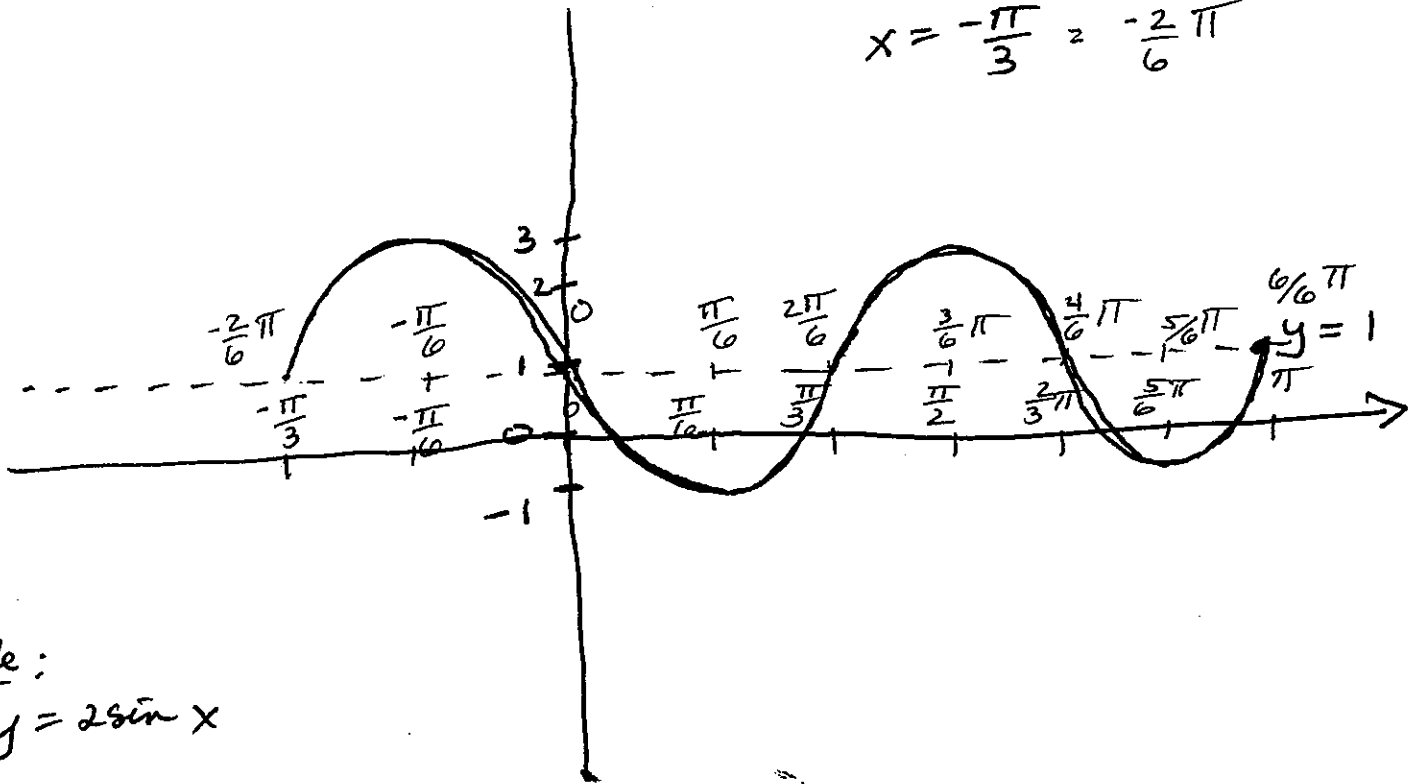
Starting Point:

$$3\left(x + \frac{\pi}{3}\right) = 0$$

$$x + \frac{\pi}{3} = 0$$

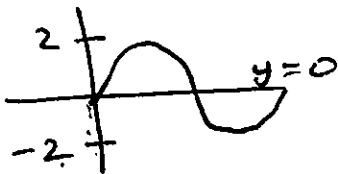
$$x = -\frac{\pi}{3} = -\frac{2\pi}{6}$$

usual starting for  $y = \sin$



Aside:

$$y = 2 \sin x$$

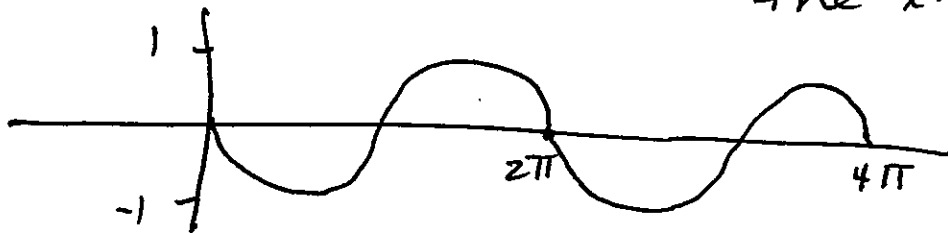


# Reflections about the x-axis.

EXAMPLE

$$y = -\sin x$$

Reflect about  
the x-axis



Summary:  $y = A \sin(B(x-C)) + D$

Amplitude:  $A$

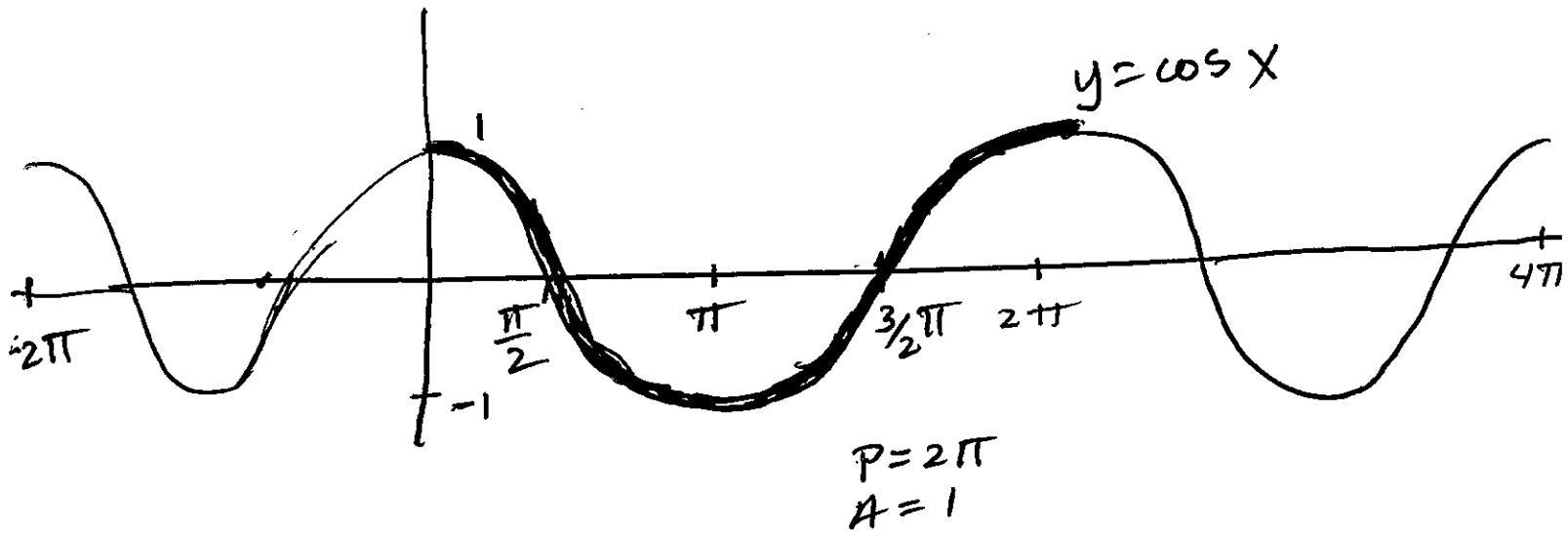
Period:  $P = \frac{2\pi}{B}$

Phase shift:  $C$

Vertical shift:  $D$

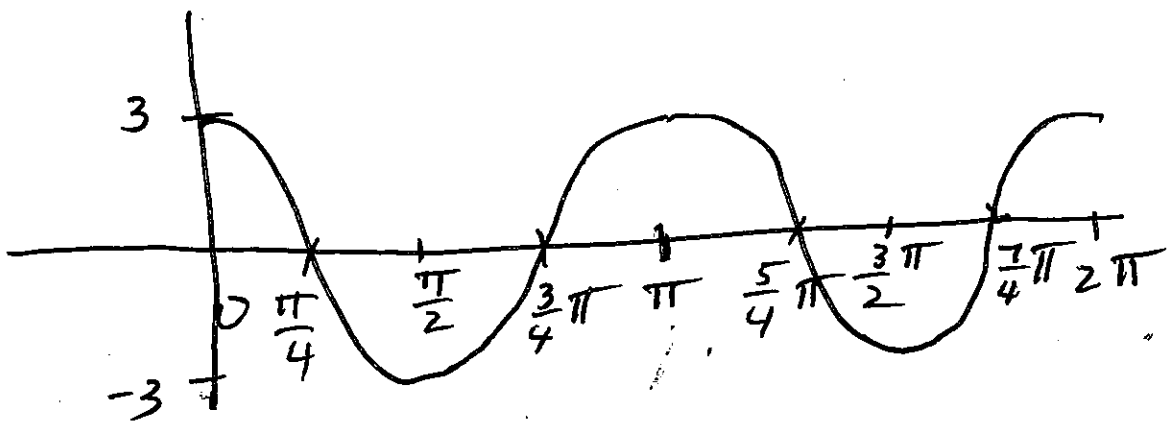
Reflection about x-axis if  $A < 0$ .

# The Cosine Graph



## EXAMPLE Sketch

①  $y = 3 \cos 2x$   
 $P = \frac{2\pi}{2} = \pi$ ,  $Q \text{ or } P = \frac{\pi}{4}$   
 $A = 3$



②  $y = \cos(x - \frac{\pi}{6})$

$P = 2\pi$

Phase shift:  $\frac{\pi}{6}$

Qtr Per:  $\frac{2\pi}{4} = \frac{\pi}{2} = \frac{3}{6}\pi$

