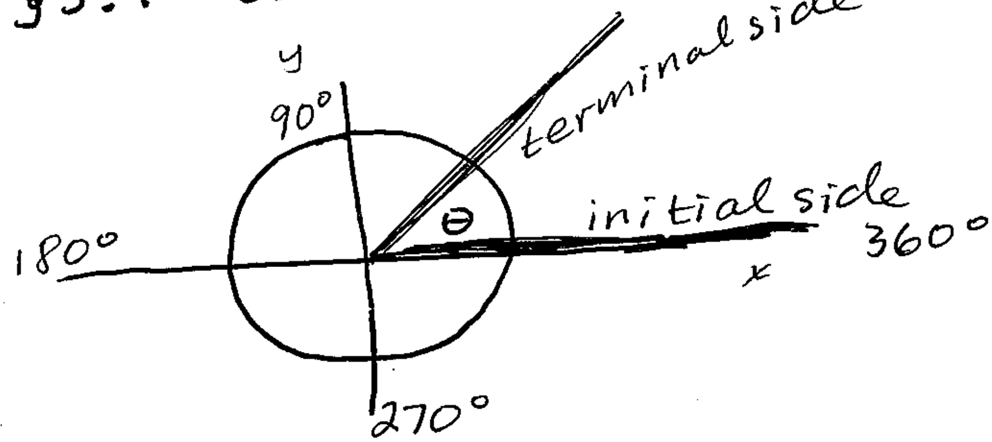
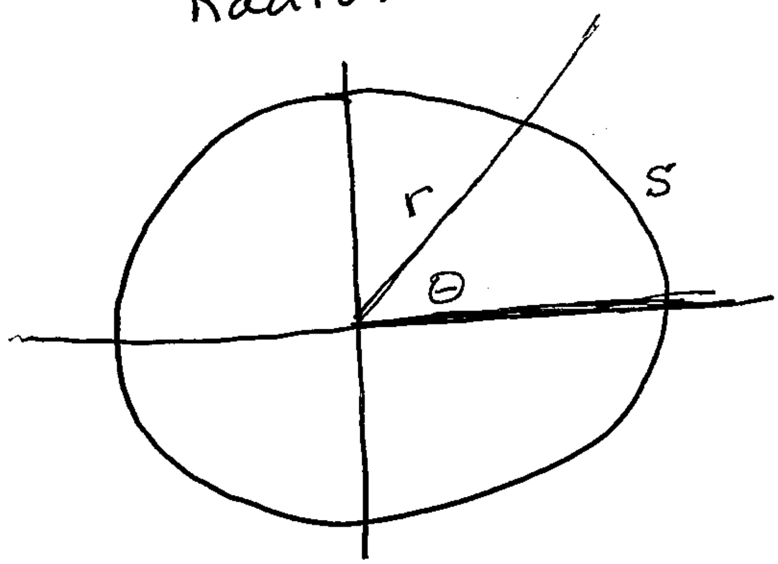
§5.1 Continued



θ theta
α alpha
β beta
γ gamma

Radians



s is the arc length.
r is radius.

The circumference of a circle is $C = 2\pi r$

We say that there are 2π radians in a circle.

The arc length s can be ~~dee~~ given in radians. We label the angle θ in terms of radians. θ radians is the length of s in radians.

We get the equation:

$$\underset{\substack{\uparrow \\ 2\pi \text{ rad} \\ \text{in circle}}}{\frac{\theta}{2\pi}} = \frac{s}{2\pi r}$$

θ rad in angle → θ

s ← arc length

$2\pi r$ ← ~~circle~~ circumference

↑ This is a fraction of the circle

$$\theta \cancel{2\pi} r = s \cdot \cancel{2\pi}$$

$s = r\theta$ where the unit of measurement for s is the same as for r.

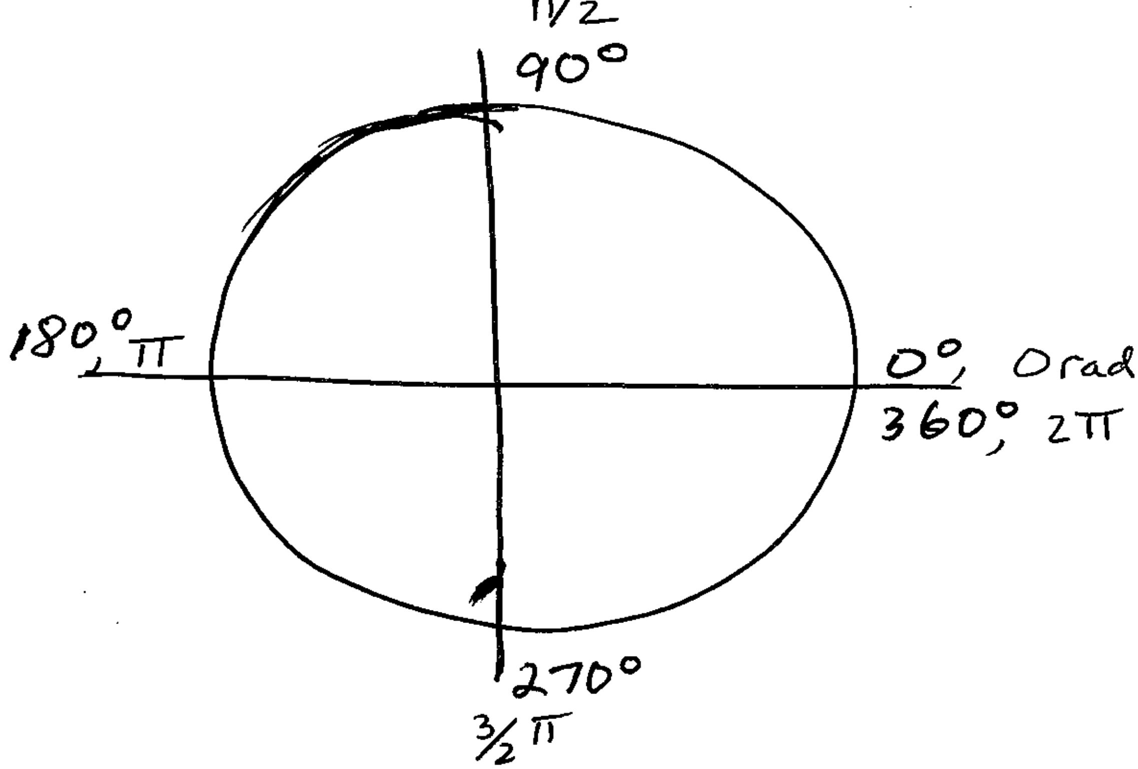
Example: Calculate the ~~arc~~ length of an arc s of the angle $\theta = \frac{\pi}{4}$ radians if $r = 7$ meters.

Solution

$$s = r\theta$$

$$s = \frac{(7\,m)}{1\text{ rad}}\left(\frac{\pi}{4}\text{ rad}\right)$$

$$s = \frac{7}{4}\pi \text{ meters}$$



There is 180° in π radians.
We have a conversion ratio of $\frac{180°}{\pi \text{ rad}}$ or $\frac{\pi \text{ rad}}{180°}$ that we can use to convert units.

EXAMPLE Convert from degrees to radians.

a) 30°

SOL $30° \cdot \frac{\pi \text{ rad}}{180°} = \frac{30}{180}\pi = \frac{3}{18}\pi$

$= \frac{\pi}{6}$ rad

b) $45° = 45° \cdot \frac{\pi}{180°} \text{rad} = \frac{45}{180} \pi \text{ rad}$

$= \frac{\pi}{4}$

EXAMPLE Convert radians to degrees.

a) $\frac{\pi}{3}$ rad

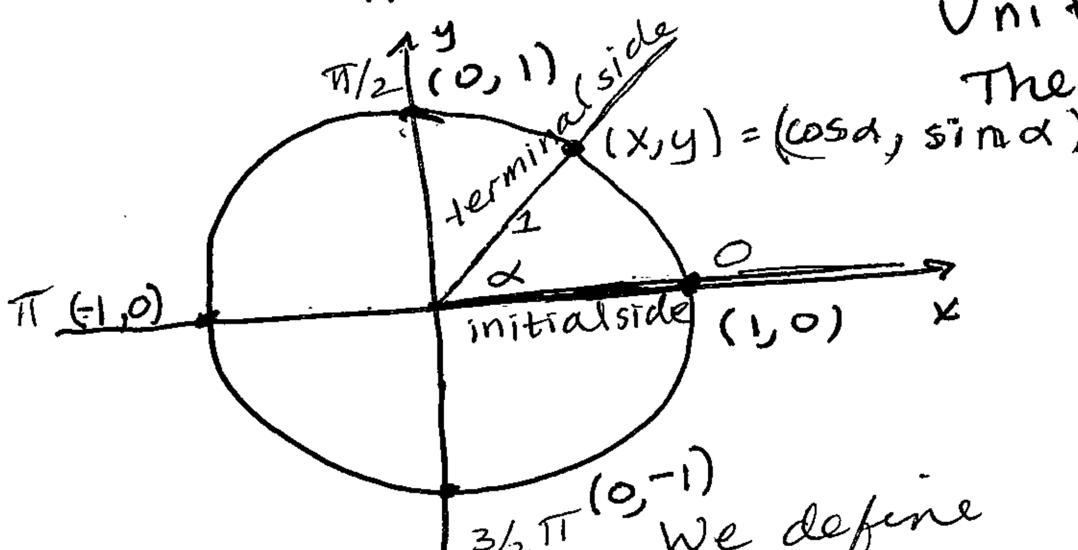
$$= \frac{\pi}{3}\text{rad}\ \frac{180^\circ}{\pi\ \text{rad}} = \frac{180^\circ}{3} = 60^\circ$$

§5.2 The Sine and Cosine Function

HW §5.2 #1-87 odd

Unit Circle.

The radius is r=1.



We define

$$\boxed{\cos\alpha = x \text{ and } \sin\alpha = y}$$

where α is an angle in standard position on the unit circle and (x, y) is the point of intersection of the terminal side.

EXAMPLE Find the exact values.

a) $\cos \frac{\pi}{2} = 0$ ← x-coord at $\pi/2$

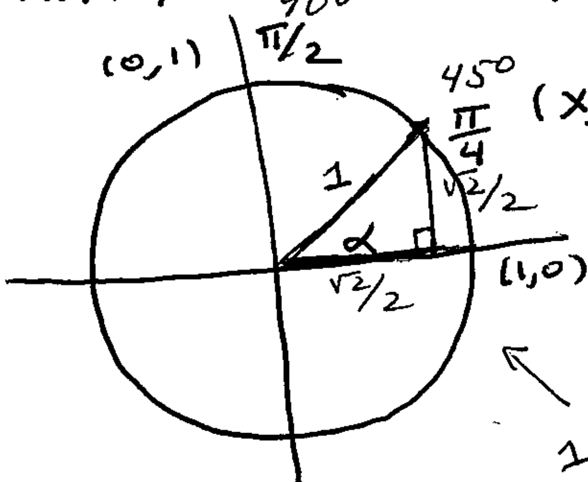
b) $\cos \pi = -1$ ← x-coord at π

c) $\sin \pi/2 = 1$ ← y coord at $\pi/2$

d) $\sin \pi = 0$ ← y-coord at π

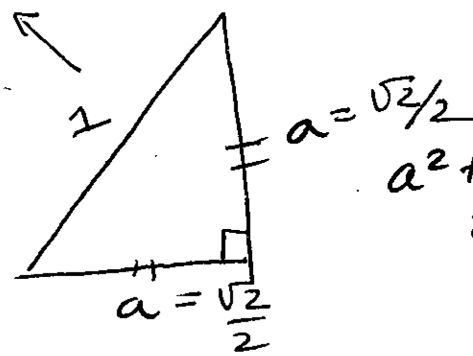
e) $\sin \frac{3}{2}\pi = -1$ ← y-coord.

Finding Cosine and Sine for Multiples of $\pi/4$. (or $45°$)



Pyth Thm

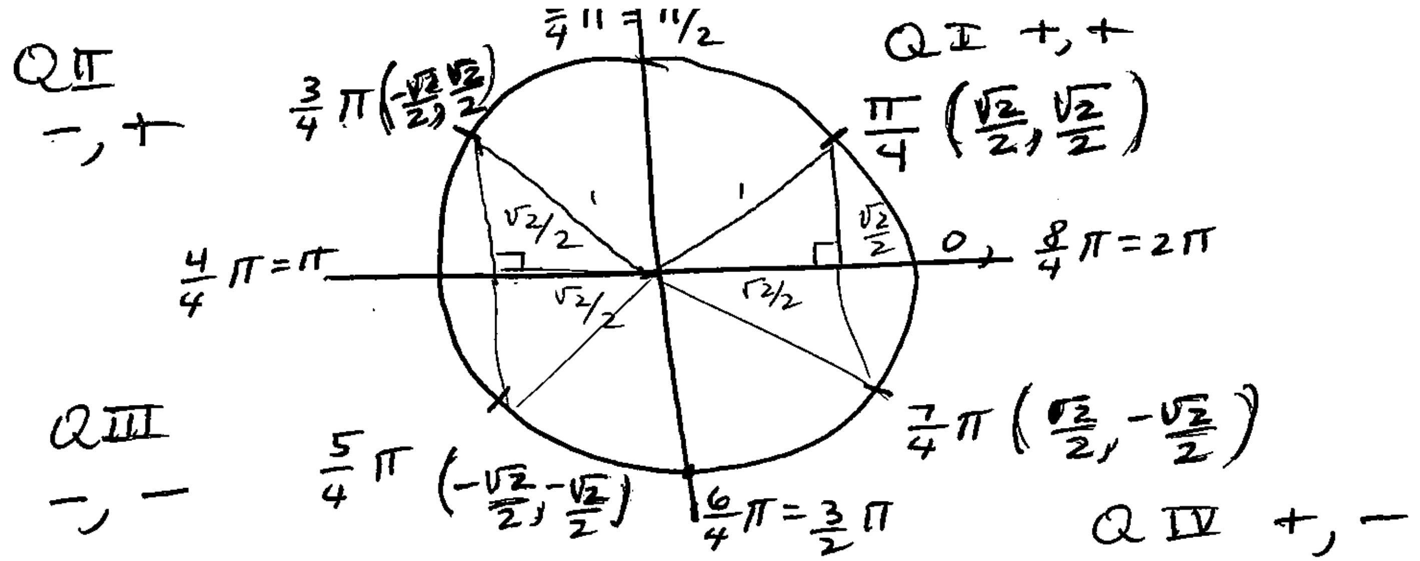
$a^2+b^2=c^2$



$$a^2+a^2=1^2$$
$$2a^2=1$$
$$a^2=\frac{1}{2}$$
$$a=\sqrt{1/2}=\frac{1}{\sqrt{2}}$$
$$=\frac{1}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$$
$$=\frac{\sqrt{2}}{(\sqrt{2})^2}=\frac{\sqrt{2}}{2}$$

we get

- $\cos \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
- $\sin \frac{\pi}{4} = \frac{\sqrt{2}}{2}$
- $\cos 45° = \frac{\sqrt{2}}{2}$
- $\sin 45° = \frac{\sqrt{2}}{2}$

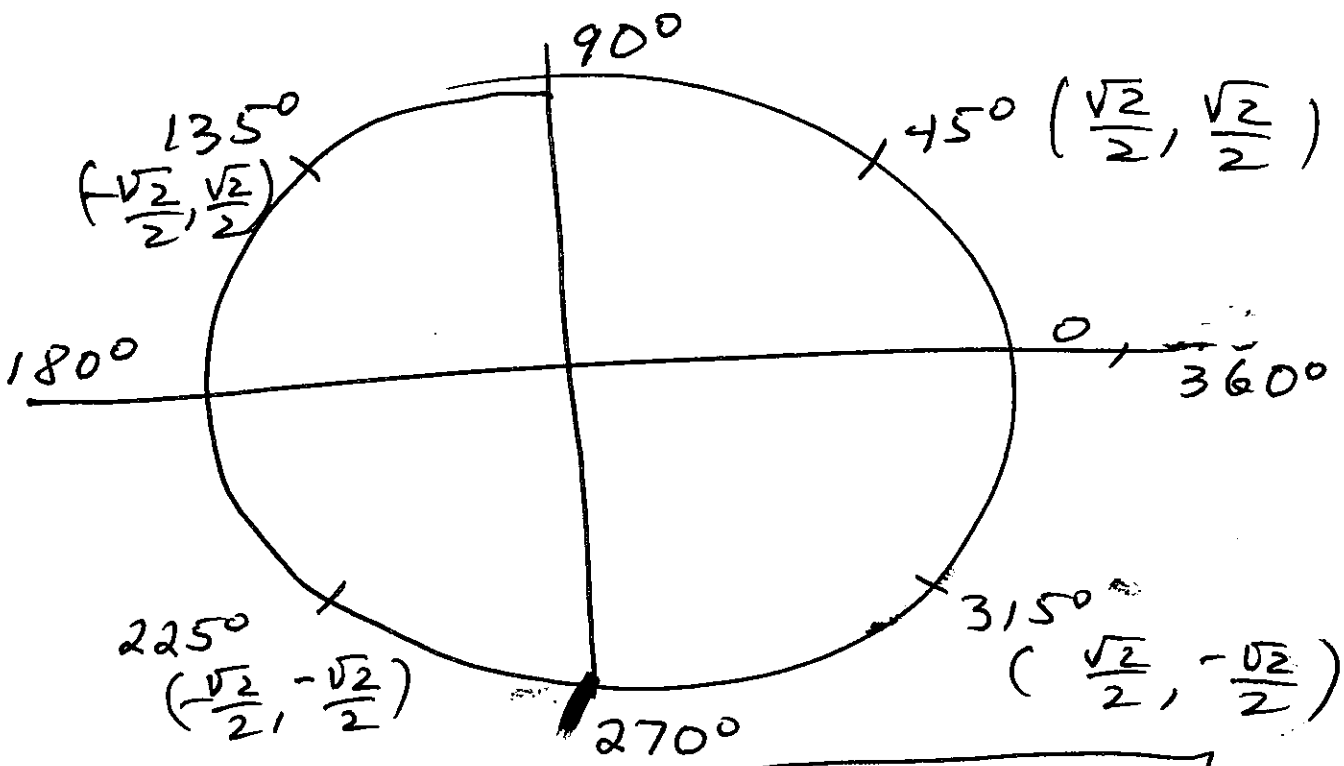


EXAMPLE Find

a) $\sin \frac{3}{4}\pi = \sqrt{2}/2$ ← y-coord

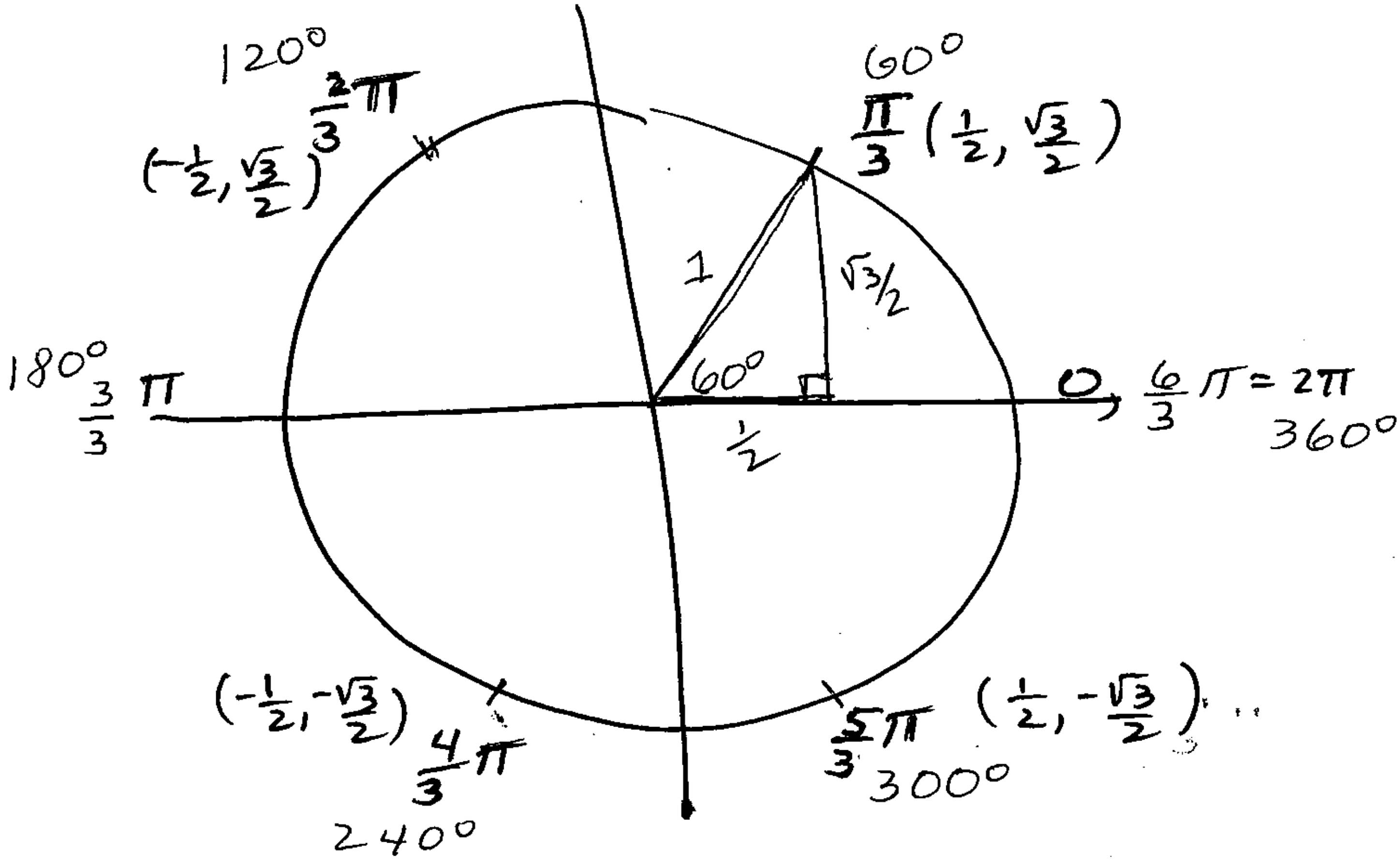
b) $\cos \frac{7}{4}\pi = \sqrt{2}/2$ ← x-coord

In degrees

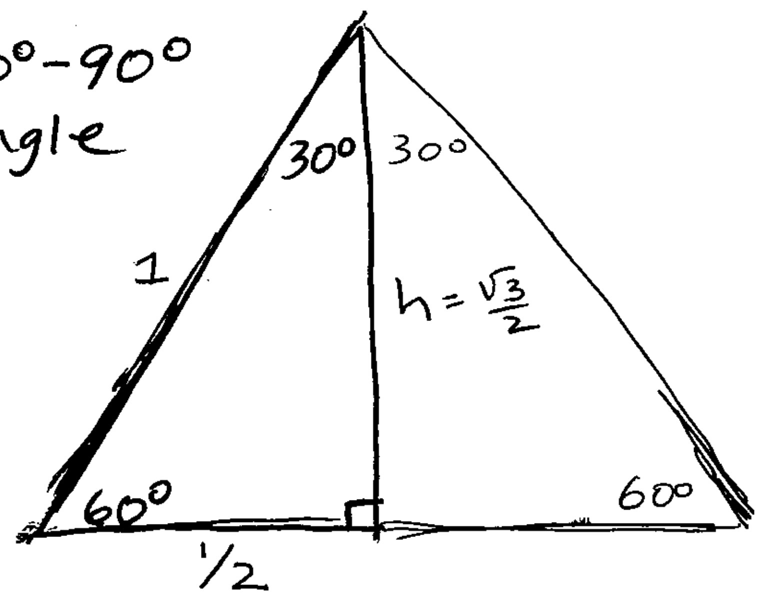


$\boxed{\text{So} \quad \cos 315° = \frac{\sqrt{2}}{2}}$

Multiples of $\frac{\pi}{3}$



30°-60°-90° Triangle



Pyth Thm

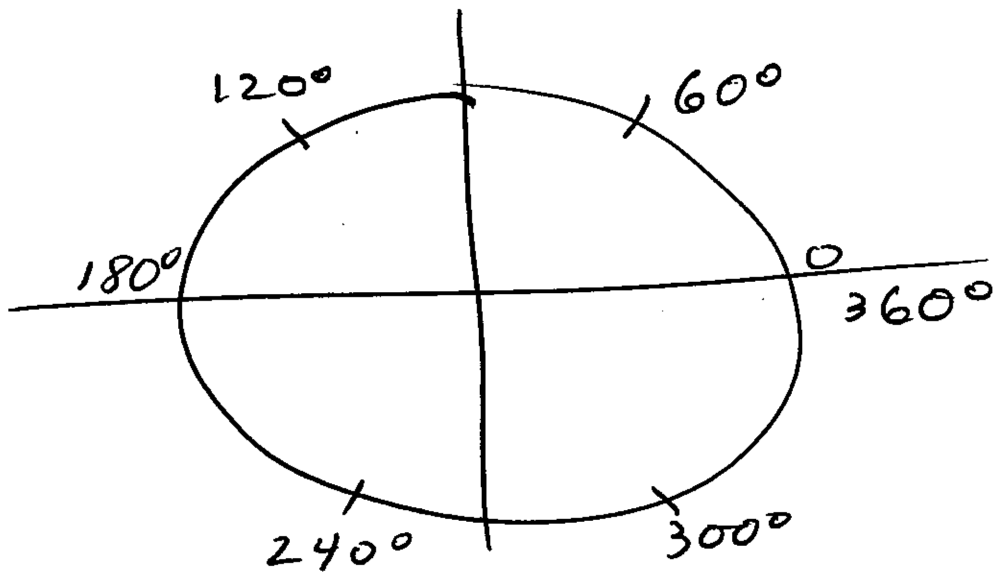
$$h^2 + (1/2)^2 = 1^2$$
$$h^2 + 1/4 = 1$$
$$h^2 = 3/4$$
$$h = \sqrt{3/4} = \frac{\sqrt{3}}{2}$$

EXAMPLE: Evaluate. Give exact values.

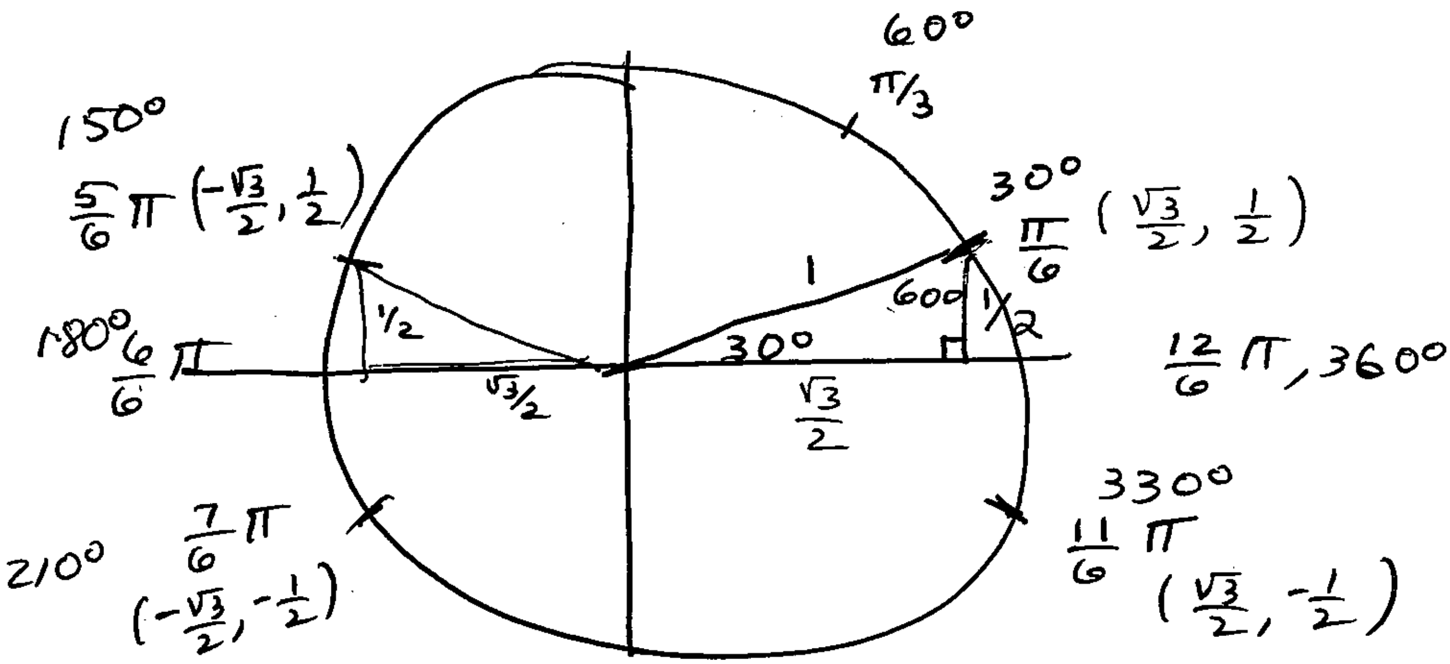
a) $\cos\frac{\pi}{3} = \frac{1}{2}$ ← x-coord

b) $\sin\frac{4}{3}\pi = -\frac{\sqrt{3}}{2}$ ← y-coord

Multiples of 60°

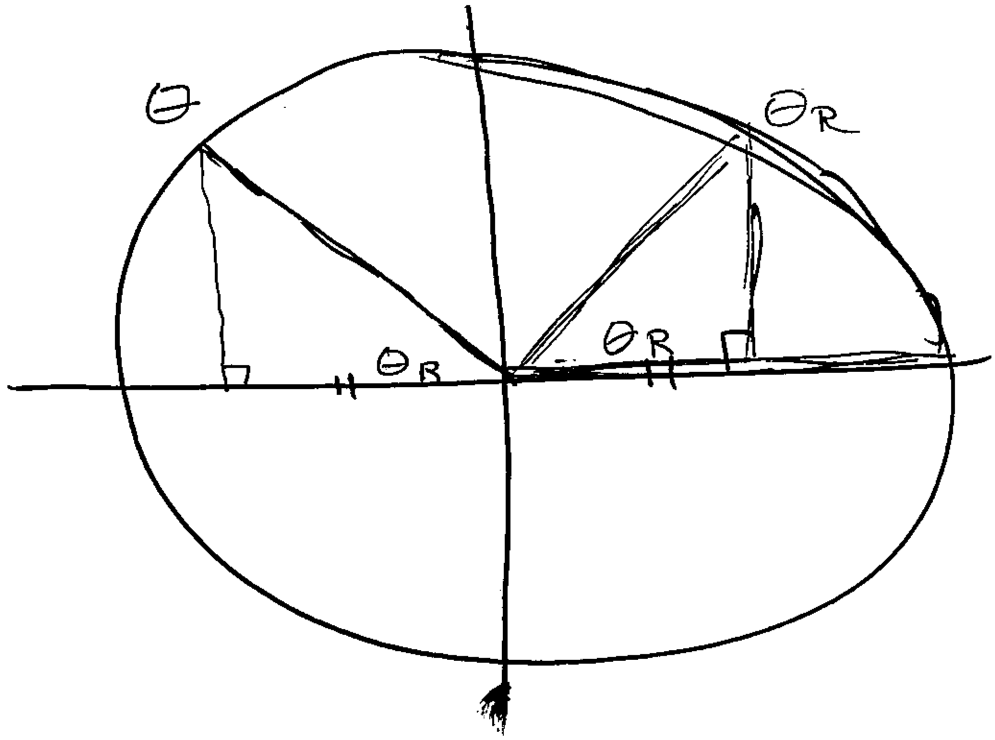


Multiples of $\frac{\pi}{6}$



Find
- $\cos \frac{5}{6}\pi$
  $= -\sqrt{3}/2$ ← x coord.
- $\sin \frac{11}{6}\pi = -1/2$ ← y-coord

Reference Angles



The reference angle θ_R of the angle θ is the acute angle between the terminal side of θ and the x-axis.

We see that

$$\cos\theta = \pm \cos\theta_R$$

adjusted to the quadrant in which θ lies

$$\sin\theta = \pm \sin\theta_R$$