Test Review

§ 4.4

① Dating a Bone: A piece of bone from an organism is found to contain 10% of the carbon 14 that it contained when it was living. How long ago was the organism alive?

$$A = A_0 e^{-kt}, \quad k = \frac{\ln 2}{h}, \quad h = 5730$$

SOLUTION

$$A_0 = 1$$
$$A = .1$$
$$k = \frac{\ln 2}{5730}$$

$$.1 = 1 \cdot e^{-\frac{\ln 2}{5730} t}$$

$$\ln(.1) = \ln e^{\frac{-(\ln 2)t}{5730}}$$

$$\ln(.1) = \frac{-(\ln 2)\, t}{5730}$$

$$t = \frac{-5730 \ln(.1)}{\ln 2}$$

$$t \approx 19{,}034.6 \text{ years.}$$

② Radioactive Waste

If 25 g of radioactive waste reduces to 20 g of radioactive waste after 8000 years, then what is the half-life, h, for the radioactive element?

SOLUTION $\quad A = A_0 e^{-kt}, \quad k = \frac{\ln 2}{h}, \quad h = ?$

$A_0 = 25$

$A = 20$

$t = 8000$

$$A = A_0 e^{-\left(\frac{\ln 2}{h}\right)t}$$

$$20 = 25\, e^{-\frac{(\ln 2)}{h}(8000)}$$

$$\frac{20}{25} = e^{-\frac{(\ln 2)}{h}(8000)}$$

$$\ln(.8) = \ln e^{-\frac{(\ln 2)(8000)}{h}}$$

$$\ln(.8) = -\frac{(\ln 2)(8000)}{h}$$

$$h = -\frac{(\ln 2)(8000)}{\ln(.8)}$$

$$h \approx 24{,}850.3 \text{ years}$$

§4.1

③ How long will it take \$10,000 invested at $2\frac{3}{4}\%$ compounded continuously to grow to \$15,000?

$A = Pe^{rt}$

<u>SOLUTION</u>

$P = 10{,}000,$

$A = 15{,}000$

$r = 2.75\% = .0275$

$A = Pe^{rt}$

$$15{,}000 = 10{,}000\, e^{(.0275)t}$$

$$\frac{15{,}000}{10{,}000} = e^{(.0275)t}$$

$$\ln(1.5) = \ln e^{(.0275)t}$$

$$\ln(1.5) = 0.0275t$$

$$t = \frac{\ln(1.5)}{0.0275} \approx 14.7 \text{ years.}$$

§4.4 Solve each equation. Give exact solutions.

(4) $\log(x+1) - \log(x) = 3$

SOLUTION

$$\log_{10}\left(\frac{x+1}{x}\right) = 3$$

$$10^3 = \frac{x+1}{x}$$

$$1000x = x+1$$

$$999x = 1$$

$$x = 1/999$$

(5) $\ln(x) + \ln(x+2) = \ln 8$

SOLUTION

$$\ln x(x+2) = \ln 8$$

$$x(x+2) = 8$$

$$x^2+2x = 8$$

$$x^2+2x-8 = 0$$

$$(x+4)(x-2) = 0$$

$$\cancel{x=}-4, \quad x = 2$$

$\ln(-4)$ undef. $\boxed{x=2}$

Solve. Give exact values.
Use natural or common log when using logorithms

(6) $7 = 2e^{3t}$

SOLUTION $\frac{7}{2} = e^{3t}$

$$\ln(7/2) = \ln e^{3t}$$

$$\ln(7/2) = 3t$$

$$t = \frac{\ln(7/2)}{3}$$

(7) $5 = 7e^{2t}$

SOLUTION $\frac{5}{7} = e^{2t}$

$$\ln(5/7) = \ln e^{2t}$$

$$\ln(5/7) = 2t$$

$$t = \frac{1}{2}\ln(5/7)$$

(8) $2^x = 3^{x-1}$

Sol
$$\ln 2^x = \ln 3^{x-1}$$
$$x \ln 2 = (x-1)\ln 3$$
$$x \ln 2 = x \ln 3 - \ln 3$$
$$x \ln 2 - x \ln 3 = -\ln 3$$
$$x(\ln 2 - \ln 3) = -\ln 3$$
$$x = \frac{-\ln 3}{\ln 2 - \ln 3}$$

§4.3

Rewrite each expression as a sum or difference of logarithms $\ln z$, $\ln x$, and $\ln y$

(9) $\ln zxy$

SOLUTION $\ln z + \ln x + \ln y$

(10) $\ln \frac{8x^5}{y^7} = \ln 8 + \ln x^5 - \ln y^7$

$= \ln 2^3 + \ln x^5 - \ln y^7$

$= 3 \ln 2 + 5 \ln x - 7 \ln y$

(11) $\ln \frac{x^7}{y^3 \sqrt{z}} = \ln x^7 - \ln y^3 - \ln z^{1/2}$

$= 7 \ln x - 3 \ln y - \frac{1}{2} \ln z$

Write as a single logarithm.

(12) $2 \ln x - 3 \ln(x+7) - 5 \ln z$

$= \ln x^2 - \ln(x+7)^3 - \ln z^5$

$= \ln \left(\frac{x^2}{(x+7)^3 z^5} \right)$

Simplify

(13) $e^{\ln\sqrt{y}}$

$= \sqrt{y}$

(14) $10^{\log(3x+1)}$

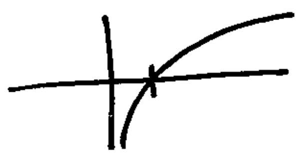
$= 3x+1$

(15) $\ln e^{7t}$

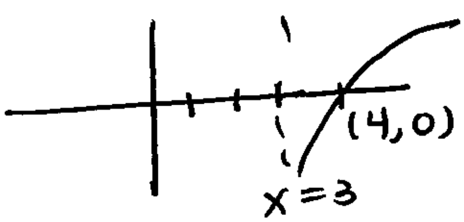
$= 7t$

§ 4.2 Sketch the graph.

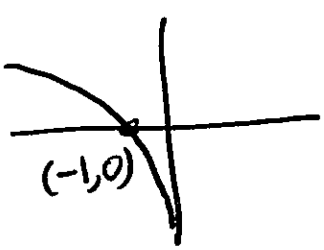
(16) $y = \ln x$



(17) $y = \ln(x-3)$



(18) $y = \ln(-x)$



Write as an exponent. Do not solve for x

(19) $\log_2 x = 5$

Sol $2^5 = x$

(20) $\ln 3x = 7$

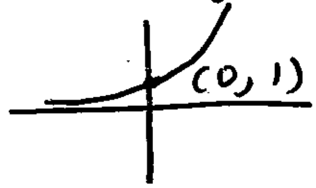
Sol $e^7 = 3x$

(21) $\log 9x = -3$

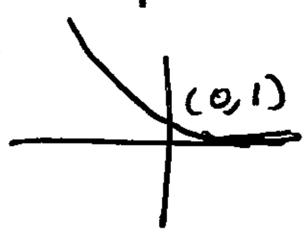
$10^{-3} = 9x$

§4.1 Sketch the graph.

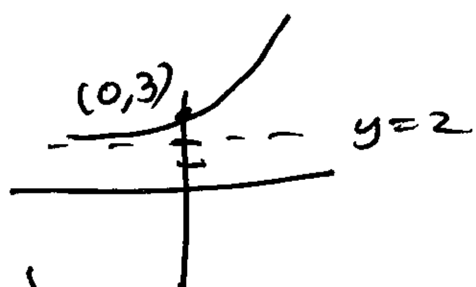
(22) $y = e^x$



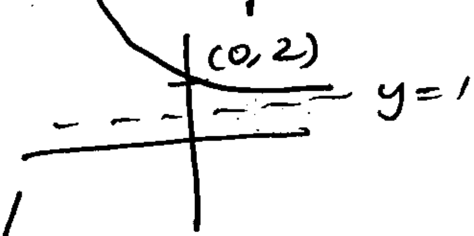
(23) $y = e^{-x}$



(24) $y = e^x + 2$



(25) $y = e^{-x} + 1$



(26) $y = 3e^x$