§4.2 Continued

EXAMPLE: Evaluate

① $\log(100)$ ← common log

SOLUTION

$x = \log_{10} 100$

$10^x = 100$ Write as exponent

$10^x = 10^2$

$x = 2$

② $\log(0.001)$

SOLUTION $x = \log_{10}(0.001)$

$10^x = 0.001$

$10^x = 10^{-3}$

$x = -3$

③ $\log(1)$

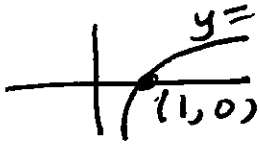
SOLUTION $x = \log_{10}(1)$

$10^x = 1$

$10^x = 10^0$

$x = 0$

Property: $\log_a(1) = 0$



EXAMPLE Use a calculator to solve for x.

① $10^x = 50$

SOLUTION

- Convert to a logarithm

$$\log_{10} 50 = x$$

- Put in calculator

$$\log 50 \approx 1.69897$$

② $2e^{0.5x} = 6$

SOLUTION

- Isolate the exponent

$$e^{0.5x} = 3$$

- Convert to a logarithm

$$\log_e 3 = 0.5x$$

- Solve for x

$$\frac{\ln(3)}{.5} = x$$

- Put into calculator

$$x = 2.197224578$$

Applications.

EXAMPLE: If $8000 is invested at 9% compounded continuously, then how long will it take for the investment to grow to $20,000?

Recall: $A = Pe^{rt}$

SOLUTION

$P = 8000$

$A = 20,000$

$r = .09$

$t = ?$

$20,000 = 8,000\, e^{.09t}$

- isolate exponent.

$\frac{20,000}{8,000} = e^{.09t}$

$\frac{5}{2} = e^{.09t}$

- write as a log

$\log_e\left(\frac{5}{2}\right) = .09t$

- solve for t

$\frac{\ln(5/2)}{.09} = t$

- $t \approx 10.2$ years

calculator

Doubling Time

If an amount P is invested at a rate of r, continuously then how long will it take for the investment to double.

SOLUTION

$$A = Pe^{rt}$$

$$A = 2P$$

$$2P = Pe^{rt}$$

• isolate exponent

$$\frac{2P}{P} = e^{rt}$$

$$2 = e^{rt}$$

• write as a log

$$\log_e 2 = rt$$

• Solve for t

$$t = \frac{\ln 2}{r}$$

If r is given as a decimal, say .09, then we have

$$t = \frac{\ln 2}{.09} = \left(\frac{\ln 2}{.01}\right)/9$$

$$\approx 69/9 \approx \frac{70}{9}$$

If r is given in percent, then the doubling time is about $70/r$

- If your money is invested at 10% compounded continuously, then how long, approximately, does it take to double?

Sol $\frac{70}{10} = 7$

- if r = 9%?

$\frac{70}{9} = 7\frac{7}{9} \approx 8$ yrs.

- You invest $1000 at 9% for 40 years, about how much will you have at the end of the period?

$40 \div 8 = 5$

double 5 times

- increase by a factor of $2^5 = 32$
- answer $1000 \cdot 32$

$= \$32{,}000$

§3.5 #83 $x^4 - 19x^2 + 90 \leq 0$

SOLVE

SOLUTION: • Find all real zeros.

$$x^4 - 19x^2 + 90 = 0$$

$$(x^2)^2 - 19(x^2) + 90 = 0$$

$u = x^2$ $\quad u^2 - 19u + 90 = 0$

$$(u - 9)(u - 10) = 0$$

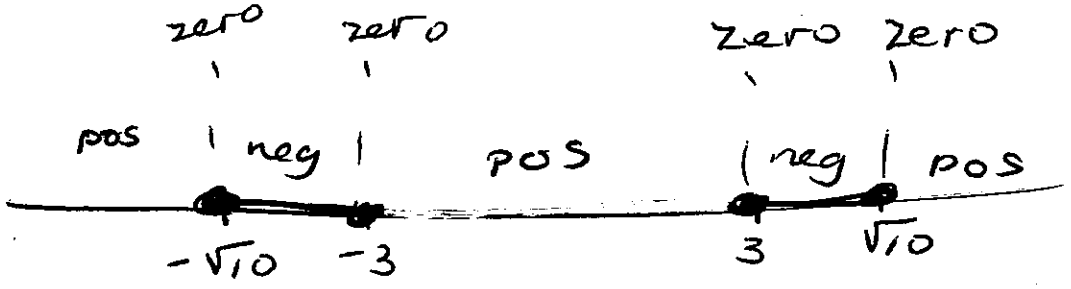
$$u = 9, \quad u = 10$$

$$x^2 = 9, \quad x^2 = 10$$

$$x = \pm 3, \quad x = \pm\sqrt{10} \approx \pm 3.16$$

$$y = (x-3)(x+3)(x-\sqrt{10})(x+\sqrt{10})$$

Sign chart



Test Point

x	$y = x^4 - 19x^2 + 90$
0	90 pos

Answer: $[-\sqrt{10}, -3] \cup [3, \sqrt{10}]$

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§3.4 #25 Solve.

$$\sqrt{2x+5} + \sqrt{x+6} = 9$$

SOLUTION $\sqrt{2x+5} = 9 - \sqrt{x+6}$

$$(\sqrt{2x+5})^2 = (9 - \sqrt{x+6})^2$$

$$2x+5 = 81 - 2\cdot 9\sqrt{x+6} + (\sqrt{x+6})^2$$

$$2x+5 = 81 - 18\sqrt{x+6} + x + 6$$

$$2x+5-81-x-6 = -18\sqrt{x+6}$$

$$x - 82 = -18\sqrt{x+6}$$

$$(x-82)^2 = (-18\sqrt{x+6})^2$$

$$x^2 - 164x + 6724 = 324(x+6)$$

$$x^2 - 164x + 6724 = 324x + 1944$$

$$x^2 - 164x - 324x + 6724 - 1944 = 0$$

$$x^2 - 488x + 4780 = 0$$

$$x = \frac{488 \pm \sqrt{(488)^2 - 4(1)(4780)}}{2(1)}$$

$$= \frac{488 \pm 468}{2}$$

$$x = \frac{488-468}{2} = \frac{20}{2} = 10$$

$$x = \frac{488+468}{2} = 478$$

1
82
82
164
6560
6724

6 18
18
144
180
324

2
324
6
1944

164
324
488

check $x = 10$

$$\sqrt{2x+5} + \sqrt{x+6} = 9$$

$$\sqrt{2 \cdot 10 + 5} + \sqrt{10+6} \stackrel{?}{=} 9$$

$$\sqrt{25} + \sqrt{16} \stackrel{?}{=} 9$$

$$5 + 4 = 9 \quad \text{yes}$$

check $x = 478$

$$\sqrt{2 \cdot 478 + 5} + \sqrt{478+6} \stackrel{?}{=} 9$$

$$53 \stackrel{?}{=} 9 \quad \underline{\text{NO}}$$

Answer: 10

§3.1 #11 Write in the form $y = a(x-h)^2 + k$ and sketch its graph.

$y = -2x^2 + 3x - 1$

SOLUTION $h = \frac{-b}{2a} = \frac{-3}{2(-2)} = \frac{3}{4}$

$k = f(h) = f(3/4)$

$= -2\left(\frac{3}{4}\right)^2 + 3\left(\frac{3}{4}\right) - 1$

$= -2\left(\frac{9}{16}\right) + \frac{9}{4} - 1$

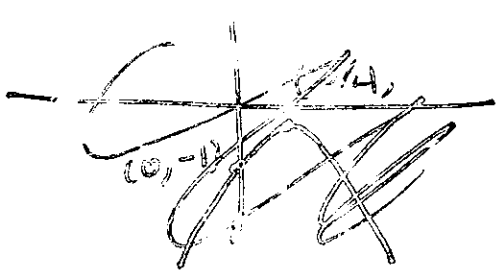
$= -\frac{9}{8} + \frac{18}{8} - \frac{8}{8} = \frac{1}{8}$

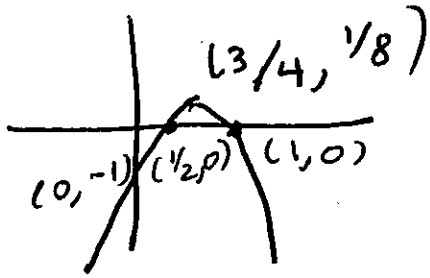
$y = a(x-h)^2 + k$

$a = -2, \quad h = 3/4, \quad k = 1/8$

$y = -2(x - 3/4)^2 + \frac{1}{8}$ vertex $(3/4, \frac{1}{8})$

y-int
Let $x = 0$
$y = -1$





x-intercepts

$$-2x^2+3x-1=0$$

$$2x^2-3x+1=0$$

$$(2x-1)(x-1)=0$$

$$2x-1=0$$

$$x=1/2, \quad x=1$$

$$x=\frac{-3\pm\sqrt{(3)^2-4(-2)(-1)}}{2(-2)}$$

$$=\frac{-3\pm\sqrt{9-8}}{-4}=\frac{-3\pm\sqrt{1}}{-4}=\frac{-3\pm1}{-4}$$

$$x=\frac{-3+1}{-4}=\frac{-2}{-4}=1/2, \quad x=\frac{-3-1}{-4}=\frac{-4}{-4}=1$$