Test Review

§1.3 Write an equation of the line in $y = mx + b$ form that

① passes through $(2, 5)$ and $(4, -9)$

SOLUTION $m = \frac{-9-5}{4-2} = \frac{-14}{2} = -7$

$x_1 = 2,\ y_1 = 5$

$y - y_1 = m(x - x_1)$

$y - 5 = -7(x-2)$

$y - 5 = -7x + 14$

$y = -7x + 19$

② Passes through $(6, 3)$ and is perpendicular to the line $y = \frac{2}{3}x + 1$

SOLUTION $x_1 = 6,\ y_1 = 3,\ m_\perp = \frac{-3}{2}$ ← negative reciprocal of $m = \frac{2}{3}$

$y - y_1 = m(x - x_1)$

$y - 3 = \frac{-3}{2}(x-6)$

$y - 3 = \frac{-3}{2}x + 9$

$y = \frac{-3}{2}x + 12$

§1.7 Solve the inequality. Write your answer in interval notation.

(3) $2|x-5|<14$

SOLUTION

$|x-5|<7$

$-7 < x-5 < 7$

$-2 < x < 12$

$(-2, 12)$

(4) $3|x+1|>12$

SOLUTION $|x+1|>4$

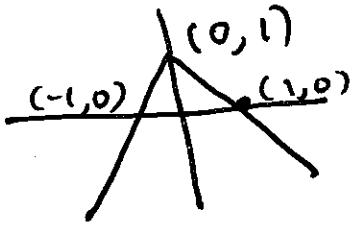
$x+1<-4$ or $4<x+1$

$x<-5$ or $3<x$

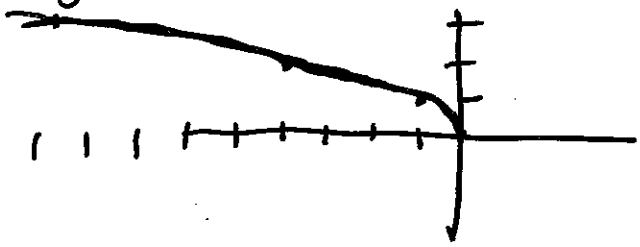
$(-\infty, -5) \cup (3, \infty)$

§ 2.2, 2.3 Sketch the graph.

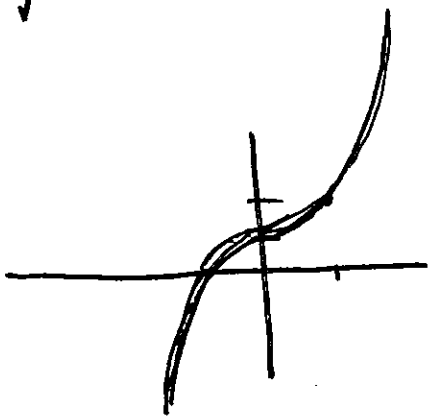
⑤ $y = -|x| + 1$



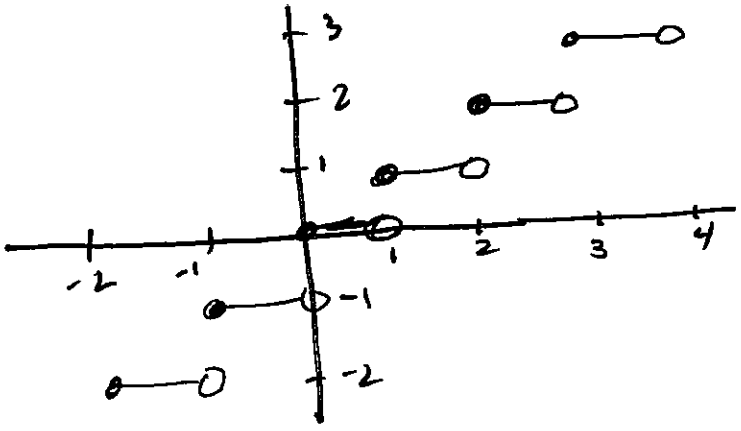
⑥ $y = \sqrt{-x}$



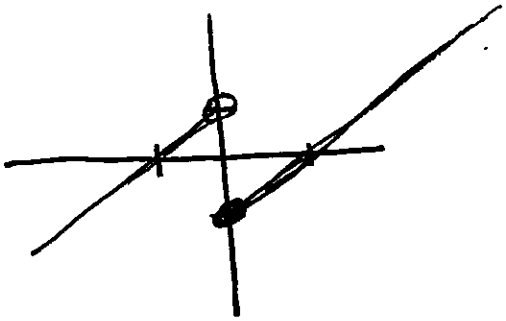
⑦ $y = x^3 + 1$



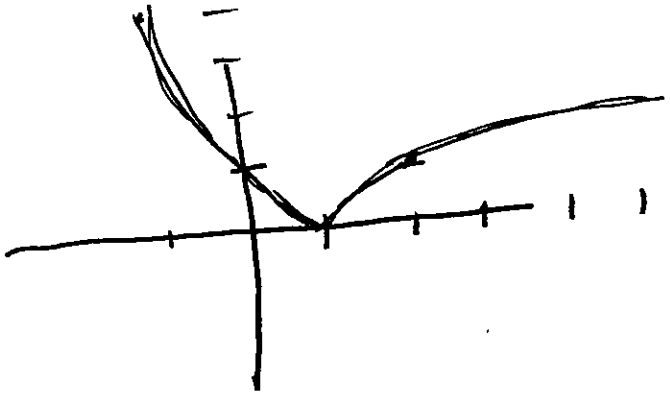
⑧ $y = [[x]]$



(9) $y = \begin{cases} x+1 & \text{if } x < 0 \\ x-1 & \text{if } x \geq 0 \end{cases}$

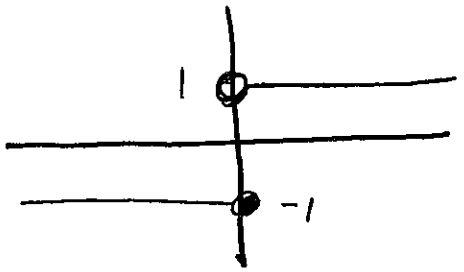


(10) $y = \begin{cases} (x-1)^2 & \text{if } x < 1 \\ \sqrt{x-1} & \text{if } x \geq 1 \end{cases}$



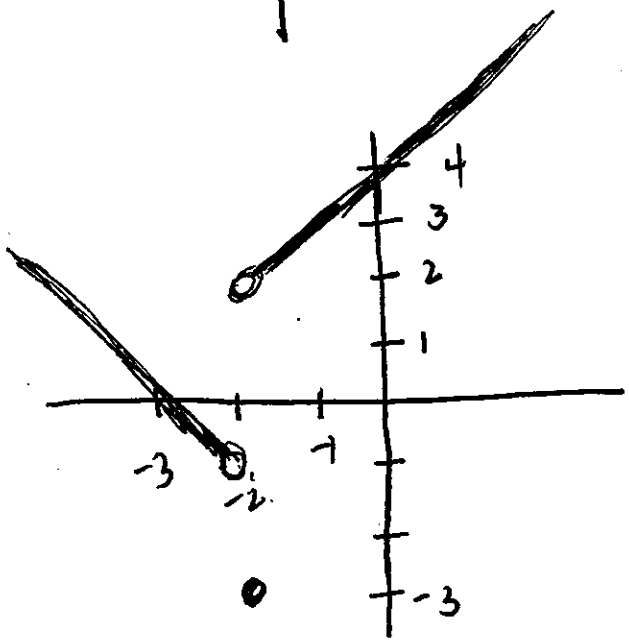
Write a piece-wise function for the graph.

(11)



$$y = \begin{cases} -1 & \text{if } x \leq 0 \\ 1 & \text{if } x > 0 \end{cases}$$

(12)



$$y = \begin{cases} x+4 & \text{if } x > -2 \\ -x-3 & \text{if } x < -2 \\ -3 & \text{if } x = -2 \end{cases}$$

§2.3 Determine whether the function is odd, even, or neither.

(13) $f(x) = x^4 - x^2$

SOLUTION $f(-x) = (-x)^4 - (-x)^2$

$= x^4 - x^2$

$= f(x)$ even

(14) $f(x) = x^3 + 7x$

SOL $f(-x) = (-x)^3 + 7(-x)$

$= -x^3 - 7x$

$= -(x^3 + 7x)$

$= -f(x)$, $f(-x) = -f(x)$ odd

(15) $f(x) = (x-1)^2$

SOL $f(-x) = ((-x)-1)^2$

$= (-x-1)^2 = (-x)^2 + 2(-x)(-1) + (-1)^2$

$= x^2 + 2x + 1$

$= (x+1)^2 \neq f(x)$

$\neq -f(x)$

neither

§2.4 Let $f(x) = |x|$, $g(x) = x-7$, $h(x) = x^2$.

Write each as a composition of functions chosen from f, g, and h.

(16) $F(x) = x^2 - 7$

SOLUTION $F(x) = h(x) - 7$

$= g(h(x))$

$F = g \circ h$

(17) $G(x) = |x| - 7$

SOL $G(x) = f(x) - 7$

$= g(f(x))$

$G = g \circ f$

(18) $Q(x) = (x^2 - 7)^2$

SOL $Q(x) = (h(x) - 7)^2$

$= (g(h(x)))^2$

$= h(g(h(x)))$

$Q = h \circ g \circ h$

(19) $S(x) = x - 14$

SOL $S(x) = (x-7) - 7$

$= g(x) - 7$

$= g(g(x))$

$S = g \circ g$

(20) $T(x) = x^4$

$= (x^2)^2$

$= h(h(x))$

$T = h \circ h$

§2.5 Find the inverse using the switch and solve method.

(21) $f(x) = 5x+2$

Sol $y = 5x+2$

$x = 5y+2$

$x-2 = 5y$

$\frac{x-2}{5} = y$ $\quad f^{-1}(x) = \frac{x}{5} - \frac{2}{5}$

(22) $g(x) = \frac{x+2}{x-3}$

Sol $y = \frac{x+2}{x-3}$

$x = \frac{y+2}{y-3}$

$x(y-3) = y+2$

$xy - 3x = y+2$

$xy - y = +3x+2$

$y(x-1) = +3x+2$

$y = \frac{+3x+2}{x-1}$ $\quad f^{-1}(x) = \frac{+3x+2}{x-1}$

(23) Find $f(g(x))$ and $g(f(x))$. Determine whether f & g are inverse functions.

$$f(x) = \sqrt[3]{\frac{x-2}{5}}, \quad g(x) = 5x^3 + 2$$

Sol

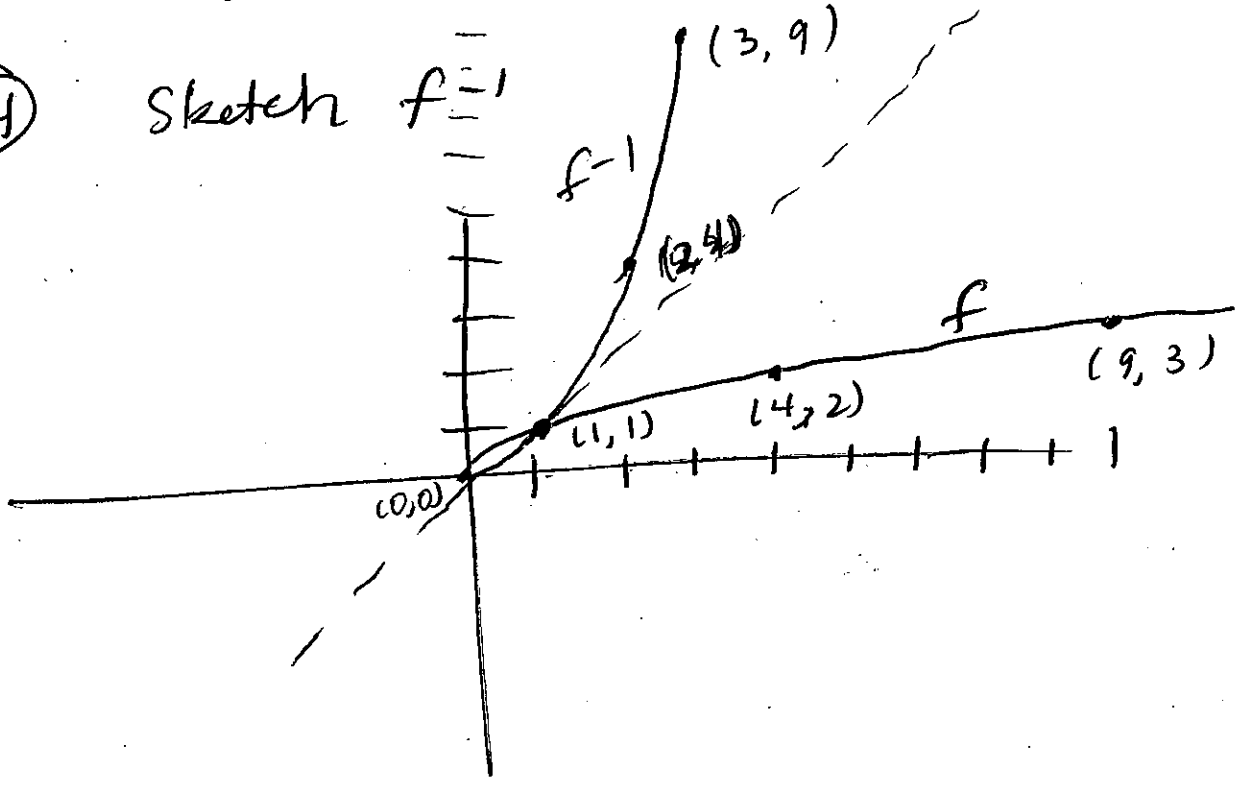
$$f(g(x)) = \sqrt[3]{\frac{(5x^3+2)-2}{5}} = \sqrt[3]{x^3} = x$$

$$g(f(x)) = 5\left(\sqrt[3]{\frac{x-2}{5}}\right)^3 + 2 = 5\left(\frac{x-2}{5}\right) + 2$$

$$= x - 2 + 2 = x.$$

Yes, they are inverses.

(24) Sketch f^{-1}



§3.1 #25 Let $f(x) = 2x^2 + 6x + 4$

a) Write $f(x)$ is the form $y = a(x-h)^2 + k$

$y = 2x^2 + 6x + 4$

$y = 2\left(x^2 + 3x + \frac{9}{4}\right) + 4 - 2\left(\frac{9}{4}\right)$

$y = 2(x + 3/2)^2 - \frac{1}{2}$

b) State the vertex.

$(-3/2, -1/2)$

c) Find the x-intercepts.

$2x^2 + 6x + 4 = 0$

$2(x^2 + 3x + 2) = 0$

$2(x+1)(x+2) = 0$

$x = -1, \; x = -2$

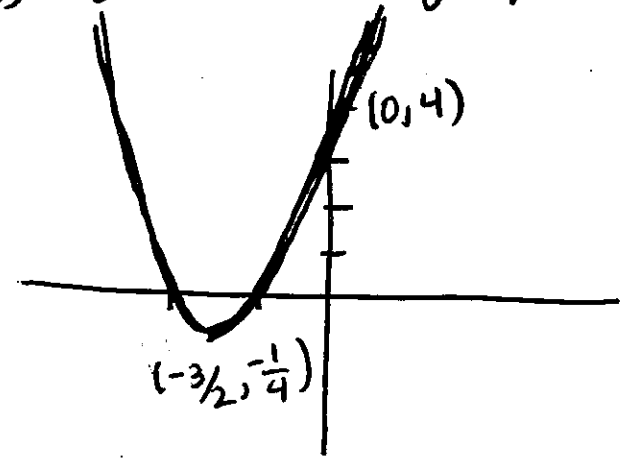
$(-1, 0), (-2, 0)$

d) Find the y-intercept.

$x = 0$

$y = 2(0)^2 + 6(0) + 4$ $\quad (0, 4)$

e) sketch the graph



§3.1 Solve the inequality.

(26) $x^2 + 5x + 6 > 0$

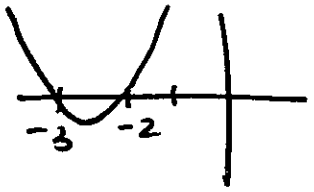
Sol

zeros $x^2 + 5x + 6 = 0$

$(x+2)(x+3) = 0$

$x = -2, \quad x = -3$

Graph



Sol $(-\infty, -3) \cup (-2, \infty)$

(27) $x^2 - x - 6 \leq 0$

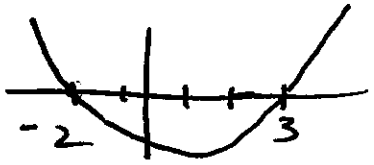
Sol

zeros $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

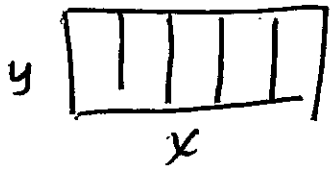
$x = 3, \quad x = -2$

Graph



Sol $[-2, 3]$

(28) Billy Bob has 3200 ft of fencing to construct a rectangular pen with four interior walls parallel to one side. What overall dimensions will maximize the area?



SOLUTION FORMULAS

$$2x + 6y = 3200$$

$$A = xy$$

Eliminate Variable

$$2x = -6y + 3200$$

$$x = -3y + 1600$$

$$A = xy = (-3y + 1600)y$$

$$A = -3y^2 + 1600y$$

Find the vertex.

$$h = \frac{-b}{2a} = \frac{-1600}{2(-3)} = \frac{800}{3} = 266\frac{2}{3}$$

$$y = 266\frac{2}{3}\text{ ft}$$

Find x

$$x = -3y + 1600$$

$$x = -3\left(\frac{800}{3}\right) + 1600$$

$$x = 800\text{ ft}$$

800ft by $266\frac{2}{3}$ ft

(29)

§3.2 Find all real and imaginary zeros of the polynomial

$$y = x^3 - 2x^2 + x - 2$$

SOL • Poss Rat Roots = $\frac{\text{factors of } 2}{\text{factors of } 1}$

$$= \pm 1, \pm 2$$

• Check

$f(1) = 1^3 - 2(1)^2 + 1 - 2$

$= 1 - 2 + 1 - 2 = -2$ no

$f(-1) = (-1)^3 - 2(-1)^2 + (-1) - 2$

$= -1 - 2 - 1 - 2 = -6$ no

$f(2) = 2^3 - 2(2)^2 + 2 - 2 = 8 - 8 + 2 - 2 = 0$ yes

$$\begin{array}{c|cccc} 2 & 1 & -2 & 1 & -2 \\ & & 2 & 0 & 2 \\ \hline & 1 & 0 & 1 & 0 \end{array}$$

$$f(x) = (x-2)(x^2+1)$$

$x^2 + 1 = 0$

$x^2 = -1$

$x = \pm\sqrt{-1}$

$x = \pm i$

Answer: $\{2, \pm i\}$

§ 3.4 Find all real solutions to the equation.

(30) $\sqrt{x-1} = x - 13$

Sol $(\sqrt{x-1})^2 = (x-13)^2$

$x - 1 = x^2 - 26x + 169$

$x^2 - 27x + 170 = 0$

$(x-17)(x-10) = 0$

$x = 10, \; x = 17$

Check $x = 10$

$\sqrt{10-1} \stackrel{?}{=} 10 - 13$

$\sqrt{9} \stackrel{?}{=} -3$

$3 \neq -3$ no

$x = 17$

$\sqrt{17-1} \stackrel{?}{=} 17 - 13$

$\sqrt{16} \stackrel{?}{=} 4$

$4 = 4$ yes

$\boxed{x = 17}$

(31) $x^{2/3} = \frac{1}{4}$

Sol $\left(x^{2/3}\right)^{3/2} = \pm \left(\frac{1}{4}\right)^{3/2}$

$$x = \pm \frac{1}{4^{3/2}}$$

$$x = \pm \frac{1}{(\sqrt{4})^3}$$

$$x = \pm \frac{1}{2^3}$$

$$x = \pm \frac{1}{8}$$

(32) $x^4 - 14x^2 = -45$

Sol $x^4 - 14x^2 + 45 = 0$

$(x^2)^2 - 14(x^2) + 45 = 0$

$u = x^2$ $\quad u^2 - 14u + 45 = 0$

$(u-5)(u-9) = 0$

$u = 5, \; u = 9$

$x^2 = 5, \; x^2 = 9$

$x = \pm\sqrt{5}, \; x = \pm 3$

$\{\pm\sqrt{5}, \pm 3\}$