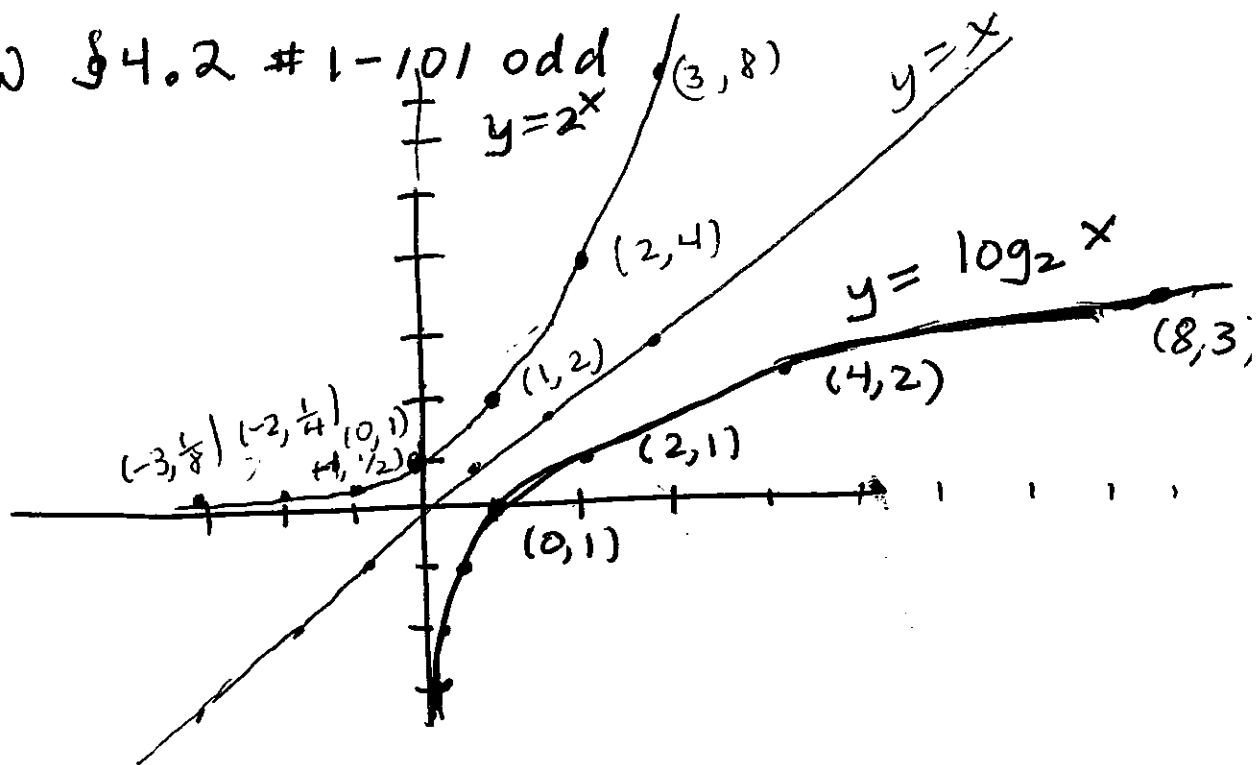


## §4.2 Logarithmic Functions and Their Applications

• HW §4.2 #1-101 odd



The inverse of  $y = 2^x$   
is  $y = \log_2 x$ .

$y = \log_2 x$  is equivalent to  $2^y = x$

$$y = \log_2 x$$

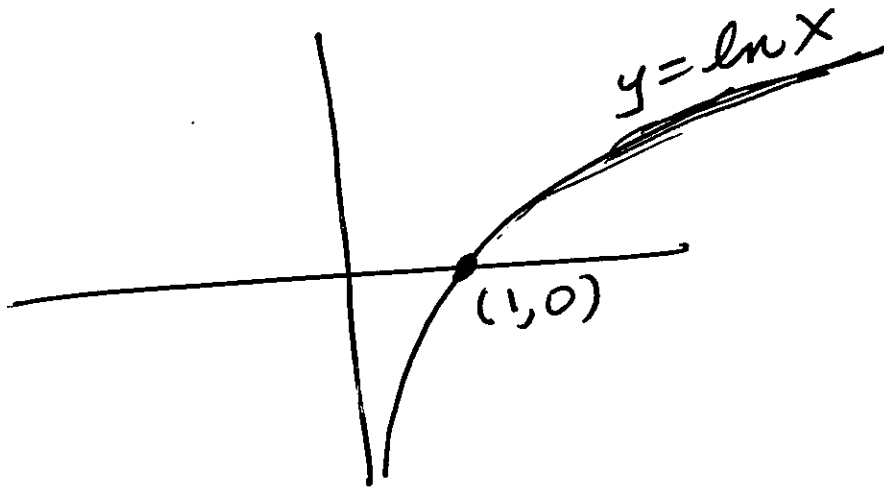
- increasing
- one-to-one
- invertible
- Domain:  $x > 0$
- Range:  $\mathbb{R}$
- Vertical Asymptote:  
the  $y$ -axis

The natural log is written  
 $y = \ln x$ .

~~It~~ •  $y = \ln x$  is the inverse  
of  $y = e^x$

•  $y = \ln x$  means  
 $y = \log_e x$

•  $y = \ln x$  means  $e^y = x$



EXAMPLE Write the logarithm  
as an exponent.

① ~~3~~  $3 = \log_2 8$   
          ↑          ↑  
          exponent base

SOLUTION  
 $2^3 = 8$

②  $-2 = \log_5 \frac{1}{25}$

$5^{-2} = \frac{1}{25}$

③  $4 = \log_3 81$

$3^4 = 81$

EXAMPLE Write the exponent as  
a ~~so~~ logarithm.

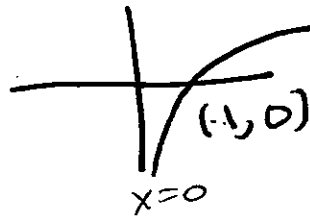
①  $7 = 2^x$   $x = \log_2 7$   
          ↑          ← exponent  
          base

②  $a = 3^x$   $x = \log_3 a$   
          ↑          ← exponent  
          base

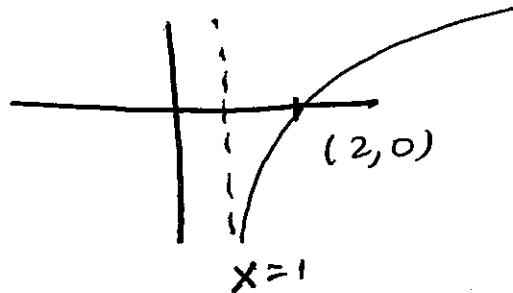
- The Common Log. is  $y = \log x$   
which means  $y = \log_{10} x$ .

EXAMPLE Sketch the graph.

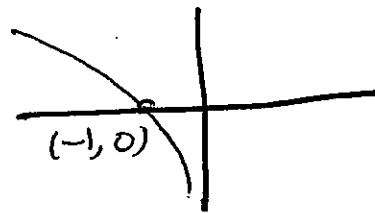
①  $y = \ln x$



②  $y = \ln(x-1)$   
          ↑  
          moves  
          right by 1



③  $y = \ln(-x)$   
          ↑  
          reflect  
          about y-axis



④ Bonus:  $y = \ln|x|$

$$= \begin{cases} \ln x & \text{if } x > 0 \\ \ln(-x) & \text{if } x < 0 \\ \text{undef} & \text{if } x = 0 \end{cases}$$

